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Notions via β*-continuous maps in topological spaces

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Abstract

The aim of this paper is to introduce the different notions of β*-continuous maps namely β*irresolute maps,strongly β*continuous maps and perfectly β*continuous maps and study their relationships.It turns out that perfectly β*continuous map is stronger than perfectly continuous map and strongly β*continuous map is weaker than strongly continuous mapsAlso it is found that the composition of two β*irresolute maps is β*irresolute maps. The applications of these maps in some topological spaces are also studied.

Key words β*-closed set, T_ω-space , $_\alpha T_{\beta*}$-space ,$_{,sp}T_{\beta*}$- space ,β*-closed set, ω^*-closed map α-irresolute,β-irresolute

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I. Introduction

Several authors ([1], [5], [8], [9]) working in the field of general topology have shown more interest in studying the concepts of generalization of continuous maps.We have studied the concept of β*-continuous maps using β*-closed sets and proved that the composition of two β*-continuous maps need not be β*-continuous maps. Irresolute maps were introduced and studied by Crossley and Hildebrand[4], R.A. Mohmoud and Abd-EL-monsef[1] have investigated β-irresolute maps.In this paper we have introduced β*irresolute maps.Further strongly β*-continuous maps and perfectly β*-continuous maps in topological spaces have been introduced and some of their properties are elucidated.

2.preliminaries:

Throughout the paper (X, τ), (Y, σ) and (Z, η) or simply X, Y and Z denote topological spaces on which no separation axioms are assumed unless otherwise mentioned explicitly.
We recall some of the definitions and results which are used in the sequel.

Definition 2.1

A subset A of a topological space (X, τ) is called

(i) A pre-open set [10] if A⊂int(cl(A)) and a pre closed set if cl(int(A))⊂A,

(ii) A semi-open set [4] if $A\subset cl(int(A))$ and a semi-closed set if $int(cl(A))\subset A$,

(iii) An α-open set[7] if $A\subset int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A)))\subset A$,

(iv) A semipre open set [5] (= β-open set [1]) if $A\subset cl(int(cl(A)))$ and a semi-pre closed set(=β closed) if $int(cl(int(A)))\subset A$

(v) A β*-closed set [2] if $spcl(A)\subset int(U)$ whenever $A\subset U$ and U is ω-open

Definition 2.2. A topological space (X,τ) is said to be

(i) A T_ω-space [12] if every ω-closed set of (X,τ) is closed.

(ii) A ${}_\alpha T_{\beta*}$-space [2] (resp ${}_{sp}T_{\beta*}$-space) if every β*-closed set of (X,τ) is α-closed (resp.semi pre closed).

(iii) A $T_{\beta*}$ -space [2] if every β*-closed set of (X,τ) is closed

Definition 2.3

Let f: (X, τ) → (Y, σ) be a map. Then f is said to be

(i) α-irresolute [7] if $f^{-1}(V)$ is α-open in (X, τ) for each α-open set V of (Y,σ)

(ii) β-irresolute [1] if $f^{-1}(V)$ is β-closed in (X, τ) for each β-closed set V of (Y, σ)

(iii) ω-irresolute [12] if $f^{-1}(V)$ is ω-closed in (X, τ) for each ω-closed set V of (Y, σ)

(iv) ω-closed [12] if f(V) is ω-closed in (Y, σ) for every closed set V of (X,τ).

(v) Pre–β-closed [6] if f(V) is β-closed in (Y, σ) for every β- closed set V of (X, τ)

(vi) ω^*-closed [12] if f(V) is ω-closed in (Y, σ) for every ω-closed V of (X, τ)

Theorem 2.4 [2]: Every closed (resp.α-closed) set of (X,τ) is β^*-closed.

3.1 – β* irresolute maps

Definition – 3.1.1: A map f: X→Y is called β*-irresolute if $f^{-1}(V)$ is β* closed in X for every β*-closed set Vof Y.

Example- 3.1.2: Let X={a,b,c}=Y, τ={φ,{a},X} and σ={φ,{a,b},Y}. Then the identity map f: X→Y is β*-irresolute.

Proposition 3.1.3: If f: X→Y is β*-irresolute, then f is β* continuous but not conversely.

Proof: Since every closed set is β*-closed, we get the proof.

Example-3.1.4: Let X={a,b,c}=Y, τ={φ,{a},{b,c},X} and σ={φ,{a},Y}. Then the identity map f: X→Y is β*continuous but not β*irresolute because the set {a, b} is β*closed in Y but f^{-1}({a, b})={a, b} is not β*closed in X.

Proposition – 3.1.5: A map f: X→Y is β*-irresolute if and only if $f^{-1}(V)$ is β*open in X for every β* open set V of Y.

Proof : Let f: X→Y be β*continuous and U be an open set in Y.Then $f^{-1}(U^c)$ is β*closed in X.But $f^{-1}(U^c) =(f^{-1}(U))^c$ and so $f^{-1}(U)$ is β*-open in X.Converse is similar.

Proposition- 3.1.6: Let f: X→Y and g:Y→Z be any two maps. Then

(a) gof is β*-irresolute if both f and g are β*-irresolute.

(b) gof is β*-continuous if g is β*-continuous and f is β* irresolute.

Proof (a): Let F be a β*closed set in Z. Since g is β*-irresolute. $g^{-1}(F)$ is β*closed in Y. Since f is β*-irresolute, f

$^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is β*closed in X. Thus gof is β*-irresolute.

Proof (b): Let F be a closed set in Z. Since g is β* - continuous, $g^{-1}(F)$ is β*-closed in Y. Since f is β*-irresolute, $f^{-1}(g^{-1}(F))= (gof)^{-1}(F)$ is β* closed in X. Hence gof is β*- continuous.

Proposition – 3.1.7: Let X be a topological space .Y be a $T_{\beta*}$ - space and f: X→Y be a map. Then the following are equivalent.

(i) f is β*- irresolute

(ii) f is β*- continuous

Proof: (i) ⇒ (ii): Follows from proposition 3.1.3

(ii)⇒(i): Let F be a β*-closed set in Y.Since Y is a $T_{\beta*}$ - space, F is a closed set in Y, and by hypothesis, $f^{-1}(F)$ is β*-closed in X. Therefore F is β*-irresolute.

Theorem- 3.1.8: If the bijective map f: X→Y is β-irresolute and ω*-open, then f is β*-irresolute**.**

Proof: Let V be a β*-closed set in Y and U be ω-open in X such that $f^{-1}(V) \subset U$. Then V⊂f(U). Since f(U) is ω-open and V is β*-closed in Y, spcl(V) ⊂ int(f(U)) and hence $f^{-1}(spcl(V))\subset f^{-1}(int(f(U))\subset int(U.)$. Since f is β-irresolute, $f^{-1}(spcl(V))$ is semi preclosed in X. Thus $spcl(f^{-1}(V)) \subset spcl(f^{-1}(spcl(V)) = f^{-1}(spcl(V)) \subset int(U)$ and so $f^{-1}(V)$ is β*- closed in X.

Theorem- 3.1.9: If the bijective map f: X→Y is pre- β irresolute and ω-irresolute and if A is β^*closed then f(A) is β*-closed.

Proof: Let U be any ω-open subset of Y containing f(A). Then $A \subset f^{-1}(U)$ and $f^{-1}(U)$ is ω-open in X since f is ω-irresolute.Therefore $spcl(A)\subset int(f^{-1}(U))$ as A is β^*closed which implies $f(spcl(A)) \subset f(int(f^{-1}(U)) \subset int(U)$. Since f is pre β-closed and spcl(A) is semi preclosed in X ,f(spcl(A)) is semi preclosed. Thus $spcl(f(A)) \subset spcl(f(spcl(A)) = f(spcl(A)) \subset int(U)$ and so f(A) is β*- closed in X.

Theorem-3.1.10: Let f: X→Y be β*-irresolute . Then f is α-irresolute if X is a ${}_{\alpha}T_{\beta*}$-space.

Proof: Let V be a α-closed subset of Y. Then V is β*closed in Y byTheorem 2.4. Since f is β*-irresolute $f^{-1}(V)$ is β* closed in X. Also X is a ${}_{\alpha}T_{\beta*}$space. Therefore $f^{-1}(V)$ is α-closed in X. Hence f is α irresolute.

Theorem-3.1.11: Let f: X→Y be a surjective, β*-irresolute and pre β-closed map. If X is a ${}_{sp}T_{\beta*}$ space, then Y is also ${}_{sp}T_{\beta*}$ space.

Proof: Let F be a β*closed subset of Y. Since f is β*-irreso;ute, $f^{-1}(F)$ is β*closed in X. Also X is ${}_{sp}T_{\beta*}$. Therefore $f^{-1}(F)$ is semipre closed in X. Since f is pre β-closed and surjective, F is semi pre closed in Y. Hence Y is a ${}_{sp}T_{\beta*}$ space.

3.2.Strongly β*-continuous maps

Definition – 3.2.1: A map f: X→Y is said to be strongly. β*-continuous if the inverse image of every β*-open set of Y is open in X.

Proposition – 3.2.2: If a map f: X→Y is strongly β*-continuous, then f is continuous but not conversely.

Proof: Since every open set is β*-open, the proof follows.

Example- 3.2.3.: Let X=Y= {a,b,c}, τ={φ,{a},{a, b}, {a, c}, X} and σ={φ,{a},Y}. Define a map f: X→Y by f(a)=a, f(b)=c, f(c)= b, clearly f is continuous, but not strongly β*continuous, since the set {b} is β*open in Y But $f^{-1}(\{b\})$ ={c} is not open in X.

Proposition – 3.2.4: Let X be a topological space, Y be a $T_{\beta*}$ space and f: X→Y be a map. Then the following

are equivalent.
(i) f is strongly β*-continuous.
(ii) f is continuous

Proof: (i)⇒(ii): Follows from **proposition 3.2.2.**

(ii) ⇒(i): Let V be any β*-open set in Y. Since Y is a $T_{\beta*}$ space, V is open in Y. By (ii) $f^{-1}(V)$ is open in X. Therefore f is strongly β*-continuous.

Proposition – 3.2.5: Every strongly β* continuous map is α-continuous, but not conversely.

Proof: The proof follows from the fact that every α-open set is β*-open.

Example – 3.2.6: Let X=Y={a, b, c}, τ={φ,{a},{a,b},X} and σ = {φ,{a},{a,b},{a,c},Y}. Then the identity map f: X→Y is α-continuous but not strongly β*-continuous. Since the inverse image of the β*open set {a, c} in Y namely {a, c} is not open in X.

Proposition – 3.2.7: If a map f: X→Y is strongly continuous, then f is strongly β* continuous.

Proof: Follows from Definitions.

The following example supports that the converse of **proposition 3.3.7** is not true.

Example 3.2.8: Let X=Y= {a, b, c}, τ={φ,{a},{b},{a, b}, {a, c}, x} and σ= {{a},{b}, {a, b}, {b, c}}.Let f: X→Y be a map defined by f(a)=b, f(b)= a and f(c)=c then f is strongly β* continuous but not strongly continuous since f^{-1}({a,c})={b,c} is not open in X though it is closed in X.

3.3Perfectly β*-continuous maps

Definition – 3.3.1: A map f: X→Y is called perfectly β*continuous if the inverse image of every β*-open set in Y is both open and closed in X.

Proposition – 3.3.2: If a map f: X→Y is perfectly β*continuous then f is perfectly continuous(resp. continuous) but not conversely.

Proof: Let V be an open set in Y. Then V is β* -open in Y. Since f is perfectly β* continuous $f^{-1}(V)$ is both open and closed in X. Thus f is perfectly continuous (resp. Continuous).

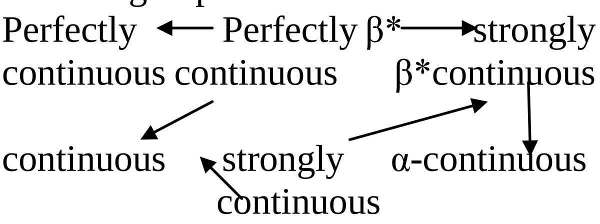
Example-3.3.3: Let X=Y={a,b,c}, τ={φ,{a},{b,c},X} and σ={φ,{a},Y}. Clearly the identity map f: X→Y is perfectly continuous and continuous but not perfectly β*-continuous, since the set{c} is β*-open in Y but f^{-1}({c})={c} is not clopen in X.

Proposition – 3.3.4: If f: X→Y is perfectly β*-continuous then it is strongly β*-continuous but not conversely.

Proof: Similar to proposition 3.3.2

Example-3.3.5: Let (X, τ) and (Y, σ) be defined as in example 3.2.8. Then f is strongly β*-continuous but not perfectly β* continuous. Since the set {b} is β* open in Y but f^{-1}({b})={a} is open but not closed in X.

From the above discussion we have following implications.



Proposition: 3.3.6: Let X be a discrete topological space, Y be any topological space and f: X→Y be a map. Then the following are equivalent.

(i) f is perfectly β*-continuous
(ii) f is strongly β*-continuous

Proof: (i)⇒(ii) follows from proposition 3.3.4.

(ii)⇒(i): Let U be any β*-open set in Y. By hypothesis $f^{-1}(U)$ is open in X. Since X is a discrete space, $f^{-1}(U)$ is also closed in X and so f is perfectly β*continuous.

Proposition – 3.3.7: If f: X→Y and g:Y→Z are perfectly β*continuous, then their composition gof: X→Z is also perfectly β*-continuous.

Proof: Let V be a β*-open set in Z. Since g is perfectly β*-continuous, $g^{-1}(V)$ is both open and closed in Y. Since f is perfectly β*continuous $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is both open and closed in X. Thus gof is perfectly β*-continuous.

Proposition – 3.3.8: If f: X→Y and g: Y→Z be any two maps then their composition gof: X→Z is

(i) β*- irresolute if g is perfectly β*continuous and f is β*-continuous.

(ii) strongly β*-continuous if g is perfectly β* continuous and f is continuous.

(iii) Perfectly β*-continuous if g is strongly continuous and f is perfectly β*-continuous.

Proof: Similar to proposition 3.3.7.

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