

Design of FIR Digital Filters using Least Square Method Based On Neural Network

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Abstract—This paper presents an approach for digital filter design based on least squares neural network. Here the concept of minimization of sum of the square errors between the amplitude response of the desired filter and the filter designed by neural network is used. A couple of examples are simulated and the result of design examples shows that the proposed neural network method is capable of achieving better performance for the filter design.

Keywords- Least Square Neural networks, finite impulse response (FIR) filter, square error.

I. INTRODUCTION

In past two decades, the problem of designing of Finite Impulse Response (FIR) filter is solved by various techniques such as windowing methods, frequency sampling method [1],[2] and the optimization methods. FIR filters are used when there is a requirement for a linear phase characteristics within the pass-band of the filter. The design procedure of FIR filter is easier and posses exact linear phase, constant time or group delay. Its stability and flexibility in the desired frequency response were used in many signal processing applications, such as wavelet transform [3], channel equalization [4] etc. Both the window method and frequency sampling method belong to the usual designing approaches of FIR filters and cannot accurately control border frequencies of pass-band and stop-band in the practical applications as a result, many researchers have presented some optimal design approaches.

This paper presents a new least square method neural network algorithm to design the linear phase FIR filter. The designing is entirely based on the approximation of a magnitude response. The main idea is to minimize the sum of squared error between the amplitude responses of the desired FIR filter and that of designed by the single layer feed-forward neural network algorithm. In section II, linear phase FIR filter is described. Then in section III, the neural network model is discussed. After that, the design examples

and simulation results are described in section IV. Finally, the conclusions are stated in section V.

II. LINEAR PHASE FIR FILTER

The frequency response $H(e^{j\omega})$ of a FIR filter with length N can be expressed as

$$H(e^{j\omega}) = \sum_{i=0}^{N-1} h(m)e^{-jm\omega} \dots\dots\dots (1)$$

Where, $h(m)$ is the impulse response. If N is odd integer, and

$$h(m) = h(N - 1 - m) \dots\dots\dots (2)$$

The magnitude response can be expressed as

$$H(\omega) = \sum_{i=0}^{\frac{N-1}{2}} a(m)\cos(m\omega) \dots\dots\dots (3)$$

Where,

$$a(0) = h\left(\frac{N-1}{2}\right), \quad a(m) = 2h\left(\frac{N-1}{2} - m\right),$$

$$m = 1, 2, \dots, \frac{N-1}{2}$$

Now, to design the linear phase FIR filter we have to find coefficients $a(m)$ in (3) such that the magnitude response $\{H(\omega)\}$ is close to a given magnitude response in some optimal sense.

III. NEURAL NETWORKS MODEL

Figure 1 illustrates the proposed neural network model for FIR filter design with linear activation function based on cosine basis. The output of neural network from (3) can be expressed as

$$H_N = C^T A \dots\dots\dots (4)$$

In vector form,

$$H_N^k = C^T A_k \dots\dots\dots (5)$$

Where, H_N^k is the output vector of the neural network, C^T is the transformation matrix of the hidden units of neural network, A_k is the weight vectors of the neural network defined as

$$A = [a_0, a_1, \dots, a_{\frac{N-1}{2}}]^T,$$

$$\omega = [\omega_0, \omega_1, \dots, \omega_{\frac{N-1}{2}}]^T,$$

$$H_N = [H_N(\omega_0), H_N(\omega_1), \dots, H_N(\omega_{\frac{N-1}{2}})]^T,$$

$$C = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \cos(\omega_0) & \cos(\omega_1) & \dots & \cos(\omega_{\frac{N-1}{2}}) \\ \vdots & \vdots & \dots & \vdots \\ \cos(\frac{N-1}{2}\omega_0) & \cos(\frac{N-1}{2}\omega_1) & \dots & \cos(\frac{N-1}{2}\omega_{\frac{N-1}{2}}) \end{bmatrix}$$

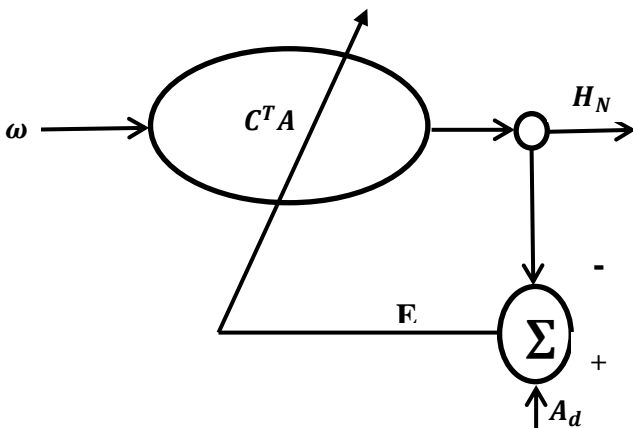


Figure 1. Neural Networks Model

Now, we define error function as

$$E(k) = A_d - H_N^k \dots\dots\dots (6)$$

Where, $E(k)$ is the error vector between the desired and actual outputs of the neural network.

And the performance index P as

$$P(E) = \frac{1}{J} \sum_{n=1}^{\frac{N-1}{2}} W E^2(l) \dots\dots (7)$$

Where, $W > 0$, and $J = \sum_{n=1}^{\frac{N-1}{2}} W$.

To minimize P we recursively calculated A as

$$A_{k+1} = A_k - \eta \frac{\partial P}{\partial A_k} \dots\dots\dots (8)$$

Where, η is a learning rate. After differentiating (7), we have

$$\frac{\partial P}{\partial A_k} = \frac{\partial H_N^k}{\partial A_k} \cdot \frac{\partial E(k)}{\partial H_N^k} \cdot \frac{\partial P}{\partial E(k)} = -CE(k) \dots\dots\dots (9)$$

Substituting (9) into (8), we have

$$A_{k+1} = A_k + \eta CE(k) \dots\dots\dots (10)$$

In order to ensure the convergence of neural network, it is important to select a proper learning rate η .

The design procedure is summarized as follows:

1. Sample the desired magnitude response $A_d(\omega)$ uniformly to obtain the training sample set of the neural network, the frequencies sample point is $\omega_l = \frac{2\pi}{N-1} l$, $l = 0, 1, \dots, \frac{N-1}{2}$.
2. Produce an initial random weights vector A using random function in MATLAB. Define an arbitrary small positive real number ϵ , and select learning rate η .
3. Specify weight coefficient vector W according to the following scheme:

$$W = \begin{cases} w_p > 1 & \text{in the passband} \\ w_{pc} \geq 1 & \text{when } \omega_l \text{ on the passband edge} \\ w_t = 1 & \text{in the transition band} \\ w_s \geq 1 & \text{in the stopband} \end{cases}$$
4. Produce the transformation matrix C of the hidden units of neural network. Produce new predicted output vector H_N of the neural network using (4).
5. Calculate error vector $E(k)$ and P via (6) and (7).
6. Update the weight vector A_{k+1} using (8).
7. If $P > \epsilon$, go to step 4, otherwise, stop the training of the neural network training.

IV. DESIGN EXAMPLES

In this section, design examples are considered to illustrate the effectiveness of the proposed design algorithm. The design examples are simulated on MATLAB 7.1.

Example 1: In this design a linear phase low pass filter is simulated with the parameters as given. The design parameters are as following $L=189$, $N = 37$, $\omega_p = 0.24$, $\omega_s = 0.3$, neurons = 19 . The proposed algorithm magnitude response as shown in Figure1 The maximum ripples in pass-band and stop-band for proposed method are 0.0390 and 0.0632 respectively and is shown in Table I.

V. CONCLUSION

This paper suggests the neural network technique for designing linear phase FIR filter. A design example is given that, we have seen that the proposed neural network technique gives better result. The method is not involved in operation of inverse matrix, and the pass-band and stop-band ripples in the proposed method is minimum. This shows that proposed method is effective and gives better approach to design FIR filters.

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TABLE I. PERFORMANCE OF LOW-PASS FIR FILTER DESIGN.

Type of Filter	L	N	$\omega_p(\pi)$	$\omega_s(\pi)$	δ_p	δ_s	Method	Neurons
Low-pass (Example 1)	189	37	0.24	0.3	0.0390	0.0632	LSM	19

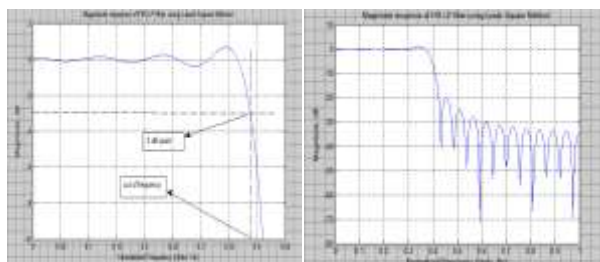


Fig. 1 Magnitude response of Low pass filter



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