ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 8, August 2013

A window function for FIR filter design with an improved frequency response and its comparison with the Taylor window

Ankan Bhattacharya[1]

***Abstract*—In this paper a window function has been defined and simulated. In the initial section the frequency responses of the four basic types of Finite Impulse Response (FIR) filters have been simulated using the window function. Then the FIR filter responses are compared with those using the Taylor window function. Improved frequency responses have been observed in each case in perspective of the stop band attenuation level which is justified by relevant simulations and plots. The frequency response of the new window function will be compared with that of other common window functions in near future with generalized and quantized experimental results.**

***Index Terms*—FIR, window function, Taylor window, frequency response, stop band attenuation**

I. INTRODUCTION

Digital filters can be classified either as finite duration unit impulse response (FIR) filters or infinite duration unit impulse response (IIR) filters. In FIR systems, the impulse response sequence is of finite duration or in other words, if the impulse response of a digital filter is determined for some finite number of sample points, then these filters are known as Finite Impulse Response or FIR digital filters.

Let us consider that the digital filter which has to be designed is to be designed has the frequency response, $H_d(\omega)$. This is also known as desired frequency response. Let the corresponding unit sample response be $h_d(n)$ *i.e.* [1]-[3]

$$H_d(\omega) = \sum_{0}^{\infty} h_d(n)\, e^{-j\omega n} \quad (1)$$

[1]***Ankan Bhattacharya**, Assistant Professor, Electronics and Communication Engg. Dept., Mallabhum Institute of Technology, Bishnupur, P.O.: Gosaipur, P.S.: Bishnupur, Dist.: Bankura, West Bengal, India.*

II. DESIGN METHODS

There are mainly three methods used for FIR filter design:

- FIR filter design using windows.
- FIR filter design using frequency sampling method.
- Optimal or minimax FIR filter design.[1]-[3]

III. WINDOW METHOD

In signal processing, a window functionis a mathematical function that is zero-valued outside of some chosen interval. For instance, a function that is constant inside the interval and zero elsewhere is called a rectangular window, which describes the shape of its graphical representation. When another function or waveform/data-sequence is multiplied by a window function, the product is also zero-valued outside the interval: all that is left is the part where they overlap; the "view through the window". Applications of window functions include FIRfilter design. In typical applications, the window functions used are non-negative smooth "bell-shaped" curves,though rectangle, triangle, and other functions can be used. [4], [5]

Many different window functions are used in Digital Signal Processing (DSP) for FIR filter design. Some of them are Rectangular window, Hamming window, Hanning window, Triangular window, Bartlett window, Kaiser window, Taylor window etc. Previous works of the author include generation of a new window function and its comparison with the Hamming and the Bartlett-Hann windows respectively. [15], [16]

IV. TAYLOR WINDOW

The Taylor window function for Total number of samples, N = 60 is shown in Fig. 1. The sample points (*n*) are indicated along *x-axis* and the corresponding amplitude levels along *y-axis* respectively.

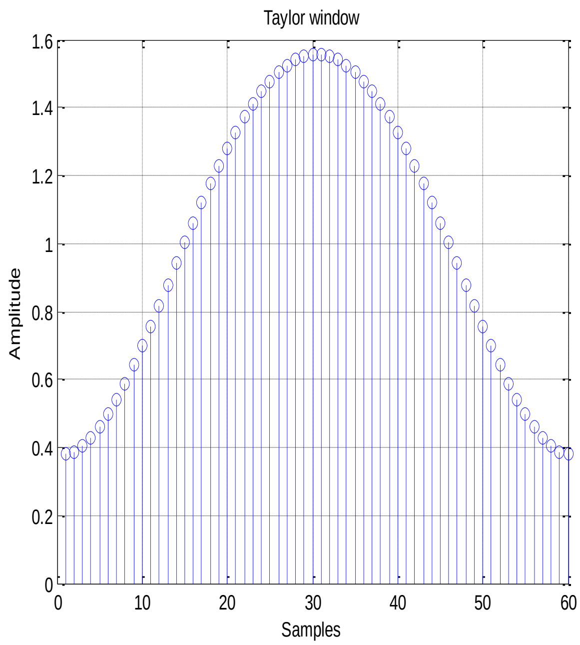


Fig. 1 *Taylor window*

V. PROPOSED WINDOW

In this context, a new window function has been proposed which is defined as,

$$w(n) = a - b\cos(2\pi n/N) + c\cos(4\pi^{x} n/N) - d\cos(6\pi^{y} n/N) + e\cos(8\pi^{z} n/N) \quad (2)$$

where,

n = Number of samples

N = Total number of sample points

and

$0 \leq n \leq N$

and the co-efficients are,

a = 0.3650

b = 0.4399

c = 0.2770

d = 0.0830

e = 0.0010

and

x = -2.19

y = -2.00

z = -2.00

The window function has been generated using MATLAB and is displayed in Fig. 2

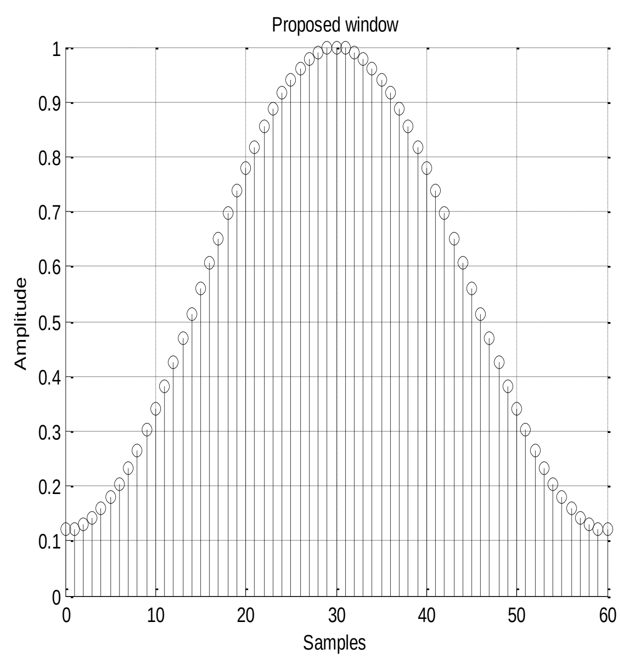


Fig. 2 *Proposed window*

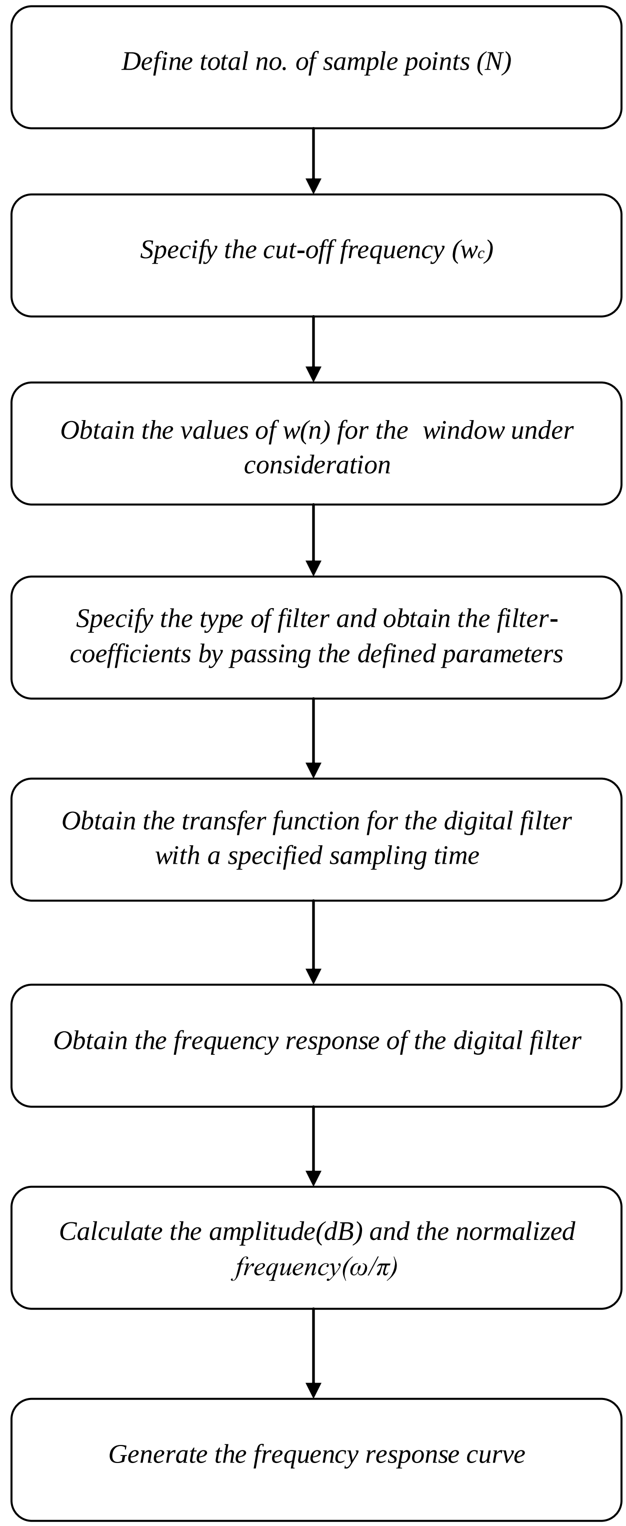
The values of *w(n)* for $0 \leq n \leq 60$ obtained from MATLAB simulation are displayed in TABLE I.

TABLE I

Values of w(n) for $0 \leq n \leq 60$

Sl. no.	w(n)	Value	Sl. no.	w(n)	Value
1	w(0)	0.1201	32	w(31)	0.9975
2	w(1)	0.1225	33	w(32)	0.9903
3	w(2)	0.1297	34	w(33)	0.9784
4	w(3)	0.1416	35	w(34)	0.9619
5	w(4)	0.1581	36	w(35)	0.9410
6	w(5)	0.1790	37	w(36)	0.9159
7	w(6)	0.2041	38	w(37)	0.8870
8	w(7)	0.2331	39	w(38)	0.8544
9	w(8)	0.2657	40	w(39)	0.8186
10	w(9)	0.3014	41	w(40)	0.7800
11	w(10)	0.3401	42	w(41)	0.7390
12	w(11)	0.3811	43	w(42)	0.6960
13	w(12)	0.4241	44	w(43)	0.6515
14	w(13)	0.4686	45	w(44)	0.6060
15	w(14)	0.5140	46	w(45)	0.5601
16	w(15)	0.5600	47	w(46)	0.5141
17	w(16)	0.6060	48	w(47)	0.4686
18	w(17)	0.6515	49	w(48)	0.4241
19	w(18)	0.6960	50	w(49)	0.3811
20	w(19)	0.7389	51	w(50)	0.3401
21	w(20)	0.7800	52	w(51)	0.3015
22	w(21)	0.8186	53	w(52)	0.2657
23	w(22)	0.8544	54	w(53)	0.2331
24	w(23)	0.8869	55	w(54)	0.2041
25	w(24)	0.9159	56	w(55)	0.1790
26	w(25)	0.9410	57	w(56)	0.1581
27	w(26)	0.9619	58	w(57)	0.1416
28	w(27)	0.9784	59	w(58)	0.1297
29	w(28)	0.9903	60	w(59)	0.1225
30	w(29)	0.9975	61	w(60)	0.1201
31	w(30)	1.0000			

VI. SUCESSIVE STEPS FOR FREQUENCY RESPONSE CURVE GENERATION



VII. FREQUENCY RESPONSES

The proposed window function has been utilized to compute the frequency response of four different types of FIR filter i.e. low pass, high pass, band pass and band stop respectively and their behavior is justified by appropriate plots.

Low pass filter response:

A low pass filter is one which passes all frequencies below a specified cut-off frequency and blocks all other frequencies above the cut-off frequency. In this case the selected cut-off frequency is 0.3. Presence of stop band ripples is at moderate level. From Fig. 3 we can observe that the maximum stop band attenuation level is –150 dB. Details are given in TABLE II.

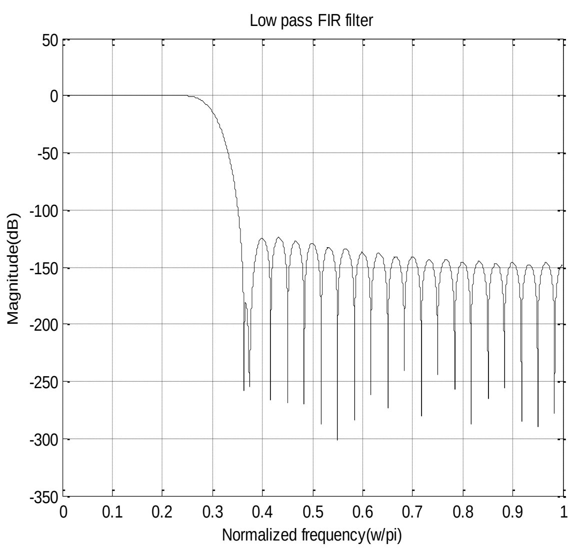


Fig. 3 *Low pass filter*

TABLE II

Low pass FIR filter specifications

Low pass filter specifications	
Cut-off frequency	0.3
Max. stop band attenuation(ripple top)	-150 dB

High pass filter response:

A high pass filter is one which blocks all frequencies below a specified cut-off value and passes all other frequencies above the specified value. The cut-off frequency selected in this case is 0.3. From Fig. 4 we can observe that ripples are present at a moderate level. Maximum stop band attenuation observed is -125 dB. Details are provided in TABLE III.

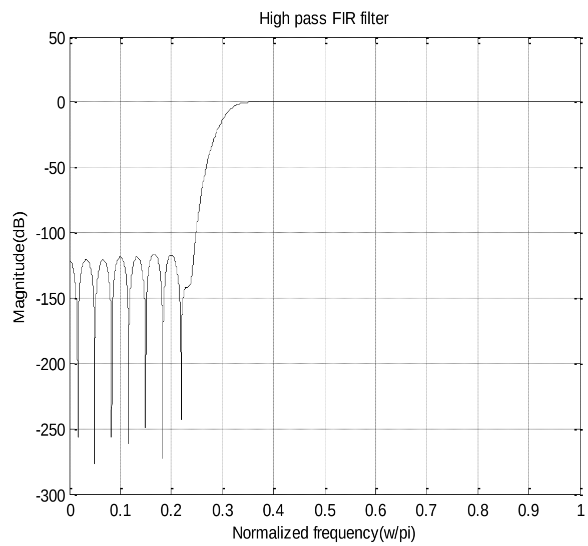


Fig. 4 *High pass filter*

TABLE III

High pass FIR filter specifications

High pass filter specifications	
Cut-off frequency	0.3
Max. stop band attenuation(ripple top)	-125 dB

Band pass filter response:

A band pass filter is one which passes all frequencies within specified limits and rejects all other frequencies. In this case, the lower cut-off frequency specified is 0.4 and the upper cut-off frequency specified is 0.6. Maximum stop band attenuation observed in this case is -148 dB with the presence of ripples at a moderate level which is clear from Fig. 5. Details are given in TABLE IV.

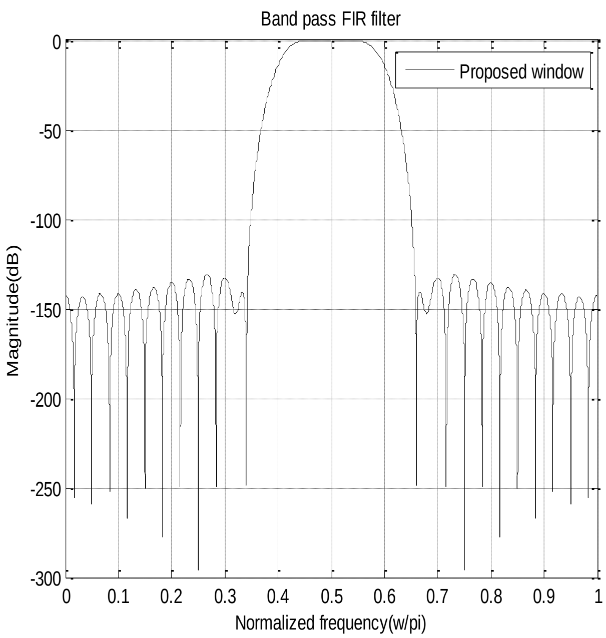


Fig. 5 *Band pass filter*

TABLE IV

Band pass FIR filter specifications

Band pass filter specifications	
Upper cut-off frequency	0.4
Lower cut-off frequency	0.6
Max. stop band attenuation(ripple top)	-148 dB

Band stop filter response:

A band stop filter is one which stops all frequencies within specified cut-off values and passes all other frequencies outside the specified limits. The lower cut-off frequency selected in this case is 0.4 and the selected upper cut-off frequency is 0.6.Maximum stop band attenuation observed in this case is -125 dB which is clear from Fig. 6. Details are given in TABLE V.

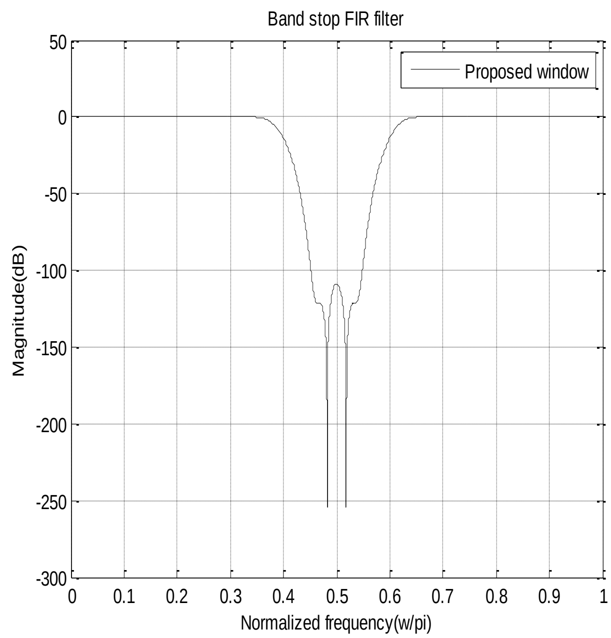


Fig. 6 *Band stop filter*

TABLE V

Band stop FIR filter specifications

Band stop filter specifications	
Upper cut-off frequency	0.4
Lower cut-off frequency	0.6
Max. stop band attenuation(ripple top)	-125 dB

VIII. A COMPARISON

In this section, the Taylor window which has almost an identical geometrical structure but with a greater amplitude level is compared with the proposed window displayed in Fig. 7. Details are provided in TABLE VI.

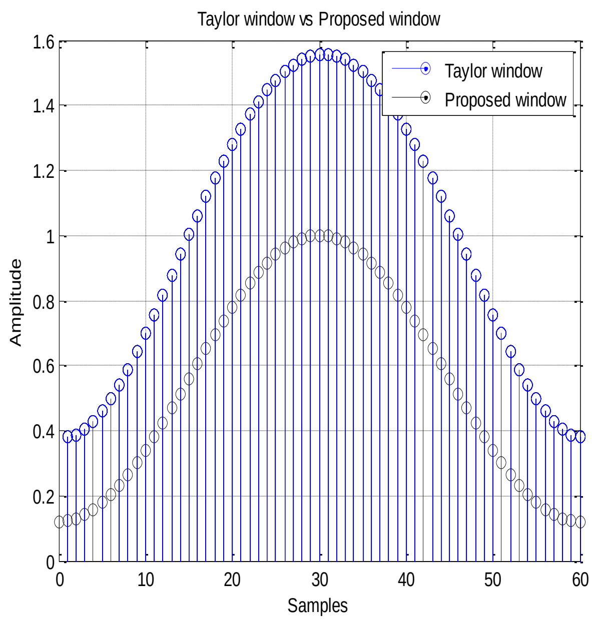


Fig. 7 *Taylor vs. Proposed window*

TABLE VI

Taylor vs. Proposed window

Specifications	***Taylor win.***	***Proposed win.***
Max. amplitude	1.58	1.00
Min. amplitude	0.39	0.12

Low pass filter response:

The low pass filter responses of FIR filter using Taylor window and the proposed window are compared. From the simulation result it is observed that the frequency response of the filter using the proposed window is much more effective than that using the Taylor window in perspective of stop band attenuation level. The maximum attenuation level using Taylor window is -140 dB whereas the maximum attenuation level using the proposed window is -150 dB. It is observed from Fig. 8 that an attenuation level difference of 10 dB is maintained throughout. Details are provided in TABLE VII.

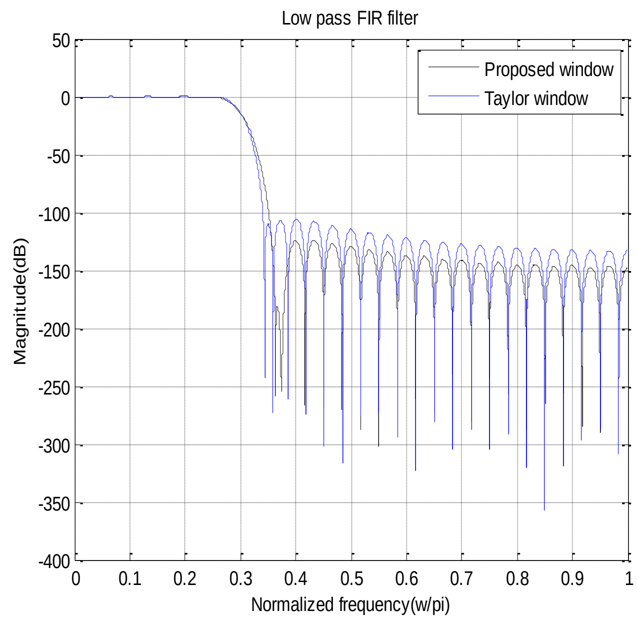


Fig. 8 *Low pass FIR filter comparison*

TABLE VII

Low pass FIR filter comparison

Low pass filter specs.	***Taylor win.***	***Proposed win.***
Cut-off frequency	0.3	0.3
Max. stop band attenuation(ripple top)	-140 dB	-150 dB
Presence of ripples	Moderate	Moderate

High pass filter response:

The high pass filter responses of FIR filter using Taylor window and the proposed window are compared with appropriate simulation results displayed in Fig. 9. Still an effective FIR filter response is observed in this case also. From the figure it is observed that the maximum attenuation level is -105 dB in case of Taylor window whereas it is equal to -120 dB using the proposed window. An attenuation level difference of about -15 dB is maintained throughout. Details are given in TABLE VIII.

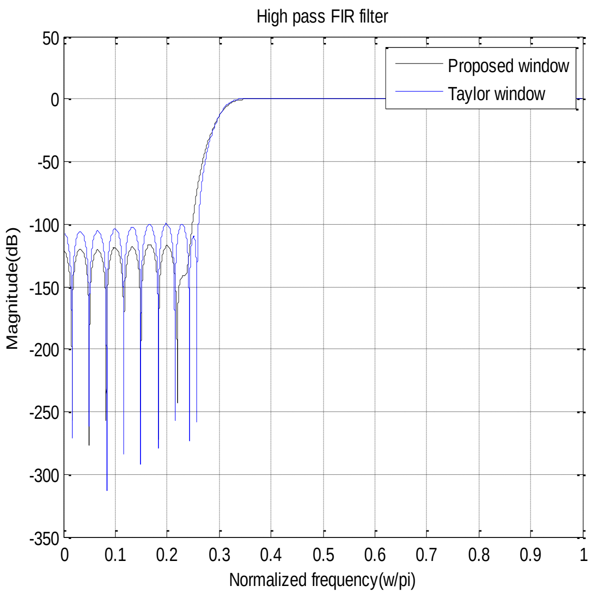


Fig. 9 *High pass FIR filter comparison*

TABLE VIII

High pass FIR filter comparison

High pass filter specs.	***Taylor win.***	***Proposed win.***
Cut-off frequency	0.3	0.3
Max. stop band attenuation(ripple top)	-140 dB	-150 dB
Presence of ripples	Moderate	Moderate

Band pass filter response:

In this section the FIR filter response is generated using the Taylor and the proposed window functions respectively and they are compared with the help of appropriate plots. It is observed that the filter response is comparatively more effective while using the proposed window function. From Fig. 10 it is observed that the maximum attenuation level is almost -130 dB using the Taylor window whereas it is down to -140 dB while using the proposed window function. An attenuation level difference of 10 dB is maintained throughout. Details are provided in TABLE IX.

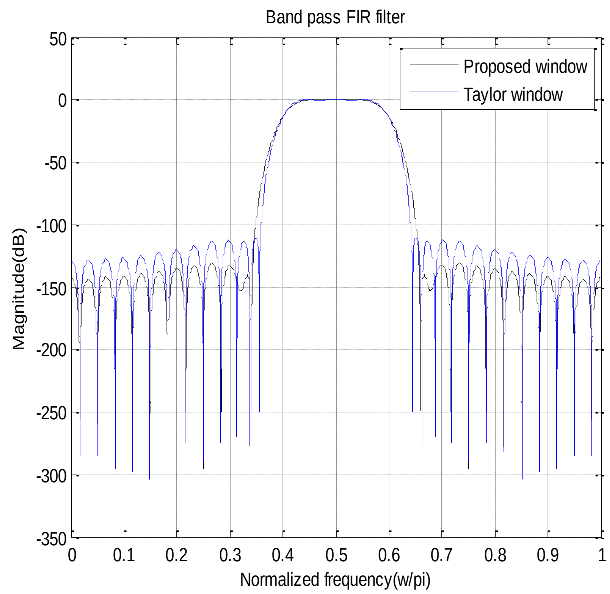


Fig. 10 *Band pass FIR filter comparison*

TABLE IX

Band pass FIR filter comparison

Band pass filter specs.	***Taylor win.***	***Proposed win.***
Upper cut-off frequency	0.4	0.4
Lower cut-off frequency	0.6	0.6
Max. stop band attenuation(ripple top)	-140 dB	-150 dB
Presence of ripples	Moderate	Moderate

Band stop filter response:

More effective filter response is observed when the proposed window function is utilized instead of the Taylor window. Maximum attenuation level observed form Fig. 11 is -110 dB using the Taylor window whereas it is almost down to -125 dB while using the proposed window function. Ripples are present at a moderate level in both the cases. Details are given in TABLE X.

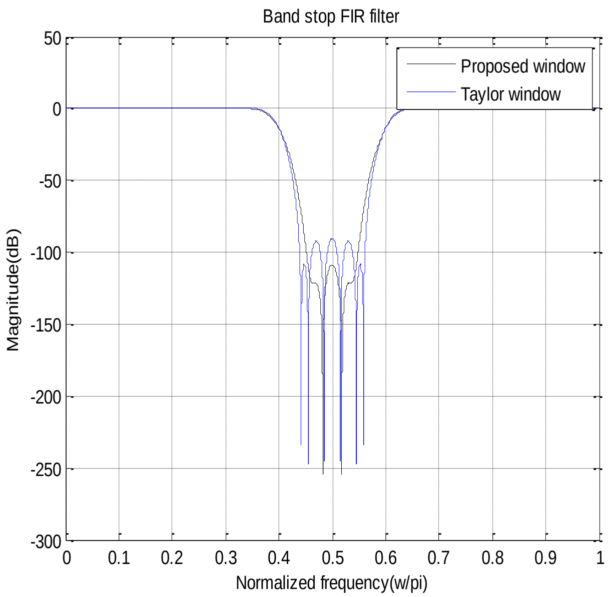


Fig. 11 *Band stop FIR filter comparison*

TABLE X

Band stop FIR filter comparison

Band stop filter specs.	***Taylor win.***	***Proposed win.***
Upper cut-off frequency	0.4	0.4
Lower cut-off frequency	0.6	0.6
Max. stop band attenuation(ripple top)	-140 dB	-150 dB
Presence of ripples	Moderate	Moderate

IX. FUTURE WORK AND CONCLUSION

From the simulation results it is observed that the proposed window function produces comparatively much effective type of filter response. In future stress is likely to be given to decrease the presence of stop band ripples and to decrease the stop band attenuation level further. Furthermore the proposed window function filter response will be compared with those of other common window functions with quantized and generalized results.

X. REFERENCES

1. J.G. Proakis and D.G. Manolakis; *"Digital Signal Processing: Principles, Algorithms and Applications,"* Prentice-Hall, Englewood cliffs NJ, second edition, 1992.
2. Sanjay Sharma; *"Digital Signal Processing";* Katson Books.
3. P. Ramesh Babu; *"Digital Signal Processing";* Scitech.
4. Eric W. Weisstein; (2003). "*CRC Concise Encyclopedia of Mathematics*", CRC Press. ISBN 1-58488-347-2.
5. Curtis Roads; (2002). "*Microsound".* MIT Press. ISBN 0-262-18215-7.
6. Atul Bhargava; *"A Survey Report for Design of FIR Filter with different method",* ISSN: 2278 – 7798, International Journal of Science, Engineering and Technology Research (IJSETR); Volume 2, Issue 2, February 2013
7. Sonika Gupta, Aman Panghal; *"Performance Analysis of FIR Filter Design by Using Rectangular, Hanning and Bartlett-Hann Windows Methods"*, Volume 2, Issue 6, June 2012 ISSN: 2277 128X International Journal of Advanced Research in Computer Science and Software Engineering (IJARCSSE).
8. Ha, Y.H., and J.A. Pearce; *"A New Window and Comparison to Standard Windows."* IEEE® Transactions on Acoustics, Speech, and Signal

Processing. Vol. 37, No. 2, (February 1999). pp. 298-301.

9. Carrara, W.G., R.M. Majewski and R.S. Goodman; "*Spotlight Synthetic Aperature Radar: Signal Processing Algorithms*", Artech House Publishers, Boston, 1995, Appendix D.2.

10. Brookner, Eli; "*Practical Phased Array Antenna Systems"*, Lex Book, Lexington, MA, 1991.

11. Oppenheim, A.V., and R.W. Schafer; "*Discrete-Time Signal Processing",* Upper Saddle River, NJ: Prentice-Hall, 1999, p. 468.

12. Oppenheim, A.V., and R.W. Schafer, "*Discrete-Time Signal Processing"*, Prentice-Hall, 1989, pp. 447-448.

13. Gautam J.K, Kumar A, Saxena R; *"On the modified Bartlett-Hanning window (family) ";* (IEEE Transactions on Signal Processing).

14. Atul Bhargava, Ravindra Pratap Narwaria; *"Design of FIR Digital Filters using Least Square Method Based On Neural Network";* ISSN: 2278 – 7798; International Journal of Science, Engineering and Technology Research (IJSETR); Volume 1, Issue 5, November 2012.

15. Ankan Bhattacharya; *"A modified window function for FIR filter design with an improved frequency response and its comparison with the Hamming window";* ISSN: 2278 – 7798; International Journal of Science, Engineering and Technology Research (IJSETR); Volume 2, Issue 5, May 2013.

16. Ankan Bhattacharya; *"A window function with modified co-efficients for FIR filter design with an improved frequency response and its comparison with the Bartlett-Hann window";* ISSN: 2278 – 7798; International Journal of Science, Engineering and Technology Research (IJSETR); Volume 2, Issue 6, June 2013.

ABOUT THE AUTHOR

Ankan Bhattacharya obtained B.Tech (2010) and M.Tech (2012) degrees in Electronics and Communication Engineering under West Bengal University of Technology. He has three International Journal publications. He is presently serving as an Assistant Professor of Electronics and Communication Engineering Department at Mallabhum Institute of Technology; Campus: Braja-Radhanagar, P.O: Gosaipur, P.S: Bishnupur, Dist: Bankura - 722122, West Bengal, India.