Homography Estimation Techniques in Image Correction Using Points and Line With Known Epipolar Geometry

Mr. Devvrat Yadav, Mr. Shashikant Agrawal

Abstract—A homography defines the mapping from points on one projective plane to the other. The homography between two views plays an important role in the geometry of multiple views. A homography is an invertible mapping relating two images of the same planar scene. Homographies play a key role in many computer vision and robotics problems, especially those that involve manmade environments typically constructed of planar surfaces, and those where the camera is sufficiently far from the scene viewed that the relief of surface features is negligible, such as the situation encountered in vision sequences of the ground taken from a flying vehicle. This paper analyses homography estimation between points and line using known epipolar geometry.

Index Terms—Epipolar geometry, Feedback, Homography, Planar surfaces,

I. INTRODUCTION

Computing homographies from point correspondences has been extensively studied in the last fifteen years and different techniques have been proposed in the literature that provide an estimate of the homography matrix. The quality of the homography estimate depends strongly on the algorithm used and the size of the set of points considered [5]. In a recent paper by the authors [5] a nonlinear observer for homography estimation was proposed based on the group structure of the set of homographies, the Special Linear group SL(3). A key advance on prior work by the authors is the formulation of a point feature innovation for the observer that incorporates point correspondences directly in the observer without requiring reconstruction of individual image homographies. The proposed approach has a number of advantages. Firstly, it avoids the computation associated with the full homography construction. This saves considerable computational resources and makes the proposed algorithm suitable for embedded systems with simple point tracking software. Secondly, the algorithm is well posed even when there is insufficient data for a full reconstruction of a homography. Finally, even if a homography can be reconstructed from a small set of feature correspondences, the estimate is often unreliable and the associated error is difficult to characterize.

II. PROJECTOR CAMERA MODEL

The idea to project an image on a surface was first visualized in a drawing by Johannes de Fontana In 1421. It sketched a monk holding a lantern. On the side of the lantern was a small translucent opening that had an image of a devil having a lance in his hands. The image was probably drawn on thin sheet of bone and was projected on the wall by the flame in the lantern. The idea was simple but it was the first in the path towards the creation of modern day technology. In mid nineties, with scientists and engineers working really hard, a new technology was invented that eventually became the first multi-media projector. The most recent technology of the day was digital processing. The development of digital light processing (DLP) was led by applying digital principles to projectors. DLP technology uses more than 1.3 million microscopic mirrors, and hinges them onto a digital micro mirror device (DMD) chip. The technology has greatly improved since the first DLP projectors that produced grainy images and now even high resolution images can be produced in a projector. Projector camera systems are basically systems that can serve as improvement to projector based applications by addition of visual feedback (camera) to the projector. A method for projecting images onto non-planar screens by using projector-camera systems eliminating distortion in projected images. In this system, point-to-point correspondences in a projector image and a camera image should be extracted. For finding correspondences, the epipolar geometry between a projector and a camera is used. By using dynamic programming method on epipolar lines, correspondences between projector image and camera image are obtained. Furthermore, in order to achieve faster and more robust matching, the non-planar screen is approximately represented by a B-spline surface. The small number of parameters for the B-spline surface are estimated from corresponding pixels on epipolar lines rapidly. Experimental results show the proposed method works well for projecting images onto non-planar screens [1]. we consider a projector-camera system, in which the relative geometry between a projector and a camera is fixed. The fundamental matrix for the camera and the projector is calibrated in advance by using the calibration method for projector camera systems. The arbitrary images are projected from the projector onto a non-planar screen, and they are observed by the camera. Hence, we do not use specific
calibration patterns as projector images. We assume that images observed by the camera is same as images observed by the user, and thus the users can observe proper images, if the camera observes proper images By providing a feedback camera to the projector we can generate or get the output that looks correct from the point of view of the audience. These systems find a wide range of applications like projecting on irregular surface, curved surface, intersection of two or more walls, and still be able to get a result close to projection on a planar screen and appear to the human eye in original form.

As projectors are passive devices they can’t sense much how the image projected is displayed on the screen. Combining the projector with a camera we are actually adding feedback to the projector and giving it information about the display environment and thus making it active device [1]. The system mainly consists of three main parts projector, feedback camera and surface [2].

The pin-hole camera model is most widely used and performs a perfect perspective transformation of 3D space on to a retinal plane [6]. In the general case, we must also account for a change of world coordinates, as well as for a change of retinal coordinates, the camera performs a projective linear transformation rather than a mere perspective transformation.

Pixel coordinates u and v are the only information we have if the camera is not calibrated. In a calibrated camera, the relation between the world Point M with coordinates (X,Y,Z) and its projection in image m with coordinates (u,v) is given by

\[ m = \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \equiv PM \] (1)

where \( K \) is a 3x3 intrinsic matrix accounting for camera sampling and optical characteristics and \( G \) is a 4x4 displacement matrix accounting for camera position and orientation in the world coordinate frame [6].

Observe that \( PM = (PH)(H^{-1}M) \) and similarly for \( P' \). Thus if \( x \) and \( x' \) are matches points with respect to the pair of cameras \( (P,P') \) corresponding to a 3D point \( M \) then they are also matched points with respect to the pair of cameras \( (PH,P'H) \) corresponding to the point \( H^{-1}M \).

III. METHODOLOGY

A homography defines the mapping from points on one projective plane to the other. The homography between two views plays an important role in the geometry of multiple views. A homography \( H \) can be used to transfer other feature types on the plane from one view to the other. A point \( x \) on the plane can be transferred to its image \( x' \) in the other view using

\[ x' = Hx \] (2)

Similarly a line \( l \) may be transferred to its corresponding line \( l' \) in the other view using

\[ l' = H^{-T} \] (3)

Features like contours or parametric curves may not be transferred directly using the homography. However, the above relations hold between any pair of matching points and lines. Points on an arbitrary contour obey equation (2) even if there is no method to transfer the contour as a whole. Similarly, the sides of a polygon will obey equation (3) even when there is no directly transfer a polygon or a rectangle using the homography.

The Fundamental matrix \( F \) for two images acquired by cameras with non coincident centers is the 3x3, rank 2 homogeneous matrix which satisfies

\[ x'^T F x = 0 \] (4)

For all corresponding points \( x \) and \( x' \) are the projection into the two cameras of the same world point. The fundamental matrix is not defined for cameras that share the principal point. If the canonical cameras are \( P = [I 0] \) and \( P' = [M m]m \) then the fundamental matrix \( F \) is given by

\[ F = [e'] \times M \] (5)

Where \( e' = m \) and \( e = M^{-1}m \). The points \( e \) and \( e' \) are called the epipoles, which are the points of intersection of the line joining the camera centers with image planes. Equivalently, it is the image in one view of the other camera center. Further \( e \) and \( e' \) are the right and left null – space of \( F \) satisfying

\[ e'^T F = 0 \text{ and } Fe = 0 \] (6)

IV. ALGORITHM

In a plane a line is a combination of points and the relation between point in a plane through the epipolar geometry. The line is assumed contact points between the center of camera and the image point. In this assumption we get a point in image another point can be determined using homography. A homography defines the mapping from points on one projective plane to the other. The homography between two views plays an important role in the geometry of multiple views. A homography is an invertible mapping relating two images of the same planar scene. The point we get using homography is the image of the intersection of the line with plane.

V. HOMOGRAPHY BETWEEN TWO IMAGES

Epipolar geometry is the geometry of stereo vision. When two cameras view a 3D scene from two distinct positions, there are a number of geometric relations between the 3D points and their projections onto the 2D images that lead to
Homography between a Point and a Line

A line correspondence reduces the three parameter family of homographies compatible with given in Equation

\[ H(\mu) = [I'] \times F + \mu e' l^T \]  

Provided where is a projective parameter. The point correspondence uniquely determines the plane and the corresponding homography [10]. Thus, given the Fundamental matrix and a corresponding points and a line gives the method to compute homography as

\[ H = [I'] \times F + \frac{(x' \times e')^T (x' \times (Fx) \times l')}{\|x' \times e'\|^2 (l^T x')} e' l^T \]  

A plane in 3-space can be specified by three points. In turn these 3D elements can be specified by image correspondences. Thus, we do not need the coordinates of the plane to compute the homography. It can be computed directly from the corresponding image points that specify the plane [10]. This is a natural mechanism to use in applications. We can find homography between a point and a line by computing the epipole \( e' \) using equation

\[ e'^T F = 0 \quad \text{and} \quad F e = 0 \]  

Compute the one parameter family for \( H \) using the line correspondence using equation

\[ H(\mu) = [I'] \times F + \mu e' l^T \]  

Compute \( \mu \) and \( H \) from the point correspondence \( x \) and \( x' \) using equation

\[ H = [I'] \times F + \frac{(x' \times e')^T (x' \times (Fx) \times l')}{\|x' \times e'\|^2 (l^T x')} e' l^T \]  

VI. Homography Between a Point and A Line

In this paper, we proposed two methods for homography estimation technique to correct images with point and line using known epipolar geometry. In first method, two cameras view a 3D scene from two distinct positions, there are a number of geometric relations between the 3D points and their projections onto the 2D images that lead to constraints between the image points. These points help to find homography estimation. In second method we analyses homography between a point and a line. More precisely, the observer directly uses point correspondences from an image sequence without requiring explicit computation of the individual homographies between any two given images.

REFERENCES

[5] Homography estimation on the Special Linear Group based on direct point correspondence Tarek Hamel, Robert Mahony, Jochen Trumpf.
Pascal Morin and Minh-Duc Hua IEEE Conference on Decision and Control and European Control Conference (CDC-ECC) Orlando, FL, USA, December 12-15, 2011


Devvrat Yadav received the B.E. Degree in Electronics & Telecommunication Engg in 2009 and M.E. Scholar Specialization in Communication. His Research interest in the field of Pattern Recognition.

Shashikant Agrawal received the B.E. Degree in Electronics & Telecommunication Engg in 2007 and M.E. Specialization in Communication in 2011. He is life member of ISTE & IETE. His Research interest in the field of Medical Image Processing & Embedded System.