

Optimization of Flywheel Weight using Genetic Algorithm

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Abstract:

Optimization is a technique through which better results are obtained under certain circumstances. The present work deals with the problem of weight minimization of flywheel.

Flywheel problem has large number of multivariable and non-linear equations / in equalities. Hence traditional optimization techniques cannot be applied in these cases. Traditional optimization techniques have a possibility for the solutions to get trapped into local minima. Also the algorithm developed for one type of problem may not be suitable for another type of problem.

In this paper, a non traditional optimization technique, namely Genetic Algorithm is used. Optimal design of flywheel is solved using Genetic algorithm.

Keywords: Optimization, Flywheel, Genetic Algorithm, Weight.

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1. Introduction

Design optimization can be defined as the process of finding the maximum or minimum of some parameters which may call the objective function and it must also satisfy a certain set of specified requirements called constraints. Many methods have been developed and are in use for design optimization. All these methods use mathematical programming techniques for solutions.

In these cases it is difficult to apply traditional optimization techniques. Non-conventional techniques are applied to such cases. These are potential search and optimization techniques for complex engineering problems. Genetic algorithms are found to have a better global perspective than the traditional methods.

Dr. Shapour Azarm [1, 2] has worked on optimization of helical spring using consol-optcad and he extended his work to fly wheel using the same optimization procedure.

S.Vijayarangan, V.Alagappan [3] was applied Genetic Algorithm technique for machine component i.e. Leaf spring. By using Genetic Algorithm, the optimum dimensions of the leaf spring were found to be minimum weight with adequate strength and stiffness.

Kalyanmoy Deb [5] presented different types of optimization algorithms viz., traditional and non-traditional algorithms. Genetic and

simulated annealing algorithms were explained with examples in non traditional algorithms.

Genetic Algorithms have good potential as optimization techniques for complex problems and have been successfully applied in the area of Mechanical Engineering. Popular applications include machine elements design, heat transfer, scheduling, vehicle routing, etc.

2. Optimal Problem Formulation

The objectives in a design problem and the associated design parameters vary from product to product. Different techniques are to be used in different problems. The purpose of the formulation procedure is to create a mathematical model of the optimal design problem, which then can be solved using an optimization algorithm. An optimization algorithm accepts an optimization problem only in a particular format. Figure 1 show the common steps involved in an optimal design formulation process. The first step is to realize the need for using optimization for a specific design problem. Then the designer needs to choose the important design variables associated with the design problem. The formulation of optimal design problems requires other considerations such as constraints, objective function, and variable bounds. Usually a hierarchy is followed in the optimal design process, although one consideration may get influenced by the other.

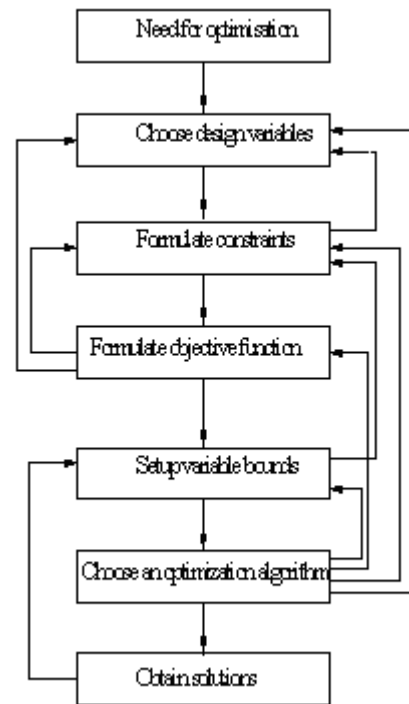


Fig.1 Flow chart for general Optimal design Procedure.

2.1. Design variables and Constraints:

The formulation of an optimization problem begins with identifying the underlying design variables, which are primarily varied during the optimization process. Other design parameters usually remain fixed or vary in relation to the design variables.

The constraints represent some-functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. Some of these considerations require that the design remain in static or dynamic equilibrium. In many mechanical engineering problems, the constraints are formulated to satisfy stress and deflection limitations. Often, a component needs to be designed in such a way that it can be placed inside a fixed housing, there by restricting size of the component. The nature and number of

constraints to be included in the formulation depend on the user.

There are usually two types of constraints that emerge from most considerations. Either the constraints are of an inequality type or of an equality type.

2.2. Objective function and Variable bounds:

The common engineering objectives involve minimization of overall cost of manufacturing or minimization of overall weight of a component or maximization of net profit earned or maximization of total life of a product or others. Instead the designer chooses the most important objective as the objective function of the optimization problem and the other objectives are included as constraints by restricting their values within a certain range. The objective function can be of two types. Either the objective function is to be maximized or minimized.

The final task of the formulation procedure is to set the minimum and the maximum bounds on each design variable. Certain Optimization algorithms do not require this information. In these problems, the constraints completely surround the feasible region. In general, all N design variables are restricted to lie within the minimum and the maximum bounds as follows:

$$X_i^{(L)} \leq X_i \leq X_i^{(U)} \quad \text{for } i = 1, 2, \dots, N.$$

In any given problem the determination of the variables bounds $X_i^{(L)}$ and $X_i^{(U)}$ may be difficult. One way to remedy this situation is to make a guess about the optimal solution and set the minimum and maximum bounds so that the optimal solution lies within these two bounds. The chosen bound may not be correct. The chosen bound may be readjusted and the optimization algorithm may be simulated again.

3. Optimization Problem Format:

The optimization problem can be mathematically written in a special format, known as nonlinear programming format. Denoting the design variables as a column vector $X = (x_1, x_2, \dots, x_N)^T$, the objective function as a scalar quantity $f(X)$,

J , inequality constraints as $g_j(X) \geq 0$, and k equality constraints and $h_k(X) = 0$;

Mathematical representation of the nonlinear programming problem:

Minimize $f(X)$

Subject to $g_j(X) \geq 0, j = 1, 2, \dots, J$;

$h_k(X) = 0, k = 1, 2, \dots, K$;

$X_i^{(L)} \leq X_i \leq X_i^{(U)}, i = 1, 2, \dots, N$;

The constraints must be written in a way so that the right side of the inequality or equality sign is zero.

3.1. Optimization Algorithms.

Certain problems involve linear terms for constraints and objective function but certain other problems involve nonlinear terms for them. Some algorithms perform better on one problem, but may perform poorly on other problems. That is why the optimization literature contains a large number of algorithms, each suitable to solve a particular type of problem. The optimization algorithms involve repetitive application of certain procedures they need to be used with the help of a computer.

3.2. Working Principle.

When the simple Genetic Algorithms is implemented it is usually done in a manner that involves the following operators such as fitness proportionate, reproduction, crossover and mutation. The Genetic Algorithm begins with an initial population of size is equal to 10 (zeroth generation) of strings which consists of binary coding selected at random. Thereafter convert the binary coding in to decimal value and calculate the actual values of design variables within their range

by using mapping rule. Next calculates the value of objective function by substituting the design variables and thereafter violation coefficient (C) and the corresponding modified objective function is obtained.

This modified objective function is used to calculate the fitness values of each individual string and also calculate the average fitness value. First, the reproduction operator makes a match of two individual strings for mating, and then crossover site is selected at random within the string length and position values are swapped between the same strings.

Finally the mutation is applied to the randomly selected strings. It is the random flipping of the bits or gene that is changing a zero to one and vice versa, so that the first cycle of operation is completed which is called as iteration. This iteration is termed as generation in genetic Algorithms. This new generation creates a second population (i.e. generation -1). This second population is further evaluated and tested for optimal solution which is obtained at zero violation and at the higher fitness value. Genetic algorithm is a population based search and optimization technique. It is an iterative optimization procedure. Genetic algorithm works as follows

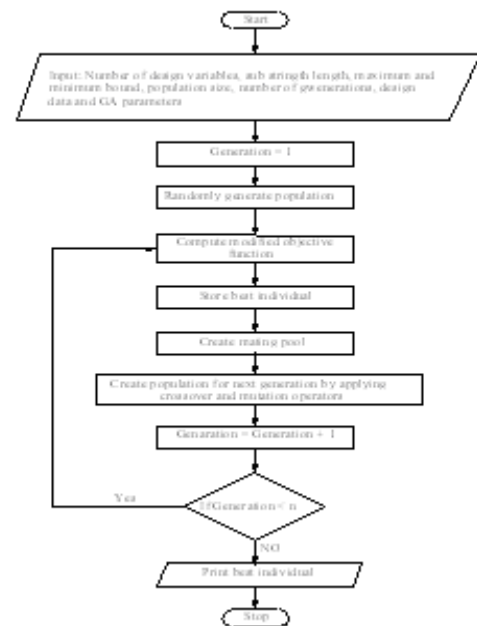


Fig.2. Flow chart of Genetic algorithm

4. Optimal Design of Flywheel.

4.1 Introduction

It is widely used as an energy storage mechanism in today's world. It is economic because the growing costs and potential scarcity of energy have increased the importance of energy storage as a conservation measure and is technological because improvements in materials make flywheel energy storage more attractive.

A punch press is one of those mechanical systems in which a flywheel is used not only for power conservation but also as a mechanical filtering element in a circuit through which power is flowing. In fact, energy is stored in a flywheel by speeding it up and is delivered to it by slowing it down. It also acts as a smoothing or equalizing element. Therefore a flywheel plays an important role in the performance of the punch press.

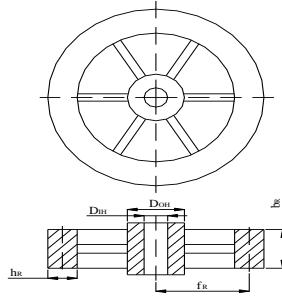


Fig.3. Flywheel in Punch press.

4.2 Objective Function

The objective function is to minimize the weight of the flywheel (W_F) which is the sum of the weights of rim (W_R), hub (W_H) and spokes (W_S).

$$f = W_F = W_R + W_H + s.W_S$$

The objective function is simplified by considering the weight of hub and spokes to be one-eighth of the weight of the rim concentrated at the mean radius of the rim.

$$W_H + s.W_S = \frac{1}{8} W_R$$

Therefore the simplified objective function is

$$W_F = \frac{9}{8} W_R = \frac{9}{8} (2 \pi \gamma_f r_R h_R b_R) g_c$$

4.3 Constraints

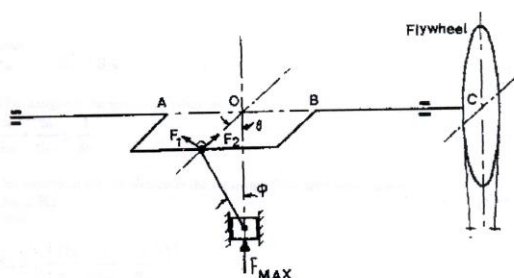
4.3.1 Weight Relation (g_1)

The weight of the hub and spokes are one eighth of the rim weight.

$$(D_{OH}^2 - D_{IH}^2) b_H + \frac{1}{2} k_s d_s^2 (D_{IR} - D_{OH}) = r_R h_R b_R$$

Where $D_{IR} = 2 r_R - h_R$

4.3.2 Shaft Stress (g_2):



At point C, only torsional stress due to punching is taken into account. Presumably this stress is much larger than any other stress present at point C, with proper overload factor. The torque producing this stress is given by

$$T = F_1 R$$

Where
$$F_1 = F_{max} \sin \theta \left[\frac{1 + \cos \theta}{\left(\left(\frac{1}{R} \right)^2 - \sin^2 \theta \right)^{0.5}} \right]$$

$$F_{max} = \pi d t_p \text{Sup } n_{OL}$$

Notice that point C is in fact under partial (half-cycle) torsional stress due to punching. Therefore the mean (τ_m) and alternating (τ_a) shear stresses are considered as

$$\tau_m = \tau_a = \frac{1}{2} \tau_{max}$$

Where
$$\tau_{max} = \frac{16 T_m}{\pi D_{IH}^3}$$

$$T_m = \frac{T}{2}$$

Since $\tau_m = \tau_a, S_{us} > S_{ys}$

Thus fatigue is the governing criterion

$$\frac{\tau_a}{S_{ns}} + \frac{\tau_m}{S_{us}} \leq \frac{1}{Sf}$$

This constraint may be written in the following form after simplification

$$D_{IH} \geq K_2$$

Where
$$K_2 = \left(\frac{4 T S_f}{\pi} \left(\frac{1}{S_{US}} + \frac{1}{S_{NS}} \right) \right)^{\frac{1}{3}}$$

4.3.3 Key Stress (g_3):

$$\tau_{k \max} = \frac{F}{A}$$

Where $F = \frac{2T}{D_{IH}}$ and $A = b_H t_k$

$$\tau_{ak} = \tau_{mk} = \frac{1}{2 \tau k_{max}}$$

$$\text{Thus } \frac{\tau_{ak}}{S_{ns}} + \frac{\tau_{mk}}{S_{us}} \leq \frac{1}{S_{fkey}}$$

This constraint may be simplified to $b_H D_{IH} \geq K_3$

$$\text{Where } K_3 = \frac{TS_{fkey}}{t_k} \left(\frac{1}{S_{us}} + \frac{1}{S_{ns}} \right)$$

4.3.4 Energy of Flywheel (g4):

In order to find the energy of the flywheel, first find the total energy which is used per stroke.

The kinetic energy (K.E) of a body rotating about a

$$\text{fixed center is } K.E = \frac{1}{2} I \omega^2$$

Since there is velocity change due to punching, the expression for change of energy is

$$\Delta K.E = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$$

This may be rewritten as

$$\Delta K.E = \frac{I V_{ave}^2 C_f}{r_R^2}$$

$$\text{Where } C_f = \frac{V_{max} - V_{min}}{V_{ave}},$$

$$V_{ave} = \frac{V_{max} - V_{min}}{2}$$

$$\text{And } I = \frac{W f r_R^2}{g_c}, \quad V = r_R \omega$$

Thus the energy constraint is $\Delta KE \geq E_f$

This can be written in the following for

$$K_4 r_R^{-3} h_R^{-1} b_R^{-1} \leq 1, \quad \text{Where}$$

$$K_4 = \frac{E_f}{9\pi\gamma_f C_f \frac{1}{4g_c} \left(\frac{2\pi N}{60} \right)^2}$$

4.3.5 Rim stresses (g5):-

The rim stresses have been developed due to combined effect of hoop tension and bending stresses. The rim is subject to high bending stresses. The maximum combined stress at the rim of the flywheel having six spokes is given

$$\text{by } S_R = \gamma_f V^2 \left[0.75 + \frac{4.935 r_R}{S^2 h_R} \right]$$

$$\text{Where } V = \omega_1 \cdot r_R \quad \text{and } \omega_1 = \frac{2\pi N}{60}$$

$$D_{OR} = 2r_R + h_R$$

$$\text{Therefore } S_R \leq S_{uf} / S_{fR}$$

4.3.6 Spokes Stresses (g6):-

In the design of spokes, it is assumed that cantilever action is predominant. The bending moment due to punching at the hub of the

$$\text{arms is taken to be } M = \frac{T(D_{OR} - D_{OH})}{S D_{OR}}$$

For which the bending stress is $S_1 = \frac{M}{Z}$ where,

$$Z = \frac{\pi k_s d_s^3}{32}$$

It is safe to assume that each arm is in tension due to one-half of the centrifugal force of that portion of the rim which it supports.

$$S_2 = \frac{3}{4} \gamma_f V^2, \quad \text{Where } V = r_R \omega$$

$$\text{Therefore } S_1 + S_2 \leq \frac{S_{uf}}{S_{fs}}$$

4.3.7. Stability (g7):- The hub length of the flywheel is usually taken greater than the shaft diameter for stability reasons.

$$D_{IH} \leq b_H$$

4.3.8. Optimal Design Model

Mathematical model for optimal design of flywheel is summarized as Minimize

$$W_F = \frac{9}{8} (2\pi\gamma_f r_R h_R b_R) g_c$$

Subject to

$$g_1: (D_{OH}^2 - D_{IH}^2) b_H + \frac{1}{2} k_s d_s^2 (2r_R - h_R - D_{OH}) - r_R h_R b_R = 0$$

$$g_2: \frac{K_2}{D_{IH}} \leq 1, \quad g_3: \frac{K_3}{D_{IH} b_H} \leq 1$$

$$g_4: \frac{K_4}{r_R^3 h_R b_R} \leq 1, \quad g_5: S_R \leq \frac{S_{uf}}{S_{fR}}$$

$$g_6: S_1 + S_2 \leq \frac{S_{uf}}{S_{fs}}, \quad g_7: \frac{D_{IH}}{b_H} \leq 1$$

The design variables r_R , h_R , b_R , D_{OH} , D_{IH} , b_H and d_s are the dimensions of the flywheel.

Method	r_R (mm)	h_R (mm)	b_R (mm)	D_{OH} (mm)	D_{IH} (mm)	b_H (mm)	d_s (mm)	W (N)
Consol- Opt cad	864	152	133	370	190	216	170	8815
G.A	864	152	127	353	228	150	159	8419

The minimum weight of Flywheel obtained by GA is reduced by 4.5 %, when compared to the weight obtained by Consol - Optcad solution.

5. Results and Discussions

Optimal design of flywheel has been discussed by using Consol-Optcad and optimization capability of Genetic Algorithms (GA).

Seven design variables and seven constraints are considered in the flywheel problem. The results obtained for flywheel are shown in table A. It was observed that the optimum dimensions and minimum weight obtained by GA are better than the values obtained by the Consol-Optcad method.

6. Conclusions: Operation in Engineering Design is the past aimed at a Design problem with single objective function with single variable and with or without constraints. But here the code is used for the optimization of flywheel to minimize the weight. It is found that the results obtained by genetic algorithms are better, search space is wide and it aims at global optimum than that the local

optimum as in a traditional method for the same input parameters.

It is found that the results obtained from genetic algorithms are less in weight as compared to consol optcad method.

7. Reference

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