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Double diffusivehydro-magnetic natural convective flow past through vertical flat plate in porous medium

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Abstract:**The steady two-dimensional laminar natural convection flow and heat transfer of a viscous incompressible, electrically conducting fluid past a vertical impermeable flat plate is considered in presence of a uniform transverse magnetic field in porous media. The governing equations are derived using the usual boundary layer approximation and accounting for applied, uniform magnetic transverse magnetic field. These equations and boundary condition are transformed into system of ODE using the non-similarity transformation. The resulting linear coupled ordinary differential equation solved analytically subjected to the transformed boundary conditions. The skin-friction and nusselt number is also calculated. Graphical result for the velocity ,concentrationand temperature profiles along with skin friction coefficient are presented and discussed with respect the appropriate physical parameter.**

Index Terms**:Natural convection,Double diffusive convection,Grashof number(Gr), Hartmann number(M) , Darcy Number (Da), Prandtl number(Pr),Schimdt number(Sc).**

1. Introduction

Convection is a mechanism of heat transfer occurring because of bulk motion of fluids. This mechanism is classified into three different process: Forced convection, Natural convection, and Mixed convection.Double-diffusive Natural convection is which the buoyant forces are due to both temperature and concentration gradients, is generally referred to as thermo-solutal convection or Double-diffusive convection. For small isobaric changes in temperature and concentration, here, the density, ρ, depends linearly on both temperature, T, and concentration,C.

Double diffusive convection in a complex geometry has attracted the interest of several researchers. It occurs in a wide range of fields such as Oceanography , astrophysics, Geology, biology, Chemical process etc. Most work about cavities of complex geometry deal with partitions fitted to insulated walls. Cavities with baffles on their active walls have been less studied . Heat transfer in partially divided enclosures has received attention primarily due to its engineering and other application like design of energy efficient building , natural gas storage tank.

S.Sivasankaran et al(2007) describe the effect of thin baffle at the hot wall of a water filled , differentially heated rectangular enclosure. Vafai and Tien (1981) numerically analyzed the boundary and inertial forces on flow and heat transfer in porous media. They concluded that effects of inertia increase with the higher permeability and the lower fluid viscosity,and the velocity gradients near the wall are bound' to found, thereby increasing the viscous resistance due to the boundary. Natural convection in horizontal Prandtl Number Effects on Mixed Convection porous layers heated from below has been studied by Kalidas and Prasad (1989). They formulated the Brinkman-Forchheimer-extended Darcy equation of motion and considered different Darcy and Prandtl numbers. Their heat transfer results indicated the existence of an asymptotic convection regime, where the Nusselt number is almost independent of the permeability of the porous matrix or Darcy number. A similar trend has also been observed by Jue (2001). Khanafer and Chamkha (1999) conducted a numerical study for mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium.

Further S.Mergui and d. Gobin (2009) did the numerical analysis of transient heat and apecies transfer by natural convection in a binary fluid vertical layer. R. C Chaudhary et al (2006) analyses the free convection flow of a viscous incompressible fluid past an infinite vertical accelerated plate embedded in porous medium with constant heat flux in the presence of transverse magnetic field. The governing equations are solved in closed form by Laplace transform technique.

Porous medium has its applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs etc. Furthermore convection through porous media may be found in fiber and granular insulation. Representative studies in this area may be found in books by Nield et al., Bejan et al. ,Ingham et al. Kim et al and Harris et al. analyzed the free convection flow past a vertical plate in porous medium with different boundary conditions. Recently, Magyari et al. discussed analytical solutions for unsteady free convection in porous media. The effect of MHD in porous media considered by Raptis et al. and Geindreau et al.

Recently S.Kapoor and P.Bera (2012) studied for non-darcy fully developed mixed convection in a vertical pipe filled with anisotropic porous media. The motion in the pipe caused by external pressure gradient and buoyancy force. Non-Darcy Brinkmen-Forchheimer extended model has been adopted in momentum equation. Ashok and Bera investigated the flow in vertical pipe filled with porous media in which they found that, In case of buoyancy-opposed flow, the velocity profile may contain point of inflection in the center zone and point of separation at the vicinity of the wall.

In the paper of S. Kapoor et al., the problem of magneto hydrodynamic free convection flow along a vertical flat plate is investigated in porous medium . The local non similarity equations governing the flow for the case of uniform viscosity and thermal conductivity are developed. The present paper is the extension of this paper .

2. The Mathematical Model

The Physical model of the problem and coordinate system are given in Fig1. Consider a steady , two dimensional laminar free convective boundary layer flow of a incompressible and electrically conducting fluid with viscosity depending on temperature past a semi infinite vertical impermeable flat plate in presence of uniformly distributed transverse magnetic field of strength H_o in the porous medium under the Bousineqess approximation. The basic equations are given below with respect to x and y axis.

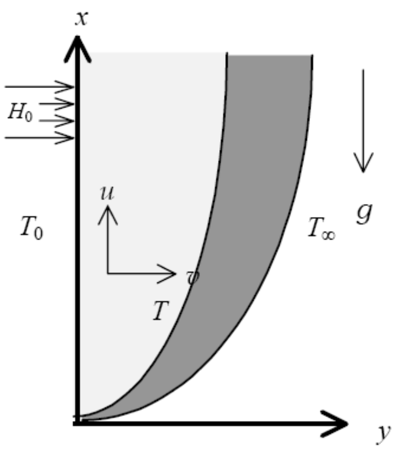

Fig 1. The Flow configuration of physical system

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3. The Governing Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)-g\beta_S(C-C_\infty)-\frac{\sigma u H_0^2}{\rho_\infty}-\frac{\mu}{k_p}u+Fu|u| (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sigma\frac{\partial^2 T}{\partial y^2} (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} (4)$$

With boundary conditions

$$u=v=0, T=T_0, C=C_0 \; at \; y=0$$

$$u\to 0, T\to T_\infty, C\to C_\infty \; ay \; y\to\infty,$$

Here $u, v$is the velocity components associated with the direction of increase of coordinates x and y measured along and normal to the vertical plate. $T$is the temperature of the fluid in the boundary layer,C is the concentration$g$ is the acceleration due to gravity, β is the coefficient of thermalexpansion, κ is the thermal conductivity, $\rho_\infty$is the density of the fluid, $C_p$is the specific heat at constant pressure and $T_\infty$is the temperature of the ambient fluid and ν the kinematics viscosity of the fluid. From the continuity equation (1) we consider the velocity normal t the plate is of the form $v=-V_0$.Now we introduce the following transformations to the equation (2) and (3)

4.Similarity Transform Parameter

$$Y=\frac{yV_0}{v}, U=\frac{u}{U_0}, \theta=\frac{(T-T_\infty)}{(T_0-T_\infty)}, Gr_T=\frac{g\beta(T_0-T_\infty)v}{U_0V_0^2}, Gr_S=\frac{g\beta(C_0-C_\infty)v}{U_0V_0^2}, , P_r=\frac{\mu C_p}{\kappa}, M=\frac{\sigma v H_0^2}{\rho_\infty V_0^2} (5)$$

5. The Transformed Governing Equation

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}-\left(M+\frac{1}{D_a}\right)U+Gr_T\theta++Gr_S\emptyset+Fu|u|=0 \qquad (6)$$

$$\frac{d^2\theta}{dY^2}+Pr\frac{d\theta}{dY}=0 (7)$$

$$\frac{d^2\emptyset}{dY^2}+Sc\frac{d\emptyset}{dY}=0 (8)$$

with the boundary conditions

$$U=0;\theta=1,\emptyset=1 \; at \; Y=0$$

$$U\to 0,\theta\to 0,\emptyset\to 0 \; at \; Y\to\infty \quad (9)$$

6. Analytical Solution

The equations (6) , (7) and (8) with the boundary condition (9) are simply ordinary differential equation. We can find the solution of that equation (6), (7) and (8) as the following form equation (10) , (11) and (12) respectively

$$U=\left(\frac{Gr_T}{Pr^2-Pr-Q}\right)\left(e^{\frac{-1+\sqrt{1+4Q}}{2}Y}-e^{-PrY}\right)+\frac{Gr_S}{Sc^2-Sc-Q}\left(e^{\frac{-1+\sqrt{1+4Q}}{2}Y}-e^{-ScY}\right),$$

$$Where \; Q=\left(\mathrm{M}+\frac{1}{D_a}\right) \quad (10)$$

$$\theta=exp(-PrY) \quad (11)$$

$$\emptyset=exp(-ScY) (12)$$

We also find the skin friction , rate of concentration and the rate of heat transfer as follows

$$C_f=\left(\frac{dU}{dY}\right)_{Y=o}=\left(\frac{Gr_T}{Pr^2-Pr-Q}\right)\left(\frac{-1+\sqrt{1+4Q}}{2}+\mathrm{Pr}\right)+\frac{Gr_S}{Sc^2-Sc-Q}\left(\frac{-1+\sqrt{1+4Q}}{2}+Sc\right) (13)$$

$$Nu=-\left(\frac{d\theta}{dY}\right)_{Y=o}=Pr (14)$$

$$Sh=-\left(\frac{d\emptyset}{dY}\right)_{Y=o}=Sc (15)$$

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7. Result and Discussion

The following section demonstrates the influence of different parameter in MHD free convectivefluid flow from a vertical flat plate in porous media with uniform viscosity and uniform thermal conductivity, in presence of uniform transverse magnetic field along an impermeable vertical flat plate. The similarity solution used for the demonstration the non-dimensionalize velocity, temperature and concentration are discussed In this section we discuss the relation taken the low prandtl number(Pr) i.e a case of liquid metals .The different prandtl number (Pr) is defined as 0.92 for ammonia 0.72 for air ,0.05 for lithium and 0.004 for sodium at 649 deg/C. we calculated the skin friction and the rate of heat transfer in the equation no.(13) and (14). For increasing values of Prandtlnumber , the local skin friction decreases monotonically. The skin friction increase at the decreasing values of the magnetic field parameter, M and increasing values of the Grashofnumber(Gr).

In the Fig:2 We demonstrate , effect of the Grashofnumber(Gr).,

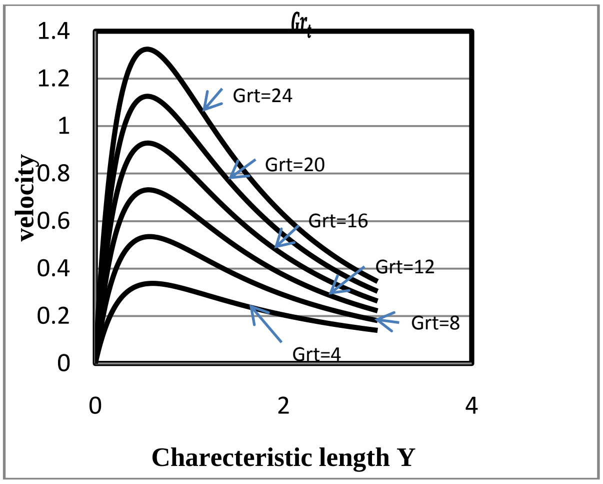

Fig2:Velocity profile for different values of Gr_t with $Gr_s = 2$,Pr = 0.72, M=1 , Da=0.1 .and Sc=0.2

Here we have taken a case in which the darcy number Da=0.1. Pr=0.72, $Gr_s = 2$and M=1. under these parameters. Pr =0.72 means there is study of air in porous zone, as we increase the value of Grashof number(Gr_t) as 4,8,12,16,20,and 24 the velocity being increasing in showing in the fig here we found that velocity reaches maximum around 0.5 in each of the case which is clearly shown in the above graph.

In the fig 3, we shows the effect of magnetic filed strength parameter say hartmenn number (M). in Fig 3 the velocity of the fluid flow is increases the hartmenn number M = 1,2, 3, 4, 5 as we fix the value of Darcy number Da=0.1 again here we found the demonstratively that velocity is reaches at the maximum around the characteristic length 0.5, but here variation in the velocity corresponding to hartmen number is nearly very small , as compare to the previous case. Orthis is a case of the zone which is nearer the porous zone magnetic effect increase the velocity of moving fluid. The graph shows the profile of velocity in which the difference between the slope is less its shows that the there is a slight improvement in the velocity w.r.t the magnetic field parameter(M).

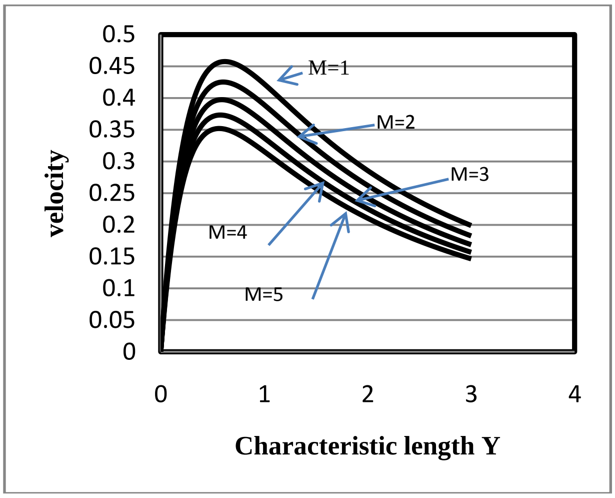

Fig 3: Velocity profile for different values of M with Pr=0.72, $Gr_t = 5$, $Gr_s = 3$,Sc=0.2 and Da=0.1

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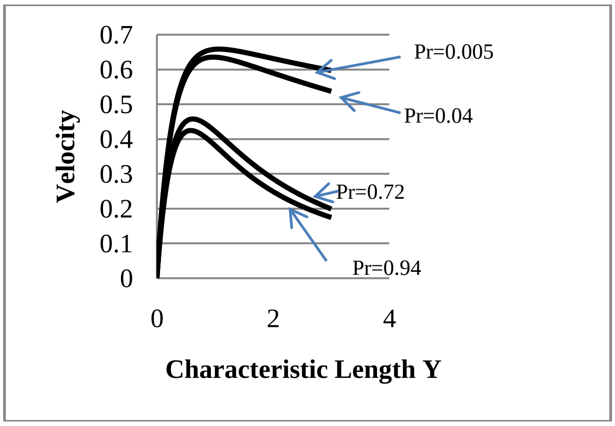

Fig :4 Velocity profile for the different values of Pr with $Gr_t = 5$, $Gr_s = 3$,Sc=0.2 M =1 and Da=0.1.

In the fig 4 we saw the velocity profile of the different fluid like ammonia , air, lithium and sodium ,In the porous zone, the graph of the different fluid is shows that the velocity of ammonia and air is almost is equal for the fixed of Darcy number Da=0.1 ,i.e near the viscous zone the velocity is same , as we decrease the value of Prandtl number (Pr) i.e moving towards the lithium and sodium here we found that the velocity of lithium is less than the sodium at the Garsh of number Grt=5. And magnetic number M=1
But as we see the dependency of the fluid flow at different Pr we found that maximum velocity of each fluid is recorded near 0.5 ,which might be different in case of Pr =.05, and .004, here it is in the neighborhood of the 0.5, but higher then air and other fluid. And another observation is that as we increased the Prandtl number the velocity being decreases.

In the present case media permeability is taken in to account

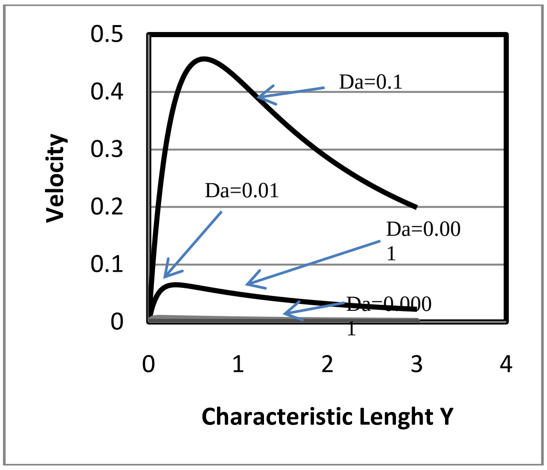

Fig: 5 Velocity profile for the different values of Da with $Gr_t = 5$,,$Gr_s = 3$, M =1 , Sc=0.2 and Pr=0.72

In the fig 5 the effect of the different darcy number has been ploted , here we see that the as we increase the darcy number the velocity is being decreases which is effective for the porous media here in the special case as Da=0.1 , the velocity is very high which is shows that the as we entered in the viscous media , velocity is higher than the porous media , now the darcy number is going decreases as Da=0.1, 0.01, 0.001, 0.0001 we saw that the velocity is vanish in the case of Da=0.001, i.e there is velocity for the air floe , i.e flow becomes zero in the porous media . when the Grt=5 and M=1.

In the fig 6. We demonstrate the heat transfer rate i.e temperature profile which is best based on the Pr .

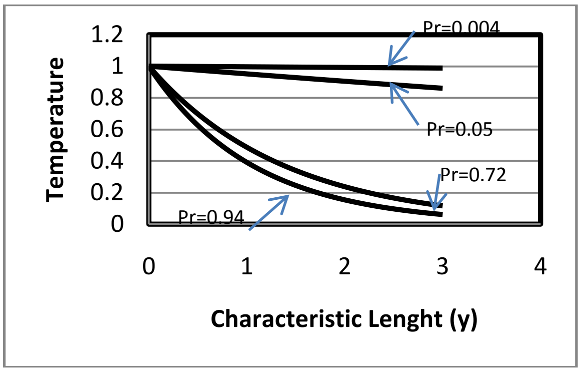

Fig: 6 Temperature profile for the different values of Pr.

Here temperature is dependent on the prandtl number only, so here we seems that as decrease the value of Pr the heat transfer rate being decreases. In case of sodium there is a less transfer as compare to the ammonia , and fig shows that there is no heat transfer is the fluid is sodium ,but in case of air heat transfer rate is better than the lithium and sodium , and the heat transfer rate for the ammonia is better i.e this fluid is good for the heat transfer .

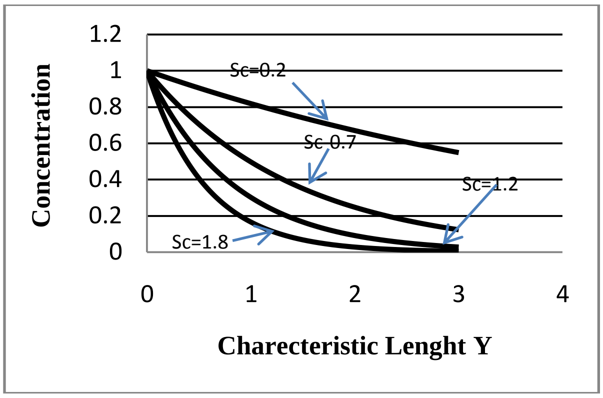

Fig :7 Concentration profile for the different values of Sc

In the fig7. We demonstrate concentration profile which is best based on the Sc .Here concentration is dependent on the schimidt number only, so here we seems that as decrease the value of sc the mass transfer rate being decreases.

Now the nusselt number is totally depended on the Pr so as we increase the Pr value of it because increase.

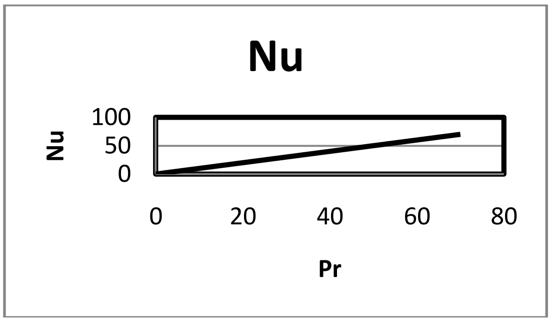

Fig: 8 Effect of nusselt number for different values of Pr

Here concentration dependent on the Schimdt number only, so here we seems that as decrease the value of Sc the heat transfer rate being decreases. Now the sherhood number is totally depended on the Sc so as we increase the Sc value of it because increase.

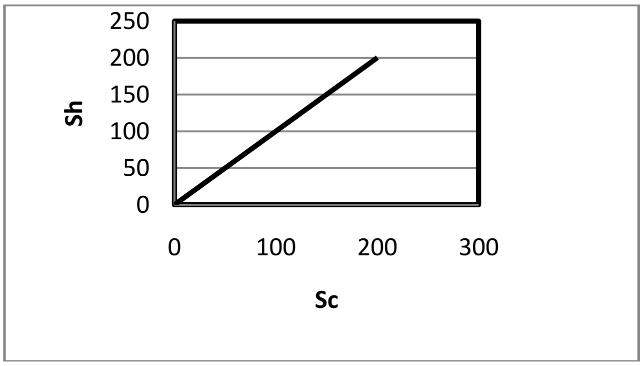

Fig: 9 Effect of Sherhood t number for different values of Sc

Fig 10 the temperature of the fluid flow is decreases as we increase the schimdt number sc=0.4, 0.8, 1.2, 1.6,2.0,2.4 as we fix the value of Darcy number Da=0, M=1 Pr=0.72, $Gr_t = 5$, and $Gr_s = 3$. Again here we found the demonstratively that temperature reaches at the maximum around the characteristic length 0.5..

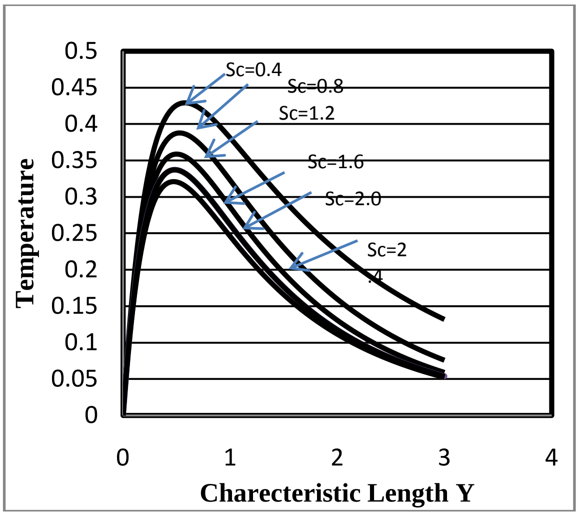

Fig :10 Temperature profile for the different values of Sc with $Gr_t = 5, Gr_s = 3$, M =1 and Da=0, Pr= 0.72.

Here in fig 11 we have taken a case in which the darcy number Da=0.1. Pr=0.72, $\mathrm{Gr_t} = 5$ and M=1. under these parameters. Pr =0.72 means there is study of air in porous zone, as we increase the value of Grashof number($\mathrm{Gr_s}$) as 3,6,9,12,15,18and 21, the temperature being increasing in showing in the fig here we found that temperature reaches maximum around 0.5 in each of the case which is clearly shown in the above graph.

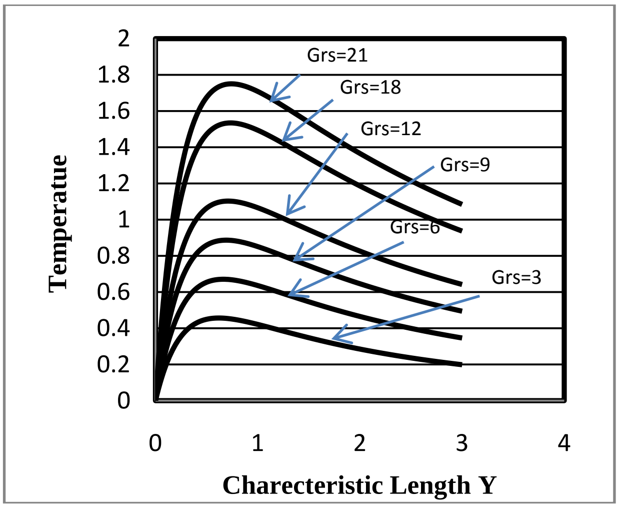

Fig :11 Temperature profile for the different values of Gr_s with $Gr_t = 5$, Sc=0.2, Pr=0.72, M =1 and Da=0.1.

In this table we calculate the following values at F=0 , where forchimer number is zero

I. TABLE

$\mathrm{Pr/Gr_s}$	2	4	6	8	10	12	14	16
0.72	-2.77	-3.49	-4.22	-4.95	-5.67	-6.4	-7.13	-7.85
0.94	-2.89	-3.62	-4.35	-5.07	-5.8	-6.53	-7.25	-7.98
0.05	-2.49	-3.22	-3.95	-4.67	-5.4	-6.13	-6.85	-7.58
0	-2.48	-3.21	-3.93	-4.66	-5.39	-6.11	-6.84	-7.57

$\mathrm{Gr_t} = 5$ Da=0.1 Sc=0.2 M=1

In this table we calculate the following values at F≠0 , where forchimer number is non zero

II. TABLE

Pr/Gr_t	2	4	6	8	10	12	14	16
0.72	-1.907	-2.723	-3.54	-4.35	-5.17	-5.99	-6.807	-7.623
0.94	-1.957	-2.824	-3.691	-4.55	-5.42	-6.29	-7.16	-8.028
0.05	-1.797	-2.503	-3.21	-3.917	-4.62	-5.33	-6.037	-6.744
0.004	-1.791	-2.492	-3.194	-3.897	-4.59	-5.29	-5.998	-6.7
	$Gr_s = 3$	Da=0.1	Sc=0.2	M=1				

8. Conclusion

In this paper, the problem of magneto hydrodynamic double diffusive convection flow along a vertical flat plate is investigated in porous medium. The local non-similarity equations governing the flow for the case of uniform viscosity and thermal conductivity are developed. The numerical computations were carried out only for the case of assisting flow for the fluids having low Prandtl number appropriate for liquid metals (Pr 0.92 for ammonia, 0.72 for air, 0.05 for lithium and 0.004 for sodium at 649^0c) and Schimdt number. we take the values 0.2 , 0.4, 0.8,1.2, 1.6, and 2.0.

The results thus we obtained for skin friction and the rate of heat transfer coefficient are presented in tabular form in the case of different properties of the liquid metals. The velocity profiles and the thermal conductivity profiles are givengraphically in the in the case of constant viscosity. Finally, followings may beconcluded from the throughout present investigations:

1. For increasing values of Prandtl number, the local skin friction decreasesmonotonically.

2. The skin friction increase at the decreasing values of the magnetic fieldparameter, M and increasing values of Grashof number Gr.

3. Profiles for the velocity as well as the thermal conductivity decrease due to theincreasing values of the Prandtl number, Pr.

4. Profiles for the velocity as well as the concentration decrease due to the increasing values of the Schimdt number Sc

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