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An EPQ Model using Weibull Deterioration for Deterioration Item with Time Varying Holding Cost

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***Abstract*— In recent era deterioration rate is pivotal to determine optimal quantity. Many researchers have been developed various models assuming various types of deterioration rate. In this paper we have developed the inventory model when the deterioration rate follows three parameter Weibull distributions, with demand and production rate is constant. Holding cost is considered as time varying function and shortages are not allowed. The numerical example supported to the developed model.**

***Keyword*— Deteriorating items, EPQ model, Time varying holding cost, Weibull distribution.**

I. Introduction

So for inventory model are used to determine optimal stock of items to satisfy the future demand and also to decide the optimality of total cost. Very wide work has been done by this point of view. Many researchers have been worked on optimal order quantity for deteriorating commodities.

Deterioration is the most affecting parameter for commodities to maintain inventory management. This is either direct or partially loss in profit. The control and maintenance of inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over time. In general, deterioration is defined as change, damage, decay, spoilage, obsolescence, evaporation, pilferage and loss of utility or loss of original value in a commodity from the original one. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.

Aggarwal and Bahari-Hashani (9), studied a model assuming that items are deteriorating at a constant rate, in which, the production rate is known but can vary from one period to another period over a finite planning period. Pakkala and Achary (8) developed a production inventory model of deteriorating items with two storage facilities and a constant demand rate. Sunil Kawale (3) considered two rates in production process for production inventory problem under constant deterioration rate.

Begum, Sahoo, Sahu and Mishra (1) developed an instantaneous replenishment policy for Weibull deteriorating items with price dependent demand and C. K. Tripathy and U. Mishra (2) used Weibull distribution for deterioration in their research for price dependent demand but holding cost is time varying. Jong-Wuu Wu at el. (5) developed, inventory model for time-varying demand with shortages under Weibull deterioration rate.

Instead to determine Economic Order Quantity some researchers focus their work to control the production to optimize total cost in inventory control. In this regard, the work has been concentrated to obtain the Economic Production Quantity in inventory management. Misra (6), firstly introduced this Economic Production Quantity model for deteriorating item. C. Sugapriya and K. Jeyaraman (4), considered the EPQ model for non-instantaneous deterioration. Choi and Hwang (7) worked in their research for optimization of production planning for deteriorating items to minimize the total cost function over a finite planning period.

In this paper, we consider EPQ model for deteriorating items, where a single product subject to instantaneous deterioration rate follows three parameter Weibull distributions. Under a production inventory policy, holding cost are expressed as linearly increasing function of time. Each unit of the item is provided with price discount decayed units.

II. Assumption and Notation

The fundamental assumptions of the model are as follows:

a. The demand rate for the product is known and finite.
b. Shortage is not allowed.
c. An infinite planning horizon is assumed.
d. Once a unit of the product is produced, it is available to meet the demand.
e. Once the production is terminated the product starts deterioration and the price discount is considered.
f. The deterioration follows the three-parameter Weibull distribution.
g. There is no replacement or repair for a deteriorated item.

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The notations that are employed here:

- $I_1(t)$: inventory level for product during the production period, i.e., $0 \le t \le T_1$.
- $I_2(t)$: inventory level of the product during the period when there is no production, i.e. $T_1 \le t \le T_2$.
- I (M) : maximum inventory level of the product.
- p : production rate per unit time.
- D : actual demand of the product per unit time.
- A : Set up cost.
- h= a+bt : inventory carrying cost per unit time, where a and b are positive constants.
- K : production cost per unit.
- r : price discount per unit cost.
- T : optimal cycle time.
- T_1 : production period.
- T_2 : time during which there is no production of the product i.e., T_1= T- T_2.
- TC (T) : total cost/unit time.

III. MATHEMATICAL FORMULATION AND SOLUTION

The distribution of the time to deterioration is random variable following the three-parameter Weibull distribution. The probability density function for three-parameter Weibull distribution function f(t) is given by.

$$f(t) = \alpha\beta(\mu-\gamma)^{\beta-1} e^{-\alpha(\mu-\gamma)^{\beta}}$$

Where, f (t) =Probability density function

- α = Scale parameter, α > 0
- β = Shape parameter, β > 0
- μ =Time of deterioration, μ> 0
- γ =Location parameter, μ≥ γ

The instantaneous rate of deterioration of the deteriorated inventory at time t, θt , can be obtained from $\theta t = \dfrac{f(t)}{1-F(t)}$ where $F(t)$ is the cumulative distribution function which is equal to $1-e^{-\alpha(\mu-\gamma)^{\beta}}$ for the three-parameter Weibull distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t) = \alpha\beta(\mu-\gamma)^{\beta-1}$ the probability density function represents the deterioration which may have a decreasing, constant or increasing rate of deterioration. The three-parameter Weibull distribution is suitable for items with any initial value of the rate of deterioration. (Begum at el. (1)).

Initially inventory is 0 at time t. The production starts simultaneously supply is also start. The production stops at t = T_1 where the maximum inventory level I(M) is reached. In the interval [0, T_1] the inventory built up at a rate is p - d. There is no deterioration in this interval. The production stop at time T_1 and then deterioration starts and supply is also with discounted rate. The inventory is finitely decreasing up to until inventory reaches zero in the time interval [T_1, T_2]. As soon as inventory level zero the next production run start. The inventory level of the product at time t over period [0, T] can be described by the following equations.

$$\frac{dI_1(t)}{dt} = p - d\,, \qquad \text{for } 0 \le t \le T_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \alpha\beta(\mu-\gamma)^{\beta-1} I_2(t) = -d\,, \text{ for } T_1 \le t \le T_2 \tag{2}$$

The boundary conditions associated with these equations are: $I_1(0)$ =0, I_2 (T_2)= 0 gives solution of (1) and (2)

$$I_1(t) = (p-d)t\,, \qquad \text{for } 0 \le t \le T_1 \tag{3}$$

$$I_2(t) = \frac{d\left[\exp\left\{\left(\alpha\beta\left(\mu-\gamma\right)^{\beta-1}\right)t\right\} - 1\right]}{\exp\left\{\left(\alpha\beta\left(\mu-\gamma\right)^{\beta-1}\right)t\right\}}\,, \qquad \text{for } T_1 \le t \le T_2 \tag{4}$$

Production Cost: The production cost per unit time is given by

$$PC = pk\frac{T_1}{T} \tag{5}$$

Setup Cost: The setup cost per unit time is given by

$$SC = \frac{A}{T} \tag{6}$$

Holding Cost: The holding cost per unit time is \

$$HC = \frac{1}{T}\left[\int_0^{T_1}(a+bt)I_1(t)dt + \int_{T_1}^{T_2}(a+bt)I_2(t)dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T_1}(a+bt)(p-d)tdt + \int_{T_1}^{T_2}(a+bt)\left(\frac{d\left[\exp\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)t-1\right]}{\exp\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)t}\right)dt\right] \tag{7}$$

Using Tailor's expansion and Assuming, $\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T<1$ and neglecting the terms of power greater than or equal to 2, resulting Equation (7) is as follows.

$$HC = a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T} \tag{8}$$

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Deterioration cost: The deterioration cost is in period $[T_1, T_2]$.

$$DC = \frac{k}{T}\left[I_2(0) - \int_{T_1}^{T_2} d.dt \right]$$

$$= \frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T_2^2}{2T} \qquad (9)$$

Price discount: Price discount is offered as a fraction of production cost for the units in the period $[0, T_2]$

$$PD = \frac{kr}{T}\int_{T_1}^{T_2} d.dt = \frac{krdT_2}{T} \qquad (10)$$

Therefore the average total cost per unit time is given by:

$$TC(T) = PC + SC + HC + DC + PD$$

$$= pk\left[\frac{T_1}{T}\right] + \frac{A}{T} + a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T}$$

$$+\frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)T_2^2}{2T} + \frac{krdT_2}{T} \qquad (11)$$

To solve this equation let us express T_1 and T_2 in terms of T. So T_1 and T_2 is as follows

$$T_1 = \frac{d}{(p-d)}\left(T_2 + \frac{(\alpha\beta(\mu-\gamma)^{\beta-1})T_2^2}{2}\right)$$

$$T_2 = \frac{(p-d)}{p}T$$

$$T = T_1 + T_2 = \frac{p}{d}T_1$$

Therefore equation (11) becomes

$$TC(T) = pk\left[\frac{d}{p}\right] + \frac{A}{T} + a(p-d)\frac{d^2T}{2p^2} + b(p-d)\frac{d^3T^2}{3p^3} + \frac{ad(p-d)^2T}{2p^2}$$

$$+\frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)(p-d)^2T}{2p^2} + \frac{krd(p-d)}{p} \qquad (12)$$

12)
Our objective is to minimize the total cost per unit time TC (T). Therefore differentiate TC (T) with respect to T and set the result equal to zero.

We get

$$\frac{dTC(T)}{dT} = -\frac{A}{T^2} + a(p-d) + \frac{d^2}{2p^2} + b(p-d)\frac{2d^3T}{3P^3} + \frac{ad(p-d)^2}{2p^2}$$

$$+\frac{kd\left(\alpha\beta(\mu-\gamma)^{\beta-1}\right)}{2p^2} = 0 \qquad (13)$$

$$\frac{d^2TVC(T)}{dT^2} = \frac{2A}{T^3} + b(p-d)\frac{2d^3}{3P^3} > 0$$

i.e. the second derivative is found to be positive.

IV. EXAMPLE

A = \$500/set up, p = 50units/unit time, d = 15units/unit time, a = 1, b =0.05, k = \$20/unit time, α = 0.02, β = 2, γ = 1, μ = 3, T = 2.35unit time, TC (T) = \$541.12, Total cost, deterioration cost are computed for five different sets of deterioration rates (α, β, γ). The results are compared. The following graphs show the total cost and deterioration cost with reference to deterioration rates (α, β, γ)

Table I: Different values of (α)

α_1	α_2	α_3	α_4	α_5
0.02	0.04	0.06	0.08	0.1

Table II: Different values of (β)

β_1	β_2	β_3	β_4	β_5
1.0	2.0	3.0	4.0	5.0

Table III: Different values of (γ)

γ_1	γ_2	γ_3	γ_4	γ_5
0.5	1.0	1.5	2.0	2.5

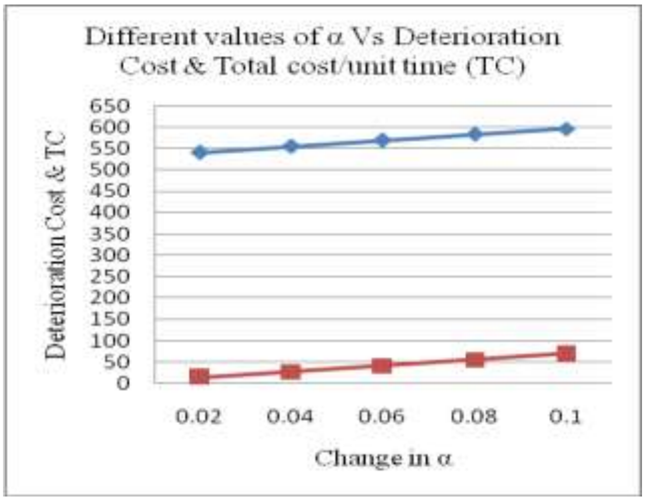

Figure.1: Comparison of DC & TC for parameter α

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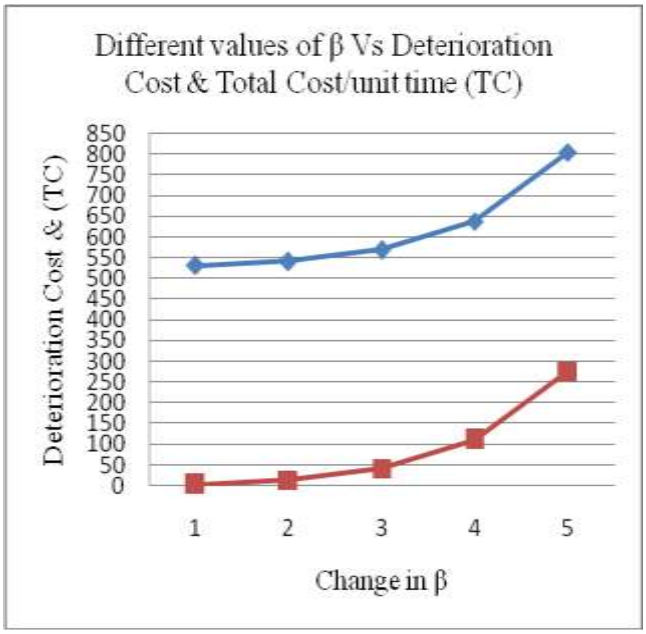

Figure.2: Comparison of DC & TC for parameter β

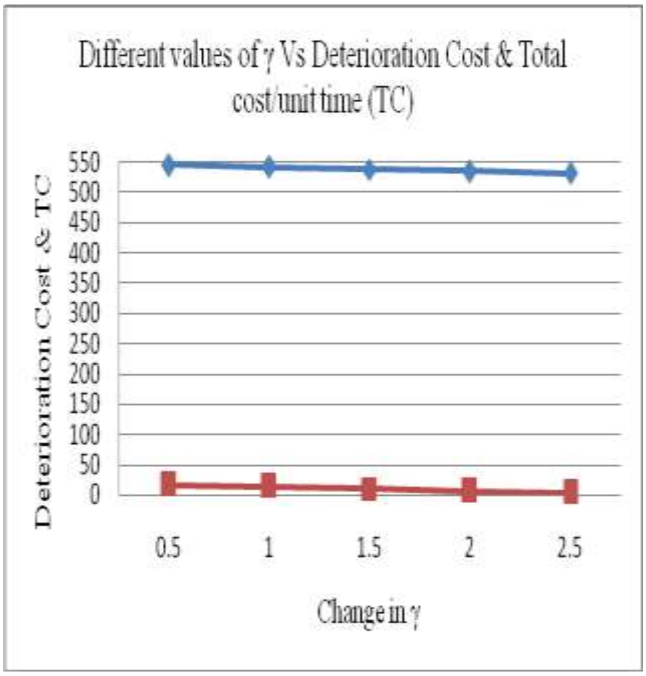

Figure.3: Comparison of DC & TC for parameter γ

In above figures, it shows the variation in total cost and deterioration cost as per change in parameter α, β and γ. From this we can obtain the optimal total cost by selecting appropriate values of α, β and γ.

Figure.1 shows that variation in scale parameter α, total cost (TC) increase as well as deterioration cost also increases. Figure.2shows that variation in shape parameter β, total cost (TC) and deterioration cost both increases.Figure.3 shows that the reverse effect of location parameter γ that is variation in γ, total cost (TC) and deterioration cost decreases.

Hence from above figure the total cost is obtained with values of α, β, and γ the optimal total cost obtained is $541.12 for α = 0.02, β = 2 and γ = 1.

V. CONCLUSION

For the single item production problem, under finite replenishment system and the Weibull deterioration with time varying holding cost an EPQ model has been developed in which price discounts offered for deteriorated items. This helps to optimize the total cost for deterioration which is follows Weibull distribution.

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