

# Theoretical Modeling of Electrical Resistivity of Metal Matrix Composite

Avik Kumar Mahata

## Abstract

All engineering materials are made up of more than one phases, with different mechanical properties, such that on a micro scale they are heterogeneous. Further, even a single phase material will show chemical segregation, and therefore properties will not be identical from point to point and it's true for all Metal Matrix Composites. Due to presence of different phases the MMCs are anisotropic. For this the computation of resistivity of MMC becomes a complex matter. The main focus of this paper is the size effect in electronic conduction and also prediction of resistivity that takes into account the effect of residual stresses and dislocations. The method described in this article accounts for these circumstances by incorporation of appropriate phenomenological models. It provides a means to analyze and estimate the resistive behavior of two phase or multi-phase micro-composites and nano-composites.

*Keywords:* Matrix-Metal Composite; Electrical Resistivity, Dislocation, Residue Stresses.

*Index Terms*—Metal Matrix Composite, Electrical Resistivity, Dislocation, Residue Stresses.

## I. INTRODUCTION

Metal Matrix Composites (MMC) are used for higher service temperature than their base metal counterparts; furthermore the reinforcement may improve specific stiffness, specific strength, abrasion resistance, creep resistance, thermal and electrical conductivity and dimensional stability. Some of the advantages of these materials over Polymer-matrix composites (PMC) include higher operating temperature, non flammability, and greater resistance to degradation by organic fluids. The aerospace industry has wide range of application of MMCs. Boron fibers are used as the reinforcement for space shuttle orbiter, and continuous graphite fibers for Hubble Telescope [1]. Copper based MMC has multiple applications in electrical contacts. One of the most important properties of metal matrix composites for electrical and electronic applications is electrical resistivity, which has been previously modeled [2–6]. However, the theoretical electrical resistivity predicted

by these models does not closely match experimental results. Also, during fabrication of composite materials, large amounts of residual stresses and dislocations form and affect their electrical resistivity. Some models have been developed to calculate the dislocation density and distribution of residual stresses [7-12]. Investigations on the effect of the microstructure development of such sub-micrometer and nanometer phase materials on their strengthening behavior have been subject to many studies [13-22]; however, the corresponding electrical conductivity, respectively, the ratio of strength and resistivity, is often taken as a somewhat unpredictable and, moreover, invariable fact. Only few publications aim to explain and model the electrical conductivity of two-phase or multi-phase micro-composites and nano-composites, e.g., [23-26].

Most of these models have in common that their check against experimental data is often unsatisfactory owing to insufficient quantitative information on the micro structural state of the specimens. In this publication, a model for the electrical conductivity is described that includes the established bulk scattering mechanisms in metals and introduces features to account for size effect and anisotropy, i.e., the

arrangement of the phases. It can be applied to two phase or multi-phase micro-composites and nano-composites and enables a detailed analysis of the electrical conductivity of each individual composite phase, as well as a prediction of the composite's overall electrical conductivity. From this information, the micro structural state that provides an optimal combination of strength and resistivity can be determined, [27], without the need for extensive experimental work.

## II. DESCRIPTION OF THE MODEL

### A. Basic Concept

The model combines three aspects: the well-known scattering mechanisms affecting electronic conduction in metals, the basic concept describing the size effect in electronic conduction, as was established for fine wires and thin layers, and a geometrical model accounting for the alignment and arrangement of all phases and subphases in the composite system. A combination of the three aspects leads to one self-consistent expression that incorporates bulk properties, interface properties, and spacial parameters. Since ample information can be found in standard literature on the scattering mechanisms in bulk metals, this aspect of the model is not treated herein in further detail. To determine the overall electrical conductivity of the composite, respectively, its inverse, the electrical resistivity, the following steps are taken:

each phase is treated as an individual resistor for which the resistive mechanisms in bulk phases apply. The bulk electrical resistivity of the phase is then corrected according to a size effect model. Finally, the electrical resistivity of the individual phases, under consideration of their order within the composite, will be summed up accounting for their volume fraction and their geometrical arrangement.

## 2.2. Modeling of the size effect in electrical resistivity

### A. Basic formulations

In composites, additional scattering of conduction electrons occurs at internal interfaces. In analogy to scattering at the surface in very fine wires, the size effect can be modeled following the approaches presented in [28-30]. The application of an internal size effect owing to internal boundaries was first suggested in [31]. Although the size effect was discussed in respect to many in situ composites, including Ag\_Cu, Cu\_Nb, Cu\_Fe, no consequential simulations are published that reveal information on interfacial scattering as a function of microstructural arrangement and temperature. The size effect was first commented on in [32], following experiments on thin, salt-deposited silver films [33]. It was suggested that the limitation of the mean free path of the conduction electrons,  $mfpe$ , by collision with the specimen surface is responsible for the observed dependence of the electrical resistivity on the wire diameter or film thickness. In [30], a comprehensive review and comparison of various previous analyses of the size effect is presented. Comparison of the solutions for various geometries yields an expression for a specimen of arbitrary cross-section as in Eq. (1). The constant,  $C$  (Eq. (2)), is a dimensionless constant that is independent of the shape of the specimen. For application of this notion to the size effect in situ composites, the phase dimensions can be incorporated according to Eq. (3) for round filaments or Eq. (4) for plate-like lamellae.

$$r = r_0 \left[ 1 + C(mfpe) \frac{P}{S} \right] \quad (1)$$

$$C = \frac{3}{16} (1 - P) \quad (2)$$

$$\frac{P}{S} = \frac{4}{D} \quad (3)$$

$$\frac{P}{S} = \frac{2}{LT} + \frac{2}{b} \quad (4)$$

where  $r$  is electrical resistivity,  $r_0$  the bulk electrical resistivity,  $p$  the interface scattering or specularly factor,  $mfpe$  the mean free path of conduction electrons in bulk phase,  $P$  the perimeter,  $S$  the cross-section,  $D$  the average diameter of round filament,  $LT$  the average lamella thickness and  $b$  is average lamella width

### B. The electron mean free path

The mean free path of the conduction electrons,  $mfpe$ , is defined as given in Eq. (5). According to the Drude model of electron theory [34], the relationship between electrical

conductivity and electron mean free path is given as in Eq. (6). Their ratio is a constant that solely depends on the electron density. When you submit your final version, after your paper has been accepted, prepare it in two-column format, including figures and tables.

$$mfpe = V_F t \quad (5)$$

$$\frac{\sigma}{mfpe} = \frac{1}{r(mfpe)} \quad (6)$$

where  $mfpe$  is mean free path of conduction electrons,  $V_F$  the Fermi velocity,  $t$  the time of relaxation,  $s$  the electrical conductivity and  $r$  is electrical resistivity.

### C. The specularly factor

A theoretical analysis of the influence of the interface scattering factor or specularly factor,  $p$ , on the size effect, based on Eq. (1). Partial specular reflection at the surface or passage through the interface ( $p/0$ ) leads to a decrease of the size effect. Brief analyses of the parameters that determine the specularly of interfaces for scattering of electrons are given in [35], however, theoretically sound explanations are not available. In [36], data on the size effect is summarized, and specularly factors on the order of 0.5 were found for a number of metals. Indication for a high specularly factor at internal interfaces can be derived from [37], where it is stated that experiments have shown the electrical resistance of two compressed thin layers of Ag to be almost half the value of the resistance of the two single layers measured in parallel. In [38], interface scattering factors of 0.74 for Ag/Ag interfaces and 0.66 for Cu/Cu interfaces were found experimentally on vapor-deposited double-layers.

### D. Determination of the mean free path in a matrix phase

Frequently, the microstructure is quantitatively described by the dimensions of the second phase only. With respect to the electrical resistivity of composite materials, this can be insufficient due to the fineness of both the second phases and the matrix and the resulting size effects. The smallest spacing or distance between the second phase particles in a heterogeneous mixture, however, does not coincide with the average distance that a conduction electron can travel in the matrix between two scattering events. It follows from simple geometric consideration that, for example, 19.6 vol.% of round second phase particles lead to  $D_{matrix} = D_{2nd Phase}$ , 78.5% lead to  $D_{matrix} = 0$  where  $D_{2nd Phase}$  is the diameter of the second phase particles and  $D_{matrix}$  is the free path in the matrix between those second phase particles. This is obviously not a valid approach. It is analytically described as in Eq. (7) for  $0.03 < v < 0.45$  and can be used to evaluate and incorporate possible size effects of the matrix phases [27].

$$D_{matrix} = 0.678 D_{2nd Phase} (v_{2nd Phase})^{-1.1861} \quad (7)$$

where  $D_{matrix}$  is free path in the matrix between second phase particles,  $D_{2nd}$  Phase the diameter of the second phase particles and  $v$  is volume fraction of the second phase.

### III. ALIGNMENT OF THE PHASES WITHIN THE COMPOSITE

The electrical resistivity of the composite is determined by combining the electrical resistivities of its phases, accounting for their volume fractions and their geometric arrangement. The phases of the composite are seen as resistors that can be treated as either serial (Eq.(8)) or parallel (Eq. (9)).

$$r_{s,composite} = \sum_i r_i v_i \quad (8)$$

$$r_{p,composite} = 1 / \sum_i r_i v_i \quad (9)$$

Where  $r$  is electrical resistivity and  $v$  is volume fraction. If the micro structural arrangement of the phases is neither purely serial nor purely parallel, this is accounted for by a factor, the anti-parallelism, AP, in Eq. (10). All phases,  $n$ , have the same AP with respect to the specimen axis or the electrical field,  $E$  In isotropic microstructures, the anti-parallelism is around  $AP = 0.5$ . From Eqs. (8)/(10) follows Eq. (11) that can be used to determine the overall electrical resistivity of the composite.

$$r = r_p + AP(r_s - r_p) \quad (10)$$

$$r = 1 / \sum_i r_i v_i + AP[ \sum_i r_i v_i - 1 / \sum_i (v_i / r_i) ] \quad (11)$$

where  $r$  is electrical resistivity,  $v$  is volume fraction and AP is anti-parallelism. This model can not only be applied to multiple phases,  $n$ , but also to multiple orders of phases, e.g., primary phase A consisting of two secondary phases A1 and A2. Here, for each set of sub-phases,  $nn$ , within a parent phase,  $n$ , Eq. (11) must be applied, before the electrical resistivity of the parent phases,  $n$ , can be combined to the overall electrical resistivity of the composite. All sub-phases,  $nn$ , within the same parent phase,  $n$ , again have the same AP, which, however, can be different from the AP of the parent phase, respectively, the composite, or the AP of same-order sub phases in a different parent phase.

### IV. EFFECT OF RESIDUAL STRESSES AND DISLOCATIONS ON ELECTRICAL RESISTIVITY

During cooling, following processing or heat treatment of composite specimens thermal stresses are generated in the matrix, in the vicinity of the reinforcement/matrix interface, due to the difference in coefficient of thermal expansion (CTE) of these two phases. Part of these stresses may be relieved through yielding of the matrix which is manifested by plastic strain and formation of dislocations. The other part of these stresses will remain in the composite as residual thermal stresses. Even for furnace-cooled specimens both residual thermal stresses and dislocations will coexist. Strain fields in the matrix adjacent to the reinforcement form energy

barriers to the flow of electrons, raising the electrical resistivity of composites. Different matrices and reinforcements result in different residual stress distribution and dislocation densities, hence in different sizes of deformation regions, leading to different effects on electrical resistivity. Arsenault and Shi [39] calculated the dislocation density,  $R_{dis}$ , in a quenched composite specimen, using a model based on a prismatic punching method. They established that:

$$r_{dislocation} = BV_f \epsilon / b(1 - V_f)d \quad (12)$$

Where  $b$  is the Burgers vector of the matrix, and  $V_f$  and  $d$  are the volume fraction and the shortest dimension of the reinforcement, respectively. The strain misfit between reinforcement and matrix,  $\epsilon$ , can be obtained by  $\Delta CTE \times \Delta T$ , where  $\Delta CTE$  is the elementary difference in CTE between reinforcement and matrix, and  $\Delta T$  is the difference between heat treatment temperature and room temperature.

The geometric parameter of the reinforcement  $B$  is equal to 12 when the dimensions of the reinforcement in all directions are the same, as in equiaxed particulates, and equal to 4 if the dimension in one direction is much larger than in the other directions, as in whiskers and fibers [39]. From Eq. (24), it is clear that the dislocation density depends on the type, volume fraction and geometry of the reinforcement, as well as on the type of the matrix.

A highly strained deformation region containing residual stresses and dislocations affects the electrical resistivity of the composite. For a reinforcement of radius  $R$ , the radius of the deformation region around the reinforcement may be assumed to be  $\alpha R$  (including the radius of the reinforcement), where  $\alpha$  is an experimental constant.

A large  $\alpha$  value corresponds to a large deformation region. The  $\alpha$  value is determined by the dislocation density, and depends on the type of matrix and the type, size and shape of the reinforcement, but not its volume fraction. It can be expressed [39] by:

$$(\alpha - 1) = B\epsilon / bd \quad (13)$$

According to calculations by Flom and Arsenault [40],  $\alpha = 2.3$  for  $SiC_p/Al$  composite waterquenched from 470 °C. For a particulate reinforcement the volume fraction of the deformation region surrounding the reinforcement,  $V_{IIP}$ , may be expressed by:

$$V_{IIP} = (\alpha^3 - 1) * V_p \quad (14)$$

where  $V_p$  is the volume fraction of particulates. For a fibrous reinforcement with a large aspect ratio, the volume fraction of the deformation region,  $V_{IIf}$ , may be expressed by:

$$V_{IIf} = (\alpha^2 - 1) * V_f \quad (15)$$

The size of the deformation region affects the electrical resistivity of the composite. A modification term  $C(V_f, \alpha)$ , or  $C(V_p, \alpha)$  must be included in the calculated theoretical electrical resistivity of composites [5]. For a fiber-reinforced composite, this term is:

$$C(V_f, \alpha) = ((\alpha^3 - 1) / \alpha) V_f \quad (16)$$

A similar expression is valid for whisker-reinforced composites, in which  $V_f$  is replaced by  $V_w$ . For a particulate-reinforced composite, the modification term is:

$$C(V_p, \alpha) = ((\alpha^3 - 1)/\alpha^2)V_p \quad (17)$$

Thus, the modified theoretical electrical resistivity of a fiber-reinforced composite, taking into account the effect of residual stresses and dislocations, can be written as:

$$r_c(f)_1 = r_c(f)[1 + ((\alpha^2 - 1)/\alpha)V_f] \quad (18)$$

The theoretical electrical resistivity of the particulate-reinforced composite can be modified as:

$$r_c(p)_1 = r_c(p)[1 + ((\alpha^2 - 1)/\alpha)V_p] \quad (19)$$

## V. CONCLUSION

A model was described for the electrical resistivity of two-phase or multi-phase micro-composites and nano composites. It incorporates the established bulk scattering mechanisms in metals and introduces features to account for size effect and anisotropy, i.e., the arrangement of the phases. It permits an analysis of the influence of the electrical resistivity of the individual phases on the composite resistivity and the determination of the specularly factor of the interface.

The magnitude of residual stresses and the dislocation density were found to depend on the type, size and shape of reinforcement, as well as the type of matrix. The effective size of the deformation regions varied due to their overlapping and better fitted the calculated curves through empirical modification.

Theoretical prediction of resistivity that takes into account the effect of residual stresses and dislocations.

The model has shown to provide an insight into the resistive effects of selected micro structural features, which allows a further optimization of high-strength conductors. In regard to materials in which electronic conduction is influenced by other effects than those considered here, modifications and additions may have to be made to the model. Moreover, the phenomenological nature of the model does not permit the development of a more detailed understanding of the underlying physical effects that are responsible for the observed behavior.

## VI. ACKNOWLEDGMENT

The authors are pleased to acknowledge Department of Material Science and Engineering, NIT Trichy for helpful discussion for this work.

## VII. REFERENCES

- [1] Callister William D., Callister's Materials Science and Engineering, Wiley India (P) Ltd. Reprint 2012, 566.
- [2] Schoutens JE. Electricity conductivity in continuous-fiber composites. In: Everett RK, Arsenault RJ, editors. Metal matrix composites: Mechanisms and properties. Boston, MA: Academic Press Inc.; 1991.
- [3] Rayleigh. Phil Mag 1982;34:481.
- [4] Davis A. J Phys 1974;D7:120.
- [5] Schoutens JE, Roig FS. J Mater Sci 1987;22:181.
- [6] Maxwell JC. In: A treatise on electricity and magnetism. Vol. 1. NY: Dover Publications; 1954, p. 435–49.
- [7] Vogelsang M, Arsenault RJ, Fisher RM. Metall Trans A 1986;17A:379.
- [8] Arsenault RJ, Fisher RM. Scripta Metall 1983;17:67.
- [9] Arsenault RJ, Shi N. Mater Sci Eng 1986;81:175.
- [10] Kim CT, Lee JK, Plichta MR. Metall Trans A1990;21A:673.
- [11] Flom Y, Arsenault RJ. Mater Sci Eng 1985;75:151.
- [12] Zhao Z, Zhijian S, Yingkun X. Mater Sci Eng A 1991;A132:83.
- [13] W.A. Spitzig, A.R. Pelton, F.C. Laabs, Acta Metall. Mater. 35 (1987) 2427.
- [14] P.D. Funkenbusch, T.H. Courtney, Acta Metall. Mater. 33 (1985) 913.
- [15] P.D. Funkenbusch, T.H. Courtney, Scr. Metall. Mater. 23 (1989) 1719.
- [16] W.A. Spitzig, Acta Metall. Mater. 39 (1991) 1085.
- [17] J.D. Embury, Scr. Metall. Mater. 27 (1992) 981.
- [18] J.D. Embury, J.P. Hirth, Acta Metall. Mater. 42 (1994) 2051.
- [19] J.T. Wood, Dissertation, McMaster University of Hamilton, Canada, 1994.
- [20] J.T. Wood, J.D. Embury, Fatigue Fracture Eng. Mater. Struct. 18 (1995) 747.
- [21] J.T. Wood, A.J.J. Griffin, J.D. Embury, R. Zhou, M. Nastasi, M. Veron, J. Mech. Phys. Solids 44 (5) (1996) 737.
- [22] F. Heringhaus, Dissertation, RWTH Aachen, Germany, 1998.
- [23] F. Lux, J. Mater. Sci. 28 (1993) 285.
- [24] D. Raabe, Comp. Mater. Sci. 3 (1995) 402.
- [25] F. Zhongyun, Acta Metall. Mater. 43 (1995) 43.
- [26] D. Mattissen, D. Raabe, F. Heringhaus, Acta Mater. 47 (5) (1998) 1627.
- [27] F. Heringhaus, S.O. Prestemon, G. Gottstein, H.-J. Schneider-Muntau, Adv. Cryog. Eng. 44A (1998) 307.
- [28] K. Fuchs, H.H. Wills, Proc. Cambridge Philos. Soc. 34 (1938) 100.
- [29] R.B. Dingle, Proc. R. Soc. London 201 (1949) 545.
- [30] E.H. Sondheimer, Adv. Phys. 1 (1952) 1.
- [31] D.K.C. MacDonald, Philos. Mag. 42 (1951) 756.
- [32] J.J. Thomson, Proc. Cambridge Philos. Soc. 11 (1901) 120.
- [33] I. Stone, Phys. Rev. 6 (1898) 1.
- [34] P. Drude, Ann. Phys. 4 (1900) 566.
- [35] P.L. Rossiter, The Electrical Resistivity of Metals and Alloys, Cambridge University Press, Cambridge, UK,
- [36] G. Bra'ndli, J.L. Olsen, Mater. Sci. Eng. 4 (1969) 61.

- [37] A. Deubner, Naturwissenschaften 23 (1935) 557.  
[38] B. Fischer, Dissertation, University of Go'ttingen, Germany, 1980.  
[39] Arsenault RJ, Shi N. Mater Sci Eng 1986;81:175.  
[40] Flom Y, Arsenault RJ. Mater Sci Eng 1985;75:151.



**Avik Kumar Mahata**, was born on January, 1, 1988 in West Bengal. He received his Bachelor of Engineering in Mechanical Engineering from Bengal Engineering and Science University, Shibpur, Howrah, West Bengal. Pursuing MTech in Material Science and Engineering from NIT Trichy. His are of interest includes mainly Mathematical and Theoretical approach to Engineering problems. Along with extremely interested in Applied Physics.