A modified window function for FIR filter design with an improved frequency response and its comparison with the Hamming window

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Abstract: In this paper a modified window function for Finite Impulse Response (FIR) filter design with an improved frequency response has been presented. The window function is generated and has been utilized to compute the frequency response of various types of FIR filter i.e. high pass, low pass, band pass and band stop types. Noticeable improvement is observed when the newly generated frequency response is compared with various other frequency responses using the other existing windows in perspective of stop band attenuation and presence of ripples, which is justified by relevant simulations and plots. In this paper the frequency response of the modified window is compared with that of Hamming window with quantized and generalized experimental results. Comparison of the modified window with the other common window functions as well as responses will be taken up in near future.

(Keywords: Finite, Impulse, Response, Hamming, filter, window, function, frequency, attenuation, ripples)

I. INTRODUCTION

Digital filters can be classified either as finite duration unit impulse response (FIR) filters or infinite duration unit impulse response (IIR) filters. In FIR systems, the impulse response sequence is of finite duration or in other words, if the impulse response of a digital filter is determined for some finite number of sample points, then these filters are known as Finite Impulse Response or FIR digital filters. [1], [2], [3], [4]

There are mainly three methods used for FIR filter design:

- FIR filter design using windows.
- FIR filter design using frequency sampling method.
- Optimal or rminimax FIR filter design.

Let us consider that the digital filter which has to be designed is to be designed has the frequency response, $H_d(\omega)$. This is also known as desired frequency response. Let the corresponding unit sample response be $h_d(n)$ i.e.

$$H_d(\omega) = \sum_{n=0}^{\infty} h_d(n) e^{-j\omega n}$$

Ref. [1], [2], [3]

In FIR filter design using window method, the common window functions that are commonly used are Rectangular window, Triangular window, Bartlett window, Hamming window, Hanning window etc.[5].

In this context the Hamming window function will be discussed in brief following which a modified window will be presented whose frequency response
characteristics will be compared with the previous one.

II. HAMMING WINDOW

The Hamming window is defined as,

\[ w(n) = 0.54 - 0.46 \cos(2\pi n/N) \] ....(2)

where,

\[ 0 \leq n \leq N \] and

n = Number of sample points

N = Order of the filter [6], [7], [8]

The Hamming window has been pictorially represented in Fig. 1 using MATLAB R2012a software package.

![Hamming window](image)

Fig. 1

III. THE MODIFIED WINDOW FUNCTION

In this section the modified window method has been presented. The window sequence has been defined as,

\[ w(n) = a - b \cos(2\pi(n+0.1)/N) + c \cos(4\pi(n+0.3)/N) \]
\[ - d \cos(6\pi(n+0.5)/N) + e \cos(8\pi(n+0.7)/N) \]
\[ - f \cos(10\pi(n+0.9)/N) \] ....(3)

where,

\[ 0 \leq n \leq N \]

and the co-efficients are,

\[ a = 0.501; \]
\[ b = 0.495; \]
\[ c = 0.001; \]
\[ d = 0.002; \]
\[ e = 0.003; \]
\[ f = 0.004. \]

The modified window sequence has been presented in Fig. 2.

![Modified window](image)

Fig. 2

Fig. 2 shows the proposed window function where the number of samples is represented along x-axis and amplitude along y-axis respectively.
IV. FREQUENCY RESPONSES

This window function $w(n)$, as in equation (3) is used to compute the frequency response of various types of FIR filter i.e. low pass, high pass, band pass and band stop.

Low pass filter:

The order of the filter has been taken as $N = 60$ and the cut-off frequency, $w_c$ as 0.3. The frequency response of a low pass FIR filter simulated using $w(n)$ is represented in Fig. 3. Significantly less pass band and stop band ripples have been observed.

Minimum stop band attenuation observed is -100 dB. Detailed specifications are given in [i].

![Low pass FIR filter](image)

Fig. 3

<table>
<thead>
<tr>
<th>Selected cut-off frequency</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. stop band attenuation</td>
<td>-100dB</td>
</tr>
<tr>
<td>Max. stop band attenuation</td>
<td>-300dB</td>
</tr>
</tbody>
</table>

[i]. Low pass filter specs.

High pass filter:

For FIR high pass filter the response is shown in Fig. 4 as

![High pass FIR filter](image)

Fig. 4

Minimum stop band attenuation observed is -100 dB.

Very accurate type of response has been observed tallying with prescribed cut-off frequency range. Detailed specifications are given in [ii].

<table>
<thead>
<tr>
<th>Selected cut-off frequency</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. stop band attenuation</td>
<td>-100dB</td>
</tr>
<tr>
<td>Max. stop band attenuation</td>
<td>-240dB</td>
</tr>
</tbody>
</table>

[ii]. High pass filter specs.

Band pass filter:

For Band pass, the cut-off frequencies are taken as 0.4 and 0.6 respectively and the simulated frequency response has been shown in Fig. 5. Accurate attenuation between prescribed cut-off frequencies.

![Band pass FIR filter](image)

Fig. 5
Fig. 5

Minimum stop band attenuation observed is -100 dB. Detailed specifications are given in [iii].

<table>
<thead>
<tr>
<th>Selected cut-off frequency</th>
<th>0.4 -- 0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. stop band attenuation</td>
<td>-100dB</td>
</tr>
<tr>
<td>Max. stop band attenuation</td>
<td>-320dB</td>
</tr>
</tbody>
</table>

[iii]. Band pass filter specs.

Fig. 6

Accurate frequency response is also observed in this case. Effective attenuation lies between 0.4 and 0.6. Detailed specifications are given in [iv].

<table>
<thead>
<tr>
<th>Selected cut-off frequency</th>
<th>0.4 -- 0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. stop band attenuation</td>
<td>-90dB</td>
</tr>
<tr>
<td>Max. stop band attenuation</td>
<td>-150dB</td>
</tr>
</tbody>
</table>

[iiv]. Band stop filter specs.

**Band stop filter:**

For band stop filter, the response is shown in Fig. 6. Minimum attenuation is -90dB in this case which is clear from the filter response.

**V. COMPARISON**

In this section the modified window will be compared with the Hamming window along with their frequency responses in case of various types of FIR filter. The window sequences have been plotted as in Fig. 7.
Low pass filter responses:

Comparatively much more effective attenuation than that using Hamming window shown in Fig. 8. We can observe that it has resulted in much more reduced side lobes in low pass filter response. From the response it is clear that the presence of stop band ripples is excessively high in case of Hamming window response which can be considered as a major draw back. [7]

Detailed specifications are given in [v].

<table>
<thead>
<tr>
<th>Specifications (Low pass FIR)</th>
<th>Hamming window</th>
<th>Modified window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected cut-off frequency</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Avg. stop band attenuation range</td>
<td>-150dB</td>
<td>-160dB</td>
</tr>
<tr>
<td></td>
<td>-350dB</td>
<td>-250dB</td>
</tr>
<tr>
<td>Presence of stop band ripples</td>
<td>Very high</td>
<td>Low</td>
</tr>
</tbody>
</table>

[v]. Low pass filter response comparison

High pass filter responses:

Much more effective response than Hamming window in designing FIR band stop filters, shown in Fig. 9. Presence of ripple effect also in case of high pass type, which is comparatively much less in the modified window response.
Detailed specifications are given in [vi].

<table>
<thead>
<tr>
<th>Specifications (High pass FIR)</th>
<th>Hamming window</th>
<th>Modified window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selected cut-off frequency</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Avg. stop band attenuation range</td>
<td>-140dB</td>
<td>-125dB</td>
</tr>
<tr>
<td></td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>-340dB</td>
<td>-230dB</td>
</tr>
<tr>
<td>Presence of stop band ripples</td>
<td>Very high</td>
<td>Low</td>
</tr>
</tbody>
</table>

[vi]. High pass filter response comparison

Attenuation range is much more compressed in the modified window response.

**Band pass filter responses:**

Noticeable improvement in FIR band pass filter design compared to Hamming window as shown in Fig. 10

From the figure we can observe that the presence of ripples is reduced to a huge extent in the modified window response. Detailed specifications are given in [vii].

<table>
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<th>Specifications (Band pass FIR)</th>
<th>Hamming window</th>
<th>Modified window</th>
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<tbody>
<tr>
<td>Selected cut-off frequency</td>
<td>0.4 -- 0.6</td>
<td>0.4 -- 0.6</td>
</tr>
<tr>
<td>Avg. stop band attenuation range</td>
<td>-140dB</td>
<td>-130dB</td>
</tr>
<tr>
<td></td>
<td>--</td>
<td>--</td>
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<td>-230dB</td>
</tr>
<tr>
<td>Presence of stop band ripples</td>
<td>Very high</td>
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</tr>
</tbody>
</table>

[vii]. Band pass filter response comparison

**Band stop filter responses:**

Likewise, superior response compared to Hamming window method for band pass FIR filter design as shown in Fig. 11 along with detailed specifications in [viii].

Fig. 10

Fig. 11
Specifications (Band stop FIR) | Hamming window | Modified window
--- | --- | ---
Selected cut-off frequency | 0.4 – 0.6 | 0.4 – 0.6
Avg. stop band attenuation range | -130dB -- -250dB | -90dB -- -140dB
Presence of stop band ripples | Very high | Low

[viii]. Band pass filter response comparison

VI. FUTURE WORK

It has been observed that the modified window function indeed produces much more effective type of frequency responses compared to the Hamming window. In future, stress will be given for the improvement of this window function for further accurate type of responses with reduced side lobes and comparatively less pass-band and stop-band ripples and frequency responses of the other common window types will be compared with the modified one.

VII. REFERENCES


ABOUT THE AUTHOR

Mr. Ankan Bhattacharya obtained B.Tech (2010) and M.Tech (2012) degrees in Electronics and Communication engineering under West Bengal University of Technology. His area of interests include design of filters for optimum response, antenna design etc. He is presently serving as an Asst. Professor of ‘Mallabhum Institute of Technology’, located at Bishnupur, West Bengal.