ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 5, May 2013

# A note on Heat Transfer to Magnetic Field Oscillatory Flow of a Viscoelastic Fluid

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## ABSTRACT

**This note deals with the magnetic field on an oscillatory flow of a viscoelastic fluid with thermal radiation, which is bounded by a vertical plane surface, have been studied. Based on some simplifying assumptions, the governing momentum and energy equations are solved and analytical solutions for fluid velocity, temperature distribution, Nusselt number and skin friction are constructed. Solutions for mean velocity and mean temperature for various combinations of parameters like Grashoff number, Prandtl number, Permeability parameter, Magnetic field parameter radiation parameter and heat source parameter are shown detail in the graphs.**

***Keywords:*** *Cooling and heating; Mass transfer; Viscoelastic; Thermal radiation; oscillatory flow*

## I. INTRODUCTION

Several studies have been conducted on the problem of flow, heat and mass transfer in saturated, homogeneous porous media with relation to different applications. For example, the dynamics of geothermal reservoirs, terrestrial heat flow through hot springs, heat exchange between soil and the atmosphere, miscible displacement in oil reservoirs, mixing of fresh and salt water in aquifers, and many others. Albeit the fact that none of these processes is derived from the existence of chemical reactions of some sort or another, relatively limited number of research works have been conducted which consider the inclusion of such chemical reactions into these processes. It is believed that chemical reaction effects should be considered in many applications of heat and mass transfer especially those encountered in chemical reactors of porous structure, geothermal reservoirs, enhanced oil recovery etc.

Coupled heat and mass transfer (or double-diffusion) driven by buoyancy, due to temperature and concentration variations in a saturated porous medium, has several important applications in geothermal and geophysical engineering such as the migration of moisture through the air contained in fibrous insulation, the extraction of geothermal energy, underground disposal of nuclear wastes, and the spreading of chemical contaminants through water-saturated soil. Bejan and Khair [2] investigated the vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium was studied by Lai and Kulacki [3] respectively.

The study of viscoelastic fluids through porous media has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through reservoir of oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. The principles of this subject are very useful in recovering the water for drinking and irrigation purposes. Many research workers Chaudhary and Jain; Gouse Mohiddin et.al; Raptis and Takhar; Ajay Kumar Singh and Khan et. al [6, 7, 8, 9, 10] has paid their attention towards the application of visco-elastic fluid flow of different category through porous medium in channels of various cross-sections. Rajgopal [11] analyzed the Stoke’s problem for a non-Netonian fluid. Hossain and Takhar [12] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature. Heat transfer enhancement for the power-law fluids through a parallel-plate double-pass heat exchangers with external recycle was examined by Gwo et.al [13].

Understanding and modelling the flows of non-Newtonian fluids are of both fundamental and practical significance in the industrial and engineering applications. The rheological characteristics of such fluids are important in the flows of nuclear fuel slurries, lubrication with heavy oils and greases, paper coating, plasma and mercury, fossil fuels, polymers etc. Further, these fluids have a nonlinear relationship between the shear stress and shear rate. The associated equations of non-Newtonian fluids are very complex and higher order than the governing equations in viscous fluid. Pascal [14] studied the rheological effects of non-Newtonian behaviour of displacing fluids on stability of a moving interface in radial oil displacement mechanism in porous media. Nakayama and Koyama [15] examined the buoyancy-induced flow of non- Newtonian fluids over a non isothermal body of arbitrary shape in a fluid-saturated porous medium. Mehta and Rao [16, 17] studied

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ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 5, May 2013

the buoyancy-induced flow of non-Newtonian fluids over a non isothermal horizontal/vertical plate embedded in a porous medium.

A fluid saturating a porous medium is induced to flow steadily by the action of buoyancy forces originated by the combined effect of both heat and concentration on the density of the saturating fluid. A heated, impermeable, semi-infinite vertical wall with both temperature and concentration are kept constant is immersed in the porous medium. As heat and species disperse across the fluid, its density changes in space and time and the fluid is induced to flow in the upward direction adjacent to the vertical plate Devika et.al [4] MHD Oscillatory Flow of a Visco Elastic Fluid in a Porous Channel with Chemical Reaction. Sudhakar and Venkataramana [5] MHD flow of viscoelastic fluid past a permeable bed. Gbadeyan [18] Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field.

Transport phenomena in materials processing flows in the presence of magnetic fields have stimulated considerable attention in recent years. Many modern technologies involve the interaction of electromagnetic fields and flowing liquids such as metal production and electrolytic manufacture. Magnetic damping involves the application of an intense, static magnetic field to suppress fluid motion. Semiconductors, smart metallic alloys, ceramics and intelligent metallo-organic liquids are often produced using electromagnetic materials processing as are ferrofluids for medical applications, laser welds, nano-scale metallic powders etc. Szekely and Chang; Yasuda; Lofgren and Akerstedt [19, 20, 21]. Important magneto-fluid dynamic phenomena arising in materials processing including the flow of metal along translating surfaces, magnetic-field control in the production of steel, aluminium, and high-performance super alloys and also magnetic stirring, where a rotating magnetic field is used to agitate and homogenize the liquid zone of a partially-solidified ingot Davidson [22].

## III. FORMULATION OF THE PROBLEM

Consider a two-dimensional, unsteady free convective flow of a viscoelastic incompressible fluid which is bounded by a vertical infinite plane surface, embedded in a uniform porous medium with heat source under the action of uniform magnetic field applied normal to the direction of the flow. The effect of induced magnetic field is neglected. The magnetic Reynolds number is assumed to be small. The terms due to electrical dissipation is neglected in energy equation. We assume that the surface absorbs the fluid with a constant velocity and the velocity far away from the surface oscillates about a mean constant value with direction parallel to $x'$ - axis. $x'$ - axis is taken along the plane surface with direction opposite to the direction of the gravity and the $y'$ -axis is taken to be normal to the plane surface. The heat due to viscous and joule dissipation are neglected for small velocities. All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. It is considered that the free stream velocity oscillates in magnitude but not in direction. Under the above stated assumptions and taking the usual Boussinesque approximation into account, the governing equations for the flow and temperature field in dimensionless form are given as under Walters [1]

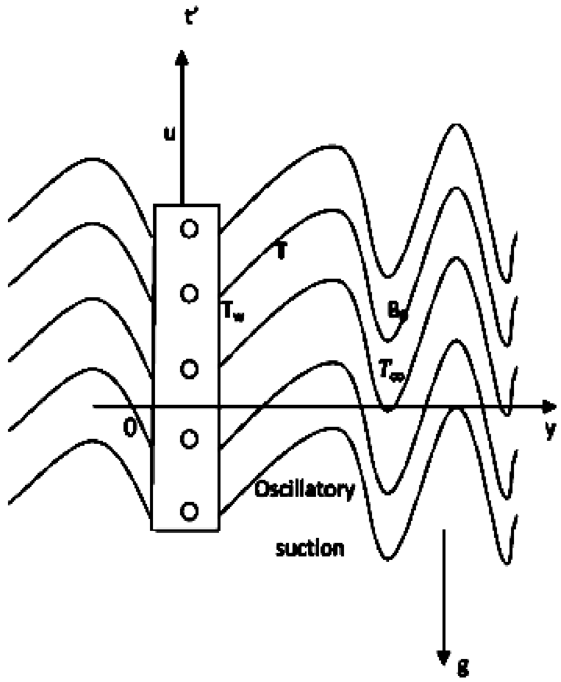


Figure (I): Geometry of the problem

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_0\left(\frac{\partial^3 u}{\partial y^2\partial t}\right)+Gr\,T-\left(M+\frac{1}{K}\right)u \quad (1)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}-\left(\frac{\partial^2 T}{\partial t^2}-\frac{\partial^2 T}{\partial t\,\partial y}\right)-\frac{1}{\Pr}\ R+Q\ T \quad (2)$$

Initial condition has been neglected as the problem is in semi-infinite region. The relevant boundary conditions in dimensionless form are

$$\begin{aligned} &u=-\ 1+\varepsilon e^{int}\ ,T=\ 1+\varepsilon e^{int} && at\quad y=0\\ &u\rightarrow 0,T\rightarrow 0 && as\quad y\rightarrow\infty \end{aligned} \quad (3)$$

The dimensionless quantities introduced in the above equations are defined as

$$u=\frac{u'}{v_0}, y=\frac{y'v_0}{\nu}, t=\frac{t'v_0^2}{\nu}, T=\frac{T'-T_\infty'}{T_w'-T_\infty'}$$
$$K=\frac{k_0 v_0^2}{\nu^2}, \quad M=\frac{\sigma B_0^2 \nu}{\rho v_0^2}, Q=\frac{\nu Q_0}{\rho C_p v_0^2} \qquad (4)$$
$$k_0=\frac{1}{\rho}k_0'\left(\frac{v_0^2}{\nu}\right), \quad Gr=\frac{\nu\beta g\ (T_w'-T_\infty')}{v_0^3}$$
$$\Pr=\frac{\nu\rho C_p}{k}, \qquad R=\frac{16\nu^2\lambda\sigma T_\infty^3}{k\,v_0^2}$$

Where $u$ is the velocity along the $x'$ -axis, is constant obtained after integration conservation of mass in pre-non dimensional form not mentioned, v is the velocity along -axis, is the kinematic viscosity, g is the acceleration due to gravity, $T$ is the temperature of the fluid, is the coefficient of volume expansion, $C_p$ is the specific heat at constant pressure, is a constant, σ is the Stefan-Boltzmann constant, $\lambda$ is the mean absorption coefficient, $T_w$ is the temperature of the surface, is the temperature far away from the surface, $\Pr$ is the Prandtl number, $Gr$ is the Grashoff number, $Q$ is Heat source parameter $q_r$ is radiative heat flux in y direction, is the density, t is the time, k is the thermal conductivity of the fluid, n is the frequency of oscillation of the fluid and $k_0$ is the elastic parameter, $B_0$ is uniform magnetic field strength, $M$ is the magnetic field parameter which is the ratio of magnetic force to the inertial force. It is a measure of the effect of flow on the magnetic field. Finally R is the radiation parameter. The effect of radiation parameter is to increase the rate of energy transport to the gas, thereby making the boundary layer becomes thicker and the fluid becomes warmer. Emissivity has been omitted in the expression of R, because for a black body the value of emissivity is unity. For Non non-Newtonian fluids like thick black paint the value of emissivity is 0.978.

## IV. SOLUTION OF THE PROBLEM

Equation (6) – (8) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (9). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

$$u=u_0(y)+\varepsilon e^{it}u_1(y)$$
$$T=T_0(y)+\varepsilon e^{it}T_1(y) \qquad (5)$$

Substituting (5) in Equation (1) – (2) and equating the harmonic and non – harmonic terms, and neglecting the higher order terms of $0(\varepsilon^2)$, we obtain

$$u_0''+u_0'-\beta^2 u_0=-Gr\,T_0 \qquad (6)$$
$$(1+ik_0)\,u_1''+u_1'-\alpha^2 u_1=-Gr\,T_1 \qquad (7)$$
$$T_0''-\Pr T_0'-(R+Q)\,T_0=0 \qquad (8)$$
$$T_1''-\Pr(1+i)\,T_1'-(i\Pr+R+Q)\,T_1=0 \qquad (9)$$

The corresponding boundary conditions can be written as

$$u_0=-1,\ T_0=1,\ T_1=1 \quad at\ y=0$$
$$u_0\to 0, T_0\to 0, T_1=0 \quad as\ y\to\infty \qquad (10)$$

Solving these differential equations from (6) – (9) using boundary conditions (10) we obtain mean velocity and mean temperature as follows.

$$u_0(y,t)=A_1 e^{m_1 y}+A_2 e^{m_2 y}$$
$$T_0(y,t)=e^{m_1 y}$$

**APPENDIX**

$$m_1=-\left(\frac{\Pr+\sqrt{\Pr^2+4(R+Q)}}{2}\right)$$
$$m_2=-\left(\frac{1+\sqrt{1+4\beta^2}}{2}\right), \beta^2=\left(M+\frac{1}{K}\right)$$
$$\alpha^2=\left(i+M+\frac{1}{K}\right), A_1=-\frac{Gr}{m_1^2-m_1-\beta^2}$$
$$A_1=-(1+A_1)$$

## V. RESULTS AND DISCUSSION

In order to get clear insight into the problem, numerical computations are carried out for various parameters like $Gr, K, R, Q, \Pr$ and $M$ are displayed .The numerical values of mean temperature has been obtained for different parameters like $R, Q$ and $\Pr$ as taken in the governing equation (2) . Similarly the numerical values of mean velocity have been obtained for different parameters like $Gr, K, R, Q, \Pr$ and $M$ as taken in the governing equation (1). Plotting the temperature and velocity profiles pictorially has been showed from figures (1) - (9). It can be seen that the Prandtl number $\Pr=0.71$ has been taken because this value corresponds to water which is known to be the Newtonian fluid. Free convection currents exist because of the temperature difference $T_p-T_\infty$ which may be positive, zero or negative. We know that the Grashof number $Gr$ is a common dimensionless group that is used when analyzing the potential effect of convection introduced by large temperature differences. So $Gr$ will assume positive. From the physical point of view, $Gr < 0$ corresponds to an externally heated plate as free convection currents are carried towards the plate. $Gr > 0$ Corresponds to an externally cooled plate and $Gr = 0$ corresponds to the absence of free convection currents. The effect of cooling and heating on the velocity for the Grashoff number can be observed from figure (1) respectively, we observe from this figure that the mean velocity increase in cases of cooling for a visco - elastic fluid. Figure (2) and (3) illustrates the effects of porous medium shape factor parameter $K$ and the radiation parameter $R$. It is observed from this figure that velocity goes on increasing with the increase of porous medium shape factor parameter $K$. The mean velocity decreases with the increase of radiation parameter $R$. Figure (4) shows the variation of mean velocity profiles for different values of $Q$. It is seen from this figure that mean velocity profiles decrease with an increasing of heat source parameter $Q$. The influence of Prandtl number $\Pr$ on mean velocity profiles have been illustrated in figure (5). It is observed that an increase in $\Pr$ results a decrease. The influence of magnetic parameter $M$ is presented graphically in figure (6). As expected, the mean velocity decreases with increasing magnetic parameter $M$. The effect of the transverse magnetic field leads to a resistive type of force similar to drag force, which tends to resist the retarding flow of visco-elastic fluid flow. The effect of thermal radiation parameter is important for temperature profiles. The mean temperature profiles for the case of the Newtonian fluid with thermal radiation are given in figure (7). It is observed that the mean temperature profiles decreases with increase of $y$. The mean temperature profiles for different parameters heat source parameter $Q$ and Prandtl number $\Pr$ are displayed in figure (8) and (9). It is observed from this figure that the mean temperature decreases with the increase of heat source parameter $Q$ and Prandtl number for the case of the Newtonian fluid.

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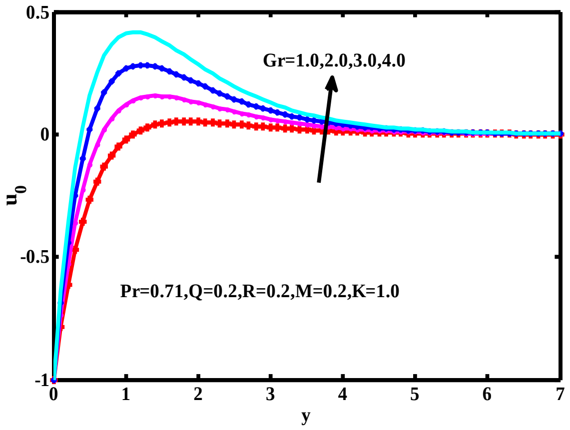


Figure (1): Mean Velocity profiles for different values of Gr

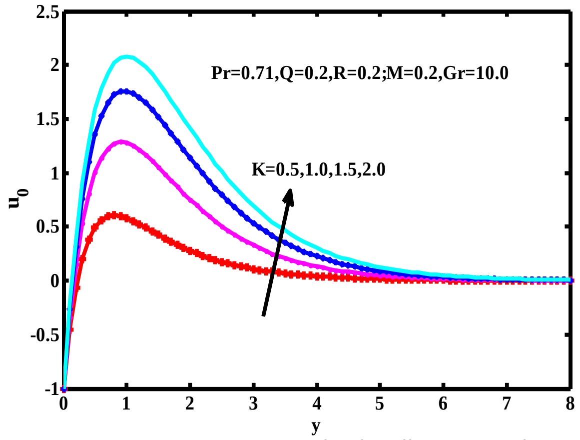


Figure (2): Mean Velocity profiles for different values of K

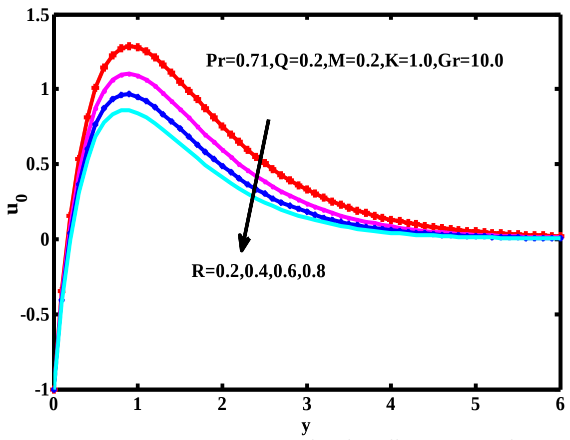


Figure (3):Mean Velocity profiles for different values of R

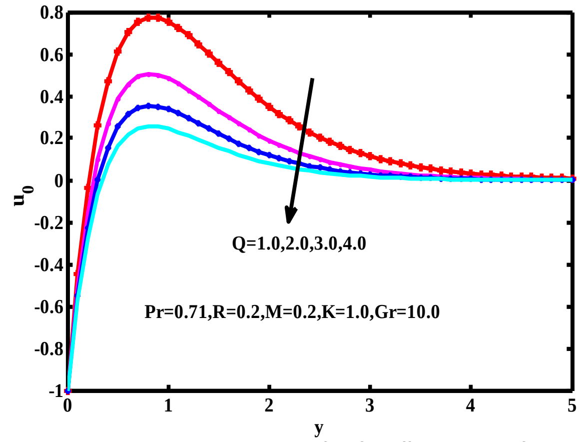


Figure (4):Mean Velocity profiles for different values of R

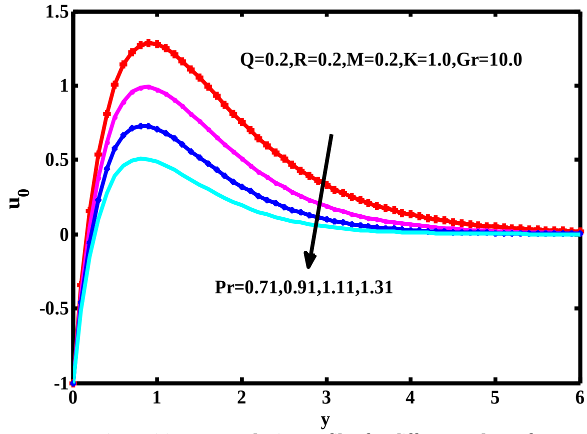


Figure (5): Mean Velocity profiles for different values of Pr

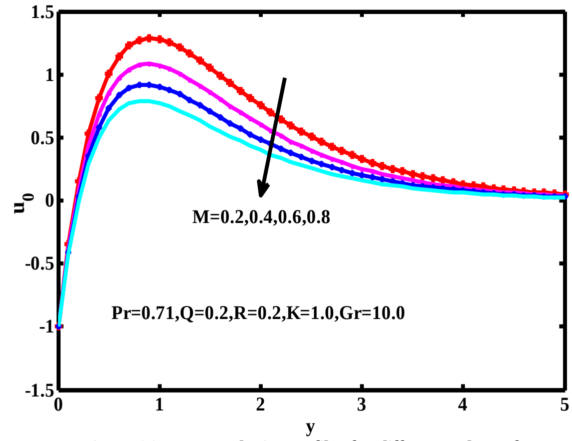


Figure (6): Mean Velocity profiles for different values of M

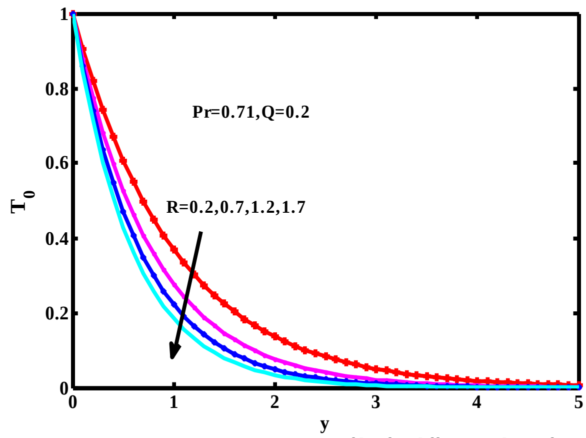


Figure (7): Mean Temperature profiles for different values of R

ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 5, May 2013

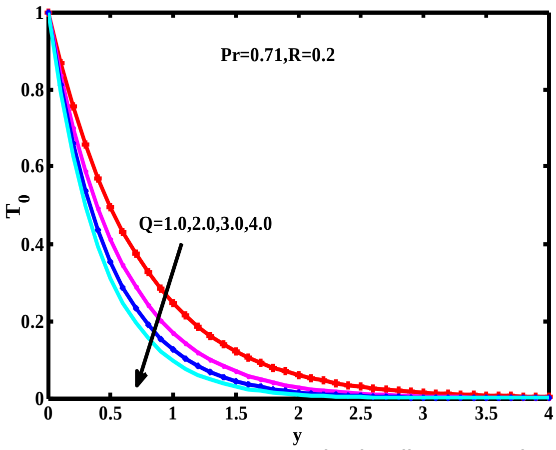


Figure (8): Mean Temperature profiles for different values of Q

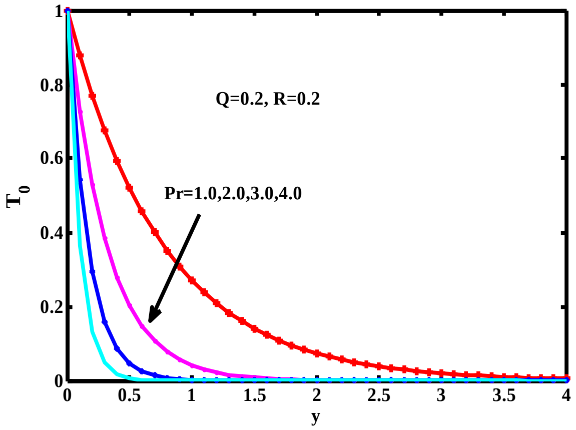


Figure (9): Mean Temperature profiles for different values of Pr

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