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ECONONICAL ORDERING STRATEGIES FOR PERISHABLE ITEMS UNDER CONDITIONS OF DEMAND DEPENDENT FINITE PRODUCTION RATE AND PARTIAL BACKORDERING

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ABSTRACT

The present paper derives an optimal policy for perishable product under conditions of price dependent demand, demand dependent production rate and partial backordering which depends on the length of waiting time and a lot sale. In a competitive market shortage acts as barrage for costumers' inflow. So in this article to avoid their dissatisfaction of waiting for long duration, a portion of the total shortage has been recovered in each cycle through backlogging process. In the inventory literature generally shortage occurs after accumulation of inventory at early stage. But in the present model, a new approach shortage followed by inventory has been deliberated than the inventory followed by shortage. In addition to the new shortage approach a demand dependent production rate has also been incorporated rather than a constant production rate taken in various models previously. Therefore the article was found to be more profitable than previous models which have been reflected through tabulated results. Hence to make a company all time profitable, the production rate of the finished goods should be under the control of the management depending on the costumers' inflow. An algorithm has been used to obtain the optimum results. The model is studied extensively numerically taking different values of varies parameters involved in this model vide sensitivity analysis.

Keywords: Price-dependent, demand, Demand dependent Production, Deterioration, Inventory, Partial back-ordering, Lot sale.

INTRODUCTION

It is a natural phenomenon that each item in any inventory has been deteriorated spontaneously in course of time. The products like food stuffs, blood bank, medicines, chemicals, radioactive substances etc. deteriorate sufficiently during their normal storage period. So while developing an optimal inventory policy for such perishable items, the loss of inventory due to deterioration may not be ignored. Hence we have considered a deterioration factor in your model.

In supermarket, it is being observed that the demand of items go up and down according to the falling and rising of prices respectively. So demand is a price-dependent variable. As the demand for the product is a price instigating, hence pricing and lot sizing decisions are very important for perishable items. Shortage is also an unavoidable situation happening in a market from time to time. It is a barrage for customers' in flow. Therefore the backlogging is a needful protection to avoid the loss of cost towards deterioration of unpreserved items. Hence perishable and backlogging are complementary conditions. Researchers have developed their inventory models taking backlogging rate to be linearly dependent on the total number of customers in the waiting line. Moreover backlogging rate depends on the waiting time spent for the arrival of the next lot. To get an up to date idea, we go through the articles of Montgomery et al [1], Park [2], Mak [3], Chang and Dye [4]. This article assumes that the partial ordering depends on the waiting time of the backlogging. It is a better alternate of the customers' unwillingness to wait for backlogging. In this regard, we referred the inventory models dealing with perishable product under conditions of finite production, fixed demand and backlogging are Raafat [6], Kim and Hwang [7], Hwang et al [8], Rubin et al [9], Burwell [10]. The optimal policy for a perishable product with shortages was considered by Giri et al [11], Jalan and Choudhury [12]. Optimal selling price and lot size with time varying deterioration with partial backlogging was discussed by Sena [13].

Most of the models discussed above have been formulated as the cost minimization problem incorporating backorders and lot sale cost. Abad [5] has proved the existence of global cost minimization problem. Lue [14] has developed a general model for finding of optimal lot size and maximum back order level. In the above models dealt with the techniques that inventory followed by shortages but the present model deals with shortages followed by inventory which leads a comparatively better result. In addition to we have also incorporated a demand dependent production rate rather than a fixed production rate reflected in varies previous models. The model is well discussed and solved optimally using Mathematica and studied through sensitivity analysis using numerical examples.

ASSUMPTIONS AND NOTATIONS

For the simplicity of the model, we have used the following notations and assumptions.

p = Selling price per unit.

$D(p)$ = Demand rate (units / period).

λD = Production rate (units/period),

Where $\lambda>1$,

θ = Constant deterioration rate, $0 < \theta < 1$. η = waiting time of the customer.

$B(\eta) = \frac{K_0}{1+k_1\eta} < 1 \forall \eta$ is the fraction of the shortage backlogged,

$0 < K_0 < 1$, $0 < K_1 < 1$.

$I(t)$ = Instantaneous inventory level.

T_0 = Stock out time span.

T_1 = Shortage recovering time span.

T_2 = Time at which the inventory level raises to peak level.

T = Cycle length.

h = Inventory carrying cost per unit period.

v = Unit cost.

Q = Lot-size ordering quantity.

$D' = \frac{dD(p)}{dp} < 0, \forall\, p \in (0, \infty)$.

Marginal revenue = $\frac{d[pD(p)]}{dp} = p + D' > 0$, strictly increasing function of p, hence $\frac{1}{D(p)}$ is a convex function of p.

A single inventory is deliberated over an infinite planning horizon.

Lead time is taken to be zero. Shortage is allowed.

MATHEMATICAL FORMULATION

The cycle starts with zero inventories and shortage is accumulated during the interval $[0, T_0]$. The production begins at $t = T_0$ to meet the current demand along with a portion of the backlogged demand. At time T_1, the shortage has been adjusted and positive level of inventory starts to build up. The stopping of the production at time T_2 results the declining of the inventory level. Consequently inventory level ends with zero stock after the cycle T. The pictorial behavior of the inventory cycle is depicted in fig. 1.

To resolve the discussion mathematically the following differential equations are taken into the contemplation.

$$\frac{dI(t)}{dt} = -DB(T_0 - t)\,, \qquad 0 \le t \le T_0\,, \quad I(0) = 0\,, \tag{1}$$

$$\frac{dI(t)}{dt} = D(\lambda - 1)\,, \qquad T_0 \le t \le T_1\,, \quad I(T_1) = 0\,, \tag{2}$$

$$\frac{dI(t)}{dt} + \theta I(t) = D(\lambda - 1)\,, \qquad T_1 \le t \le T_2\,, \quad I(T_1) = 0\,, \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D\,, \qquad T_2 \le t \le T\,, \quad I(T) = 0\,. \tag{4}$$

Solving the above differential equations from (1) to (4) under the given boundary conditions, we have

$$dI(t) = -DB(T_0 - t)\,dt = -\frac{DK_0}{1 + K_1(T_0 - t)}dt$$

$$\therefore I(t) = \frac{DK_0}{K_1} ln\left(\frac{1 + K_1(T_0 - t)}{1 + K_1 T_0}\right), \quad 0 \le t \le T_0\,, \tag{5}$$

$$I(t) = D(K - 1)(t - T_1)\,, \quad T_0 \le t \le T_1\,, \tag{6}$$

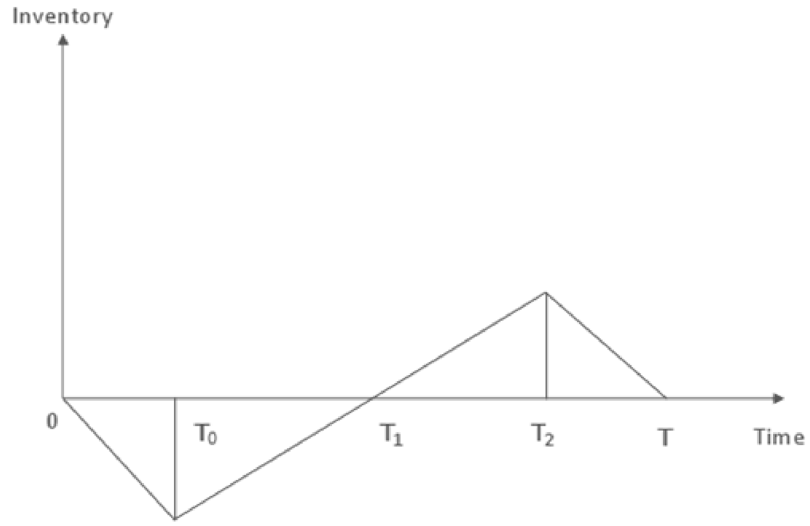
$$I(t) = \frac{D(\lambda - 1)}{\theta}\left[1 - e^{\theta(T_2 - t)}\right], \quad T_1 \le t \le T_2\,, \tag{7}$$

$$I(t) = \frac{D}{\theta}\left[e^{\theta(T - t)} - 1\right], \quad T_2 \le t \le T\,. \tag{8}$$

Since the point of intersections of the trajectories of equations (5) and (6) is at t = T_0 .

Thus, $-\frac{DK_0}{K_1} ln(1 + K_1 T_0) = D(\lambda - 1)(T_0 - T_1)$

$$\Rightarrow T_1 = T_0 + \frac{K_0 ln(1 + K_1 T_0)}{k_1(\lambda - 1)} \tag{9}$$



T_2

Figure 1 Graphical presentation of inventory system

Equating the common values of equations (7) and (8) at $t = T_2$, we have

$$\frac{D(\lambda-1)}{\theta}\left[1-e^{\theta(T_1-T_2)}\right] = \frac{D}{\theta}\left[e^{\theta(T-T_2)}-1\right] \qquad (10)$$

On simplification, we have

$$T_2 - T_1 = \frac{1}{\theta} ln\left[1+\frac{1}{\lambda}\left(e^{\theta(T-T_2)}-1\right)\right] \qquad (11)$$

Since the production of the unit is within the interval $[T_0, T_2]$, hence the lot-size

$Q = \lambda D(T_2 - T_0)$.

The revenue collected by selling the production in the interval $[T_0, T]$ is

$pD[\lambda(T_1 - T_0) + (T - T_1)]$.

As the production of the unit is within the time $[T_0, T_2]$, hence the manufacturing cost of the quantity of the material is

$$Qv = D\lambda v(T_2 - T_0) \qquad (12)$$

As $[T_2, T_1]$ is the production space where the positive inventory cycle is available and $[T_1, T]$ is demand time span when there is no shortage. Hence the number of deteriorated units throughout the positive inventory time span is the difference of the production volumes and demand quantities during this interval.

Hence the deterioration cost is

$$Nv = vD[\lambda(T_2 - T_1) - (T - T_1)], \qquad (13)$$

Where N is the number of deteriorated units.

Since the stock is available throughout the interval $[T_1, T]$, hence the holding cost in this gap is

$$H = h\int_{T_1}^{T} I(t)\,dt$$

$$= \frac{hD(\lambda-1)}{\theta}\int_{T_1}^{T_2}\left[1-e^{\theta(T_1-t)}\right]dt + \frac{hD}{\theta}\int_{T_2}^{T}\left[e^{\theta(T-t)}-1\right]dt$$

$$= \frac{hD(\lambda-1)}{\theta}\left[T_2 - T_1 + \frac{e^{\theta(T_2-T_1)}-1}{\theta}\right] - \frac{hD}{\theta}\left[T - T_2 + \frac{1-e^{\theta(T_2-T_1)}}{\theta}\right]$$

$= \frac{hD}{\theta}[(T - T_1)(T_2 - T_1) - T]$ by using equation (10) (14)

The profit of the entire cycle $[0, T]$ is $F(p, T_0, T)$ = Selling revenue- production cost- deterioration cost- holding cost- set up cost.

Thus, $F(p, T_0, T) = \frac{D\lambda K_0(p-v)ln(1+K_1T_0)}{K_1(\lambda-1)} + D(T - T_1)\left(p + v + \frac{h}{\theta}\right)$

$$-\lambda D(T_2 - T_1)\left(2v + \frac{h}{\theta}\right) - A \tag{15}$$

As $[T_2, T_1]$ is the production interval where the positive inventory cycle is available and $[T_1, T]$ is demand time span when there is no shortage.

Average profit per cycle is

$$\prod(p, T_0, T) = \frac{F(p, T_0, T)}{T}$$

$$= \Big[\frac{D\lambda K_0(p-v)\ln(1+K_1T_0)}{K_1(\lambda-1)} + D\left[T - T_0 - \frac{K_0\ln(1+K_1T_0)}{k_1(\lambda-1)}\right]\left(p + v + \frac{h}{\theta}\right)$$

$$-\lambda D\left[\frac{1}{\theta}\ln\left\{1 + \frac{1}{\lambda}\left(e^{\theta\left(T - T_0 - \frac{K_0\ln(1+K_1T_0)}{k_1(\lambda-1)}\right)} - 1\right)\right\}\right]\left(2v + \frac{h}{\theta}\right) - A\Big]/T \tag{16}$$

Here we have to maximize $\prod(p, T_0, T)$ under the conditions

$T_0 \geq 0$

$T \geq T_1$

$p \geq v$ such that p is a decision variable.

Algorithm for optimization

When the selling price p is a decision variable and $\prod(p, T_0, T)$ is not a pseudo-concave function then the profit function $\prod(p, T_0, T)$ may have multiple local maxima. Let $\prod(p/T_0, T)$ represents $\prod(p, T_0, T)$ when T and T_0 are fixed. In order to maximize the complicated unconstrained problem $\prod(p, T_0, T)$, we have used standard non-linear programming software Mathematica. The Algorithm for the solution of the problem is given below.

Step 1 Let $p = p_i$ where p_i represents some value of p.

Step 2 For this optimum$p$, let $T_0 = T_0^* \; and \; T = T^*$

Step 3 Let the optimum result $T_0 = T_0^* \; and \; T = T^*$ maximizes F$(p/T_0, T)$ locally. Let F$(p/T_0, T)$ be maximum for $p = p$*.

Step 4 Taking this $p = p$* repeated the steps 2 and 3 till $\prod(p, T_0, T)$ attains local maxima. This procedure has repeated several times taking various values of p till it attains global maxima.

Numerical example

Suppose $(p) = 1{,}600{,}000\, p^{-4}$, $v = \$6.3$ / unit, $A = \$1000$/production run, $h = \$1$/ unit/week, $\theta = 0.2$, $K_0 = 0.9$, $K_1 = 0.1$ and $\lambda = 2.4$

The optimum result was found to be $p = 8.49999,\ T_0 = 7.13,\ T = 11.002.$

With help of these solutions we also obtained

$T_1 = 10.59015,\ T_2 = 10.7659,\ \ Q = 2674.66,\ \prod(p, T_0,\ T) = 438.45882.$

A comparative study between fixed production rate and a demand dependent production rate have been summarized at the table-1 under identical numerical data.

Table 1 Optimum Value

Symbols	**Variable Production rate**	**Fixed prodution rate**	**Remarks(Present model)**
$\boldsymbol{p}$	8.49999	7.87487	Sells at higher price
$\boldsymbol{T_0}$	7.162	7.49945	Production starts earlier
$\boldsymbol{T_1}$	10.63415	11.0877	Backlog clears earlier
$\boldsymbol{T_2}$	10.79507	11.20231	Production stops earlier
T	11.011	11.35886	Cycles repeat quickly
Q	2672.58	3702.86	Orders less quantities
$\boldsymbol{N_d}$	2.56182	1.79165	Deteriorating quantity is more
$\boldsymbol{I_{max}}$	67.95496	66.64	Production stops at higher inventory
$\prod$	**438.514**	**422.325**	**About 4% more profit**
$\boldsymbol{T_2 - T_1}$	0.16092	0.27116	Inventory available for **lesser** time
$\boldsymbol{T_1 - T_0}$	3.63307	3.58825	Waiting time of the customer is less
D	306.51118	416.05085	Less demand as price is higher
R	735.62683	1000	Less production quantity makes more profit

In any business community a four percent more profit is not a small achievement. It is summarized that the present article is seemed to be a superior model in all respect due to induction of a demand dependent production rate rather than a fixed production rate taken in varies previous models.

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Table 2 Sensitivity analysis

	T_0 **Production starts**	T_1 **shortage ends**	T_2 **Production ends**	T **optimal cycle time**	p **optimal price**	Q **optimal order quantity**	D **demand**	**λD production**	$\prod(p,T_0,T)$ **profit function**
h									
0.5	7.1200	10.5764	10.76122	11.009	8.50010	2678.566	306.50974	735.63338	439.163
1.0	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.514
1.5	7.1999	10.68623	10.81937	11.000	8.43000	2752.111	316.81788	760.36291	438.310
A									
900	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
1000	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	429.3696
λ									
2.3	6.9930	10.66373	10.83268	11.060	8.50000	2706.868	306.50970	704.97230	444.1400
2.4	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
2.5	7.3410	10.64393	10.78937	10.490	8.47000	2680.033	310.81539	777.03848	433.5256
v									
5.8	7.1600	10.63140	10.78960	11.003	7.75200	3859.534	443.06240	1063.3498	587.8300
6.3	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
6.8	7.1670	10.64102	10.79374	11.000	9.09900	2031.763	233.42390	560.21736	330.3594
K_0									
0.9	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
0.8	7.3999	10.56492	10.79487	11.100	8.51000	2485.7012	305.07157	732.17177	395.6100
K_1									
0.1	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
0.11	7.3599	10.82607	10.90396	11.011	7.35990	2618.165	307.81134	738.74722	403.9495
θ									
0.2	7.1620	10.63415	10.79507	11.011	8.49999	2672.580	306.51118	735.62683	438.5140
0.1	6.8794	10.24481	10.72165	10.799	8.49000	2839.789	307.95639	739.09534	440.6471

The change in values of various parameters involved in any inventory system may occur due to uncertainties in decision making situations. Sensitivity analysis has a vital role to study the effect of optimality due to above changes. Depending upon the various results of our model reflected in the sensitivity analysis table, the following references have been drawn from.

- When the deterioration parameter θ is increasing, the optimal profit and the optimal ordering quantity are also increasing but the optimal price has decreased.
- When the amount of backlogged is reduced during the stock- out interval, both the optimal profit and optimal ordering quantity are also reduced but optimal selling price is enhanced.
- When the unit price is increased both the ordering quantity and optimal profit are decreased but the selling price is decreased.
- When the production is increased the selling price, optimal profit and ordering quantity are decreased.

CONCLUSION

The present paper derives an optimal policy for perishable product under conditions of price dependent demand, demand dependent production and partial backordering which depends on the dimension of waiting time and lot sale. As the demand depends on selling pricing, production depends on demand and the production items are perishable hence the backlogging of demand is essential to avoid the useless cost towards deterioration. In a competitive market shortage acts as barrage for costumers' inflow. In this article to avoid their dissatisfaction of waiting for long duration, a partial backlogged of total shortage has been recovered in each cycle This is given by, $B(\eta) = \frac{k_0}{1 + k_1\eta}, 0 < k_0 < 1,\ 0 < k_1 < 1,$ where η is the waiting time of the customers receiving their goods. In the inventory literature generally shortage occurs after accumulation of inventory at early stage. The consideration of a new approach, shortage followed by inventory is more profitable than inventory followed by shortage. The present model has reported a significant a more profit. It is because of the introduction of a variable demand dependent production policy rather than a constant production policy. We hope the present article definitely shall suit the world business community to control the production depending upon the customers' attitude and capability to buy in an uncertainty business world.

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