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Design and Realization of IIR Digital Band Stop Filter Using Modified Analog to Digital Mapping Technique

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Abstract- **In Digital Signal Processing (DSP), the Band-Stop filter is one of the most important filter in various application. This may be of two types, Infinite Impulse Response (IIR) Band-Stop filter and the Finite Impulse Response (FIR) Band-Stop filter. In this paper, the realization and the design of IIR Band-Stop filter is represented. The IIR filter is of recursive type, i.e. the present output sample depends on the present input samples, past input samples and past output samples. Band-Stop filter is used to eliminate a band of frequency or a single frequency(for example- Notch filter) wherever it is needed. Here the Band-Stop filter is represented with its proper realization by applying the advanced analog to digital mapping technique along with the determination of its optimum coefficients and successful simulation results of magnitude response, phase response, impulse response and the pole-zero plot using Matlab7 simulator with satisfying results.**

Index Terms— **IIR filter, Band-stop filter, Advanced analog to digital frequency mapping, coefficients.**

I. INTRODUCTION

In Digital Signal Processing environment, basically in case if signal processing, the filter is essentially required to process the signals. In digital signal processing applications, we consider only the digital filter. The digital filter performs its filtering operation in time domain [1][2][3]. There are many types of filters available for filtering operation. Band-stop filter is one of the most important filter among them. Band-stop filter is mainly used to stop the undesired frequency band or in some typical case only one frequency. The index term of the filter, capable of eliminate the frequency band is the Band-stop filter and typically the Band-stop filter is known as the Notch filter which is capable to eliminate only one frequency by proper selection [4][5][6][9][10].

The digital filter can be obtained from analog filter. Analog filter can be designed by passive elements like resistors, capacitors and active elements like voltage source, current source. The analog filter, designed by passive elements, called the passive filter, and those are designed by passive elements along with active elements, called the active filter[9][10][11][12]. The filter can also be designed using the OpAmp chip as it perform the recursive operations. The IIR filter is the recursive type filter. So, it is proper design the IIR filter, more properly, IIR Band-stop filter, with OpAmp chip[2][7][8][11].

There are a numbers of method available for the design of the digital filter. The analog to digital mapping technique is the efficient method to design the digital filter and to achieve the transfer function in digital domain or z-plane along with the required coefficients[2][4][5][12]. The digital filter have a number of features like high accuracy, sensitivity, smaller physical size, reduced sensitivity to component tolerance and drift[1][2][3][12]. Now to design a digital filter using analog to digital mapping technique, the bilinear transformation is required. In this paper, a new algorithm is introduced regarding the analog to digital mapping technique and observing the previous algorithms proposed for the designing the digital filter[1][2][3][12].

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II. IIR ANALOG FILTER

The analog filter can be constructed using resistance, capacitance and inductance. The analog filter that is designed using inductance and capacitance, is called the LC analog filter. The T-section Band-stop analog filter is shown in Fig.1 constructed using inductance and capacitance[10][13].

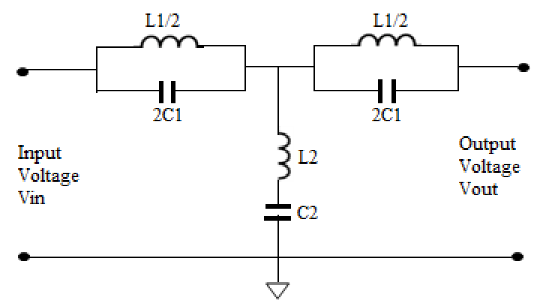

Fig.1 T-section Band stop filter (LC)

If the analog filter is designed using resistance and capacitance, it is called the RC analog filter. The T-section RC Band-stop analog filter is shown in Fig.2[9][10].

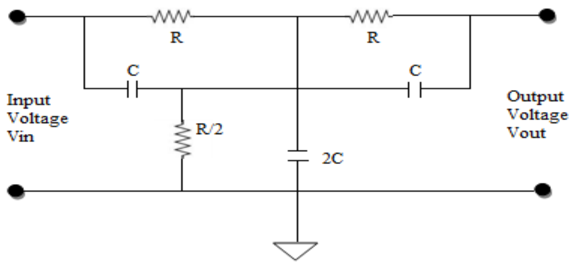

Fig.2 T-section Band stop filter (RC)

Now, by proper value selection of L, R and C, the exact Band-stop analog filter can be designed. After designing the Band-stop analog filter, it is necessary to introduce the analog to digital mapping technique to construct the IIR digital Band-stop filter.

III. ANALOG TO DIGITAL MAPPING TECHNIQUE

The calculation of the transfer function is necessary to design a digital filter,. After successful determination of the transfer function in s-domain, we apply the modified analog-to-digital mapping technique to obtain the transfer function in *z*-plane[4][5][17].

A. Transfer function in z-plane

The transfer function of a IIR digital filter can be described by[1][2][3][4][5][18][19],

$$H(z) = \frac{\sum_{n=0}^{M} b(n) z^{-n}}{1 + \sum_{n=1}^{N} a(n) z^{-n}} \qquad \ldots(1)$$

$$= \frac{B(z)}{A(z)} = \frac{b(0) + b(1)z^{-1} + b(2)z^{-2} + \ldots\ldots + b(M)z^{-M}}{1 + a(1)z^{-1} + a(2)z^{-2} + \ldots\ldots\ldots\ldots + a(N)z^{-N}} \qquad \ldots(2)$$

Where,

H(z) = Transfer function and Z-transform of impulse response h(n)
b(n) = Numerator coefficient
a(n) = Denominator coefficient

The impulse response h(n) for the realizable filter is described by[4][5][18],

$$h(n) = 0 \qquad \text{for } n \le 0 \qquad \ldots(3)$$

The satisfactory condition for a stable filter is,

$$\sum_{n=0}^{\infty} | h(n) | < \infty \qquad \ldots(4)$$

B. Derivation for transfer function H(z)

Let the impulse response of a realizable filter in time domain is h(t). We can obtain the transfer function in *s*-domain[2][4][5][18] H(s) by applying the Laplace transform on h(t), i.e.,

$$H(s) = L\{h(t)\} = \int_{0}^{\infty} h(t).e^{-st} dt \qquad \ldots(5)$$

Where,

s = complex variable
$= \sigma + j\omega$

Now, for continuous h(t), we can replace t with nT and hence the h(t) becomes,

$$h(t) = h(nT) \qquad \ldots(6)$$

Where,

T = sampling time

In the equation (6), if the sampling time T=1 second, the equation becomes,

$$h(t) = h(n) \qquad \ldots(7)$$

Where,

h(n) = Impulse response in *z*-plane

Now, if we perform the Z-Transform of h(n), we can obtain the transfer function in *z*-plane, i.e[4][5][12][19].

$$H(z) = Z\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n) z^{-n} \qquad \ldots(8)$$

C. Relationship between s-plane and z-plane

The basic relationship between *s*-plane and *z*-plane is essential for the analog-to-digital mapping. The relationship is described in equation (9),

$$z = e^{sT} \qquad \ldots(9)$$

So, for successful realization of the mapping procedure, the relationship between *s*-plane and *z*-plane is very important and without the relationship, the mapping is not practically possible.

IV. MODIFIED ANALOG TO DIGITAL MAPPING ALGORITHM

In the previous research, many of the algorithms were introduced for the analog to digital mapping, i.e. the transformation of *s*-domain to *z*-domain to construct the digital filter from the pre-constructed analog filter. In this paper, a modification of the analog-to-digital mapping technique is introduced which performs better than the earlier algorithms and

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faster in the sense of computational time. The algorithm is described in Fig. 3.

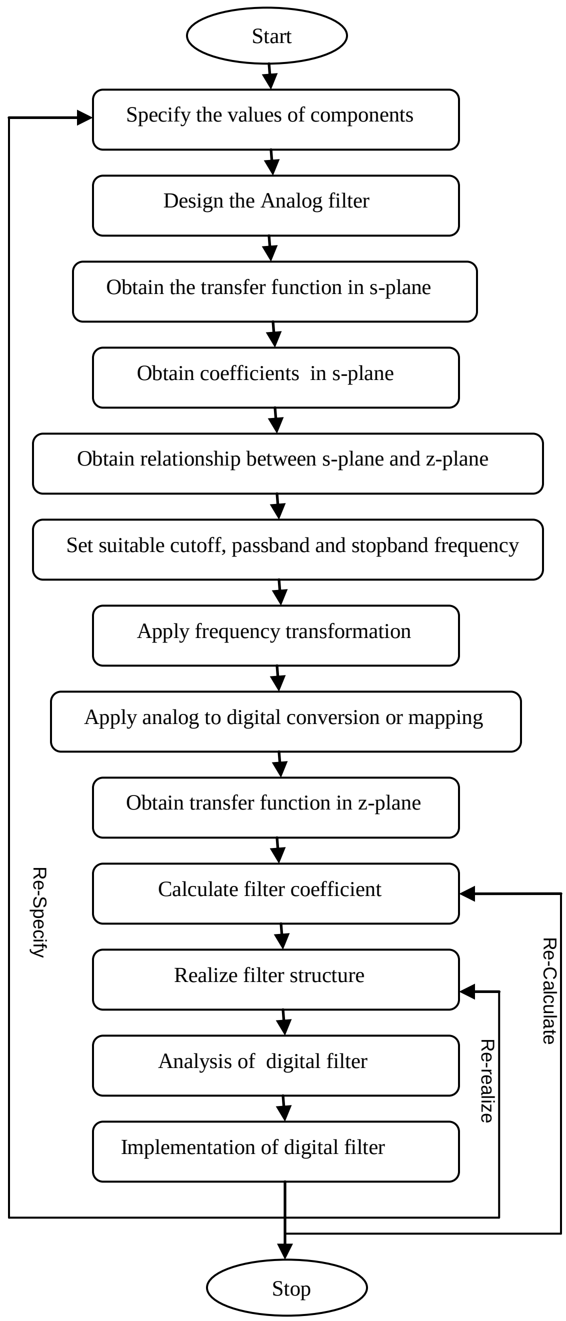

Fig.3 Modified Analog-to-Digital Mapping Algorithm

V. DESIGN OF IIR DIGITAL FILTER

The IIR Band-stop filter can also be designed by using OpAmp. In this case, the required components are the voltage source, Resistor and Capacitor. OpAmp is mainly used for designing the IIR filter because it shows the recursive effect and thus it can satisfy the characteristics of the IIR filter. After designing the IIR Band-stop filter properly with the OpAmp chip, if we use the Modified Analog-to-Digital Mapping Algorithm, the desired digital IIR Band-stop filter can be obtained. The network for the IIR Band-stop filter using OpAmp is shown in Fig. 4[11][14][15].

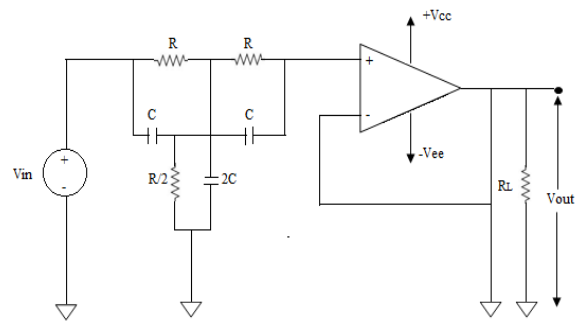

Fig.4 Active IIR Band stop filter

The transfer function of the above mentioned filter in *s*-plane can be mapped into *z*-plane by applying the Modified Analog-to-Digital Mapping Algorithm. After the mapping, the realization of the digital filter can be performed.

VI. REALIZATION OF IIR DIGITAL BAND-STOP FILTER

There are various methods available for the realization of the digital filter and they are as follows[4][5],

1. Direct form - I realization
2. Direct form - I realization
3. Transposed direct form realization
4. Cascade form realization
5. Parallel form realization
6. Lattice-Ladder structure realization

Let we consider the filter to be an Linear Time-Invariant (LTI) recursive system being x(n) is the input sequence and y(n) is the output sequence and can be described by difference equation as follows[4][5][16][18],

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad \text{....(10a)}$$

$$\begin{aligned} y(n) = &-a_1 y(n-1) - a_2 y(n-2)......-a_{N-1} y(n-N+1) \\ &-a_N y(n-N) + b_0 x(n) + b_1 x(n-1) \\ &......+ b_M x(n-M) \end{aligned} \quad \text{...(10b)}$$

Let,

$$w(n) = b_0 x(n) + b_1 x(n-1)......+ b_M x(n-M) \quad \text{...(10c)}$$

Equation (10b) can be represented as,

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) \ldots\ldots - a_{N-1} y(n-N+1)$$
$$-a_N y(n-N) + w(n) \quad \ldots(10d)$$

Now taking the Z-Transform in the both sides of equation (10b), we get

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) \ldots\ldots - a_{N-1} z^{-(N-1)} Y(z)$$
$$-a_N z^{-N} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) \ldots\ldots$$
$$\ldots + b_M z^{-M} X(z) \quad \ldots (11)$$

Rearranging the equation (11), we get,

$$Y(z) = -(a_1 z^{-1} + a_2 z^{-2} \ldots\ldots + a_{N-1} z^{-(N-1)} + a_N z^{-N}) Y(z)$$
$$+(b_0 + b_1 z^{-1} \ldots\ldots + b_M z^{-M}) X(z) \quad \ldots(12)$$

So, from equation (10b), we can see that the present output of an IIR filter depends upon the past inputs and outputs. Now, we can design the transfer function H(z)=Y(z)/X(z), and can realize the IIR digital Band-stop filter by using Direct form-I realization method as it provides separate delays for input and output.

Now, on the basis of equation (10b), the Direct form-I realization structure is described in Fig. 4[4][5][12][17].

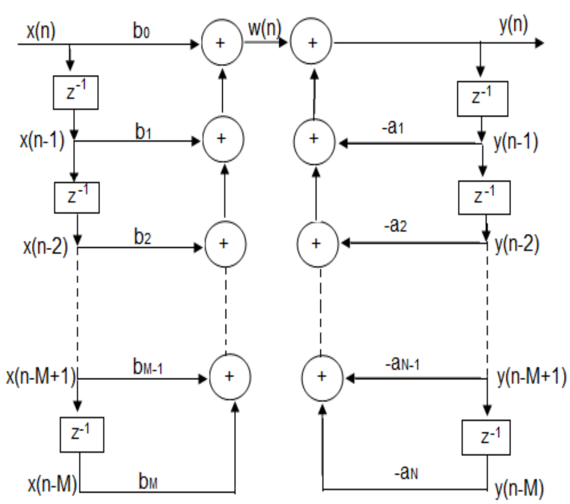

Fig.4 Direct Form-I realization

VII. SIMULATION RESULTS

The program required for designing the IIR Band-stop filter is simulated in Matlab7. In this paper, the simulation results are shown for the IIR Butterworth Band-stop filter using the Modified Analog to Digital Mapping technique. The coefficients are essential to design the filter. So, here, the coefficients are also calculated along with the simulation results. For proper simulation the specifications such as the passband and stopband ripples, passband and stopband frequencies and the sampling frequency are chosen in a proper way so that the simulation will be perfect. The pole-zero plot determines that the designed filter is stable. The figure for the magnitude response, phase response, impulse response and the pole-zero plot are shown from Fig.5 to Fig.16 . The calculated coefficients are shown in Table.1.

Filter name	Order of filter	Numerator coefficient	Denominator coefficient
Butterworth Band-Stop Filter	6	-1.197,2.569, -2.582,2.569, -1.197,0.583	-4.775,11.39, -15.01,11.39, -4.775,1.000
	8	-1.457,4.036, -5.133,7.202, -5.506,4.642, -1.798,0.872	-5.087,14.79, -26.84,33.61, -28.53,16.72, -6.126,1.287
	10	-1.724,4.721, -6.585,9.521, -8.68,8.042, -4.698,2.845, -0.877,0.316	-4.62,13.27, -25.09,35.52, -37.41,30.42, -18.39,8.314, -2.472,0.455

The simulation results for IIR Butterworth Band-stop filter for order 6, order 8 and order 10 are shown below.

A. IIR Butterworth Band-stop filter(Order=6)

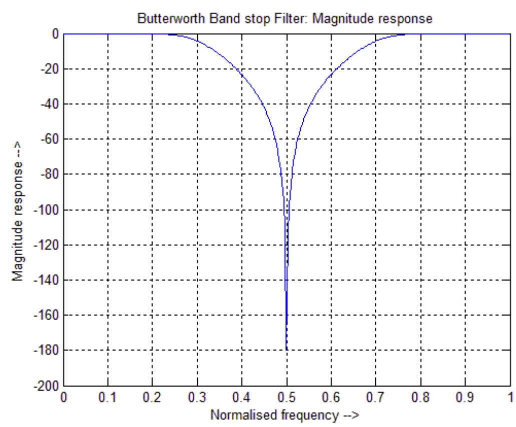

Fig.5 Magnitude response

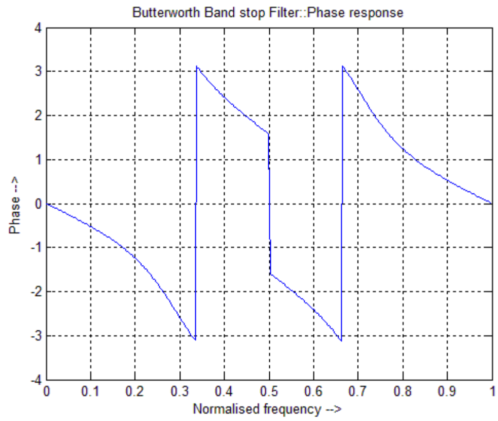

Fig.6 Phase response

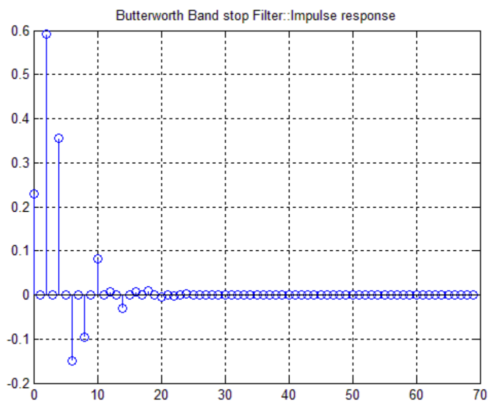

Fig.7 Impulse response

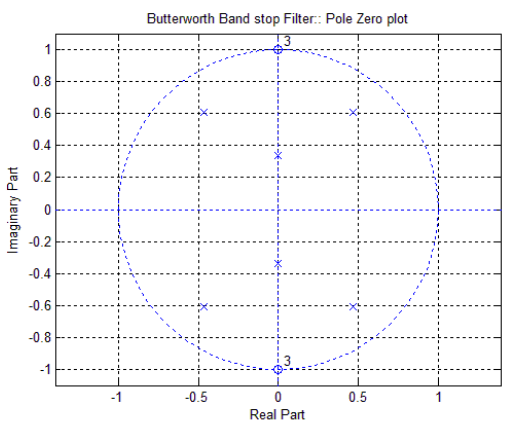

Fig.8 Pole-Zero plot

B. IIR Butterworth Band-stop filter(Order=8)

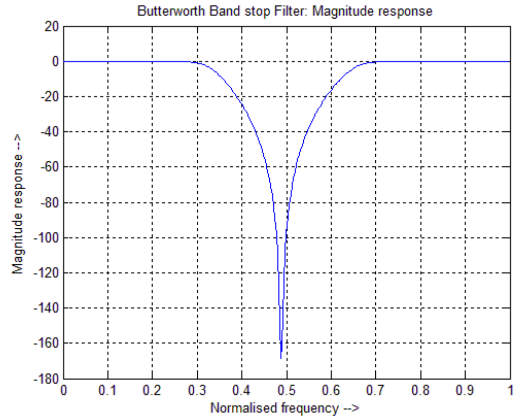

Fig.9 Magnitude response

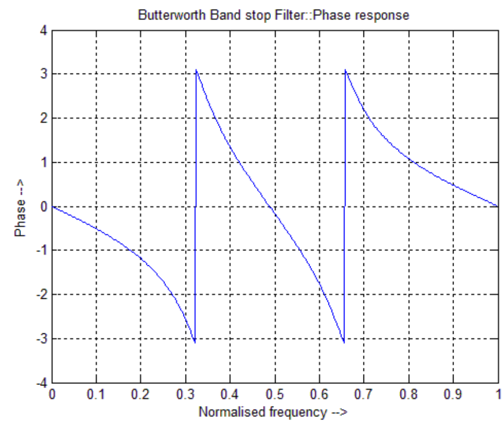

Fig.10 Phase response

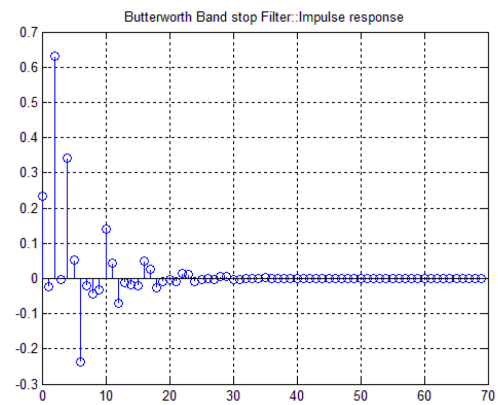

Fig.11 Impulse response

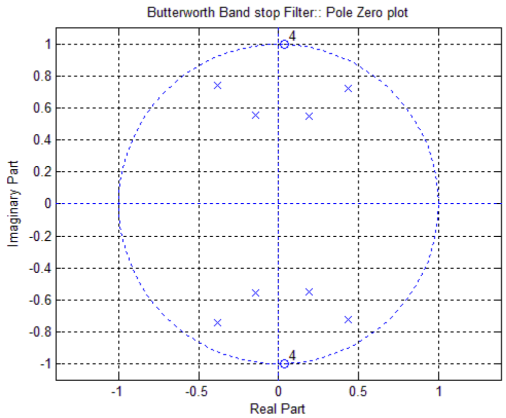

Fig.12 Pole-Zero plot

C. IIR Butterworth Band-stop filter(Order=10)

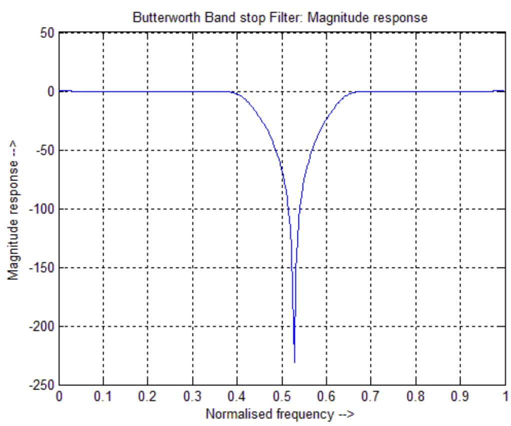

Fig.13 Magnitude response

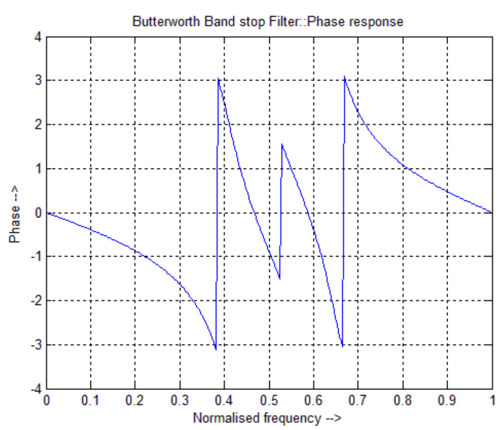

Fig.14 Phase response

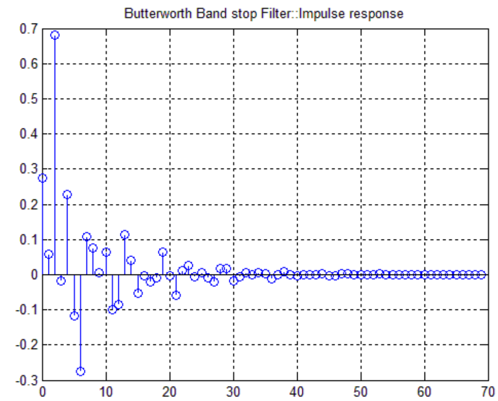

Fig.15 Impulse response

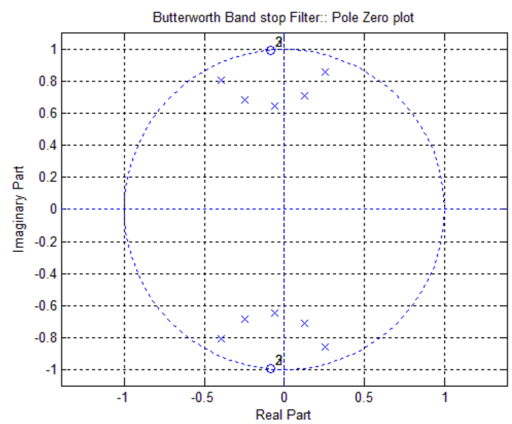

Fig.16 Pole-zero plot

VIII. CONCLUSION

The above simulation results shows that the designed filter is stable as we can see in the pole zero plots that all poles and zeros are within the unit circle which is the essential feature of a system to be stable. So, the IIR Butterworth Band-stop can be designed properly with stability using the Modified Analog to Digital Mapping Technique and applying the frequency transformation. The simulations are done using Matlab 7.6.0 version and shows the satisfactory results for designing the stable IIR Band-stop filter.

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