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STEADY STATE AERODYNAMIC ANALYSIS OF AIRCRAFT WINGS WITH CONSTANT AND VARYING CROSS SECTION

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Abstract- **The flight of an aircraft is mainly generated by Lift force and Drag force. These forces alter the relative wind producing aerodynamic forces that act on the wings of an aircraft. The present paper aims at computing the Co-efficients of Lift and Drag for a typical wing planform of Constant and Varying cross section. Numerical studies are performed to investigate the aerodynamic performance of a thin wing planform having a Constant and Varying Cross-section. The vortex lattice method (VLM) is utilized to simulate the Aerodynamic characteristics on the basis of Computational Fluid Dynamics. The performance characteristics that mainly govern the study are Co-efficient of Lift and Co-efficient of Drag. The study also deals with understanding the effect of various factors such as Aspect Ratio, Sweep angle, Vortex or Panel formation, Taper ratio, Ground Proximity and Dihedral Angle on Aerodynamic characteristics and also decide the wing dimensions for different wing configurations under Steady Subsonic flow. In the present work, the validity of the Vortex Lattice Method (VLM) for evaluating the Lift and Drag Coefficients is established by comparing the results with the available experimental data.**

Keywords: Vortex Lattice Method, Co-efficient of Lift, Co-efficient of Drag.

I. INTRODUCTION

The aerodynamic characteristics for subsonic flow about an airfoil have been widely discussed. Since the span of an airfoil is infinite, the flow is identical for each spanwise station (i.e., the flow is two dimensional). The lift is produced by the pressure differences between the lower surface and the upper surface of the airfoil section [1] and therefore the circulation (integrated along the chord length of the section), does not vary along the span.

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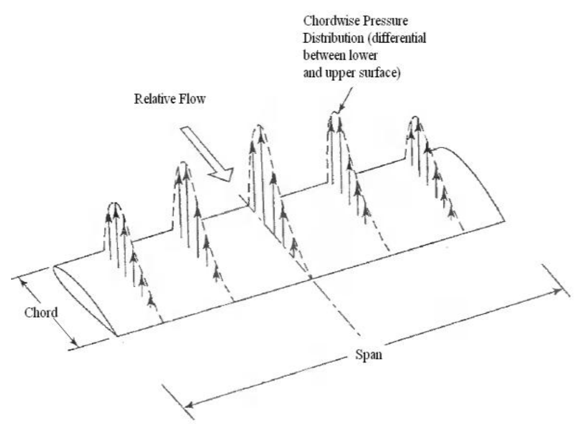

Fig.1: Aerodynamic load distribution for a rectangular wing in subsonic airstream.

Therefore determination of the Lift and Drag along the span length of an aircraft wing becomes imperative as the two dimensional results do not give an exact approximation of the lift and Drag Co-efficient. Thus, an important difference in the three-dimensional flow field around a wing (as compared with the two-dimensional flow around an airfoil) is the variation in lift. Since the lift force acting on the wing section at a given spanwise location is related to the strength of the circulation, there is a corresponding spanwise variation in circulation, such that the circulation at the wing tip is zero. Procedures that can be used to determine the vortex-strength distribution produced by the flow field around a three dimensional lifting wing is presented in this paper. The Vortex Lattice Method (VLM) [2] provides a Three Dimensional depiction for Lift and Drag along the Whole span of the Wing Planform of an aircraft.

II. OBJECTIVE

The present study aims at studying the variation in Co- efficient of lift and Co- efficient of Drag for a Rectangular Wing of constant cross section due to variation of different parameters such as Aspect Ratio (AR), Sweep angle and ground proximity [3] and also compare the Theoretical values of Co-Efficient of Lift with the available Experimental Values.

III. VORTEX LATTICE METHOD (VLM)

The first step required to solve the aeroelastic problem is the determination of the aerodynamic loads. In order to do so, one has to develop a computational program based on the Vortex Lattice Method (VLM) .This section presents a short review on the VLM method and the fundamental equations used in the numerical implementation. The VLM method is the simplest of the methods to solve incompressible flows around wings of finite span. The method represents the wing as a planar surface on which grids of horseshoe vortex are superimposed as shown in Fig 2. Each of these Horseshoe vortices posses a control point as shown in Fig 4. The computation of the velocities induced by each horseshoe vortex at each specified control point is based on the Biot-Savart law [4].

$$d\vec{V} = \frac{\Gamma_n (dl \times \vec{r})}{4\pi r^3} \tag{1}$$

A summation is performed for all control points on the wing to produce a set of linear system of equations for the horseshoe vortex strengths that satisfy the boundary conditions of no flow through the wing. The control points of each element (or lattice) are located at three-fourth of the element's chord and the vortex strengths are related to the wing circulation and the pressure difference between the upper and lower surface of the wing. The pressure differentials are then integrated to yield the total forces and moments. In the approach used here to solve the governing equations, the continuous distribution of bound vorticity over the wing surface is approximated by a finite number of discrete horseshoe vortices. The individual horseshoe vortices are placed in rectangular (or trapezoidal) panels also called finite elements or lattices. This procedure for obtaining a numerical solution for the flow is termed Vortex Lattice Method

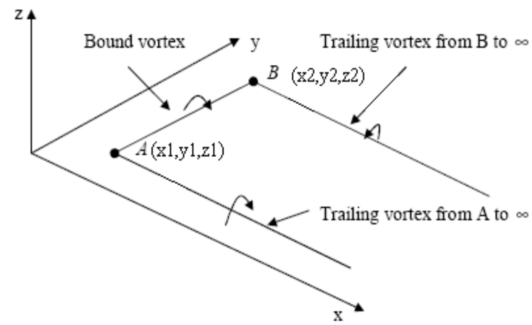

Fig. 2 "Typical" horseshoe vortex.

The basic expression for the calculation of the induced velocity by the horseshoe vortices in the VLM is

$$\vec{V} = \frac{\Gamma_n}{4\pi} \frac{\vec{r}_1 \times \vec{r}_2}{\left|\vec{r}_1 \times \vec{r}_2\right|} \left[r_0 \cdot \left(\frac{\vec{r}_1}{r_1} - \frac{\vec{r}_2}{r_2} \right) \right] \tag{2}$$

Where Γ is the circulation around the panel and

r_1 and r_2 are the vectors produced by Face AB. To calculate the velocity that is induced at a general point in space (x, y, z) by the horseshoe vortex shown in Fig. 2. The horseshoe vortex maybe assumed to represent that for a typical wing panel (e.g., the nth panel). Segment AB represents the bound vortex portion of the horseshoe system. Then the influence of each Horseshoe vortex on each of the panel is given by

$$W_{m,n} = \Gamma_n/4\pi\{((x_m-x_{1n})*(y_m-y_{2n}) - (x_m-x_{2n})*(y_m-y_{1n}))^{-1} \{[(x_{2n}-x_{1n})*(x_m-x_{1n})+(y_{2n}-y_{1n})*(y_m-y_{1n})/sqrt((x_m-x_{1n})^2+(y_m-y_{1n})^2)]-[(x_{2n}-x_{1n})*(x_m-x_{1n})+(y_{2n}-y_{1n})*(y_m-y_{1n})/sqrt((x_m-x_{1n})^2+(y_m-y_{1n})^2)]\}+(1/(y_{1n}-y_m))[1+((x_m-x_{1n})/sqrt((x_m-x_{1n})^2+(y_m-y_{1n})^2)]-(1/(y_{1n}-y_m))[1+((x_m-x_{1n})/sqrt((x_m-x_{1n})^2+(y_m-y_{1n})^2)] \tag{3}$$

where the total velocity is the sum of the contributions from the vortex segments shown in Fig 4. Assuming the point C(x,y,z) to be the control point of the *mth* panel, with coordinates (x_m, y_m, z_m) located at midspan of the element.

The methodology implemented for Vortex Lattice Method is as follows [5]

A. Choice of Singularity Element

The method by which a thin wing planform is divided into panels is shown in Fig.4 and some typical panel element.

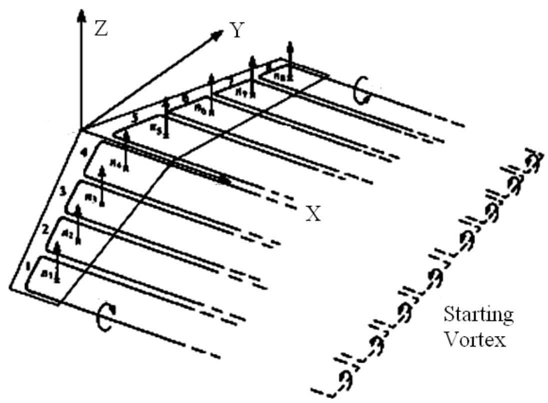

Fig.4: Horseshoe Lifting Line Model for solving a lifting line problem.

In order to solve the problem by use of Horseshoe elements as shown in Fig.3, this element should consist of a straight bound vortex segment BC that models lifting properties and two semi infinite trailing vortices that models the wake. The segment BC need not be parallel to the Y-axis, but at the element tips the vortex is shed into the flow where it must be parallel to the streamlines so that no force will act on the trailing vortices. These vortex elements are used as the near portions of vortex rings whose starting vortices extend far back so that the effect of AD is negligible. The velocity induced by such an element at a point P(x, y, z) as shown in Fig.5 can be computed by velocity induced by horseshoe element.

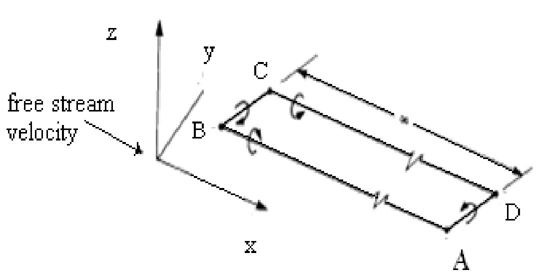

Fig.3: Horseshoe model of a lifting wing.

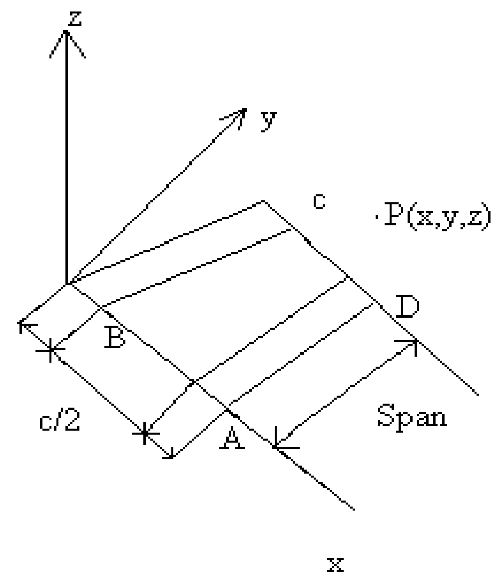

Fig.5: A spanwise Horseshoe Vortex element.

By use of Equation 3 a sub-routine is created for simplicity of the Equations.

It is possible to include these computations Equations in a subroutine such that

$$(u, v, w) = HSHOE(x, y, z, x_A, y_A, z_A, x_B, y_B, z_B, x_c, y_c, z_c, x_D, y_D, z_D, \Gamma) \quad (4)$$

B. Discretization and Grid geometry

At this phase wing is divided into N spanwise elements as shown in Fig.4. For this example the span is divided into 8 panels and also the geometric panel area like S_j, normal vector n_j and the coordinates of the collocation point (x_i, y_i, z_i) are calculated at this phase.

C. Influence co-efficients

In order to fulfil the boundary conditions specified at each collocation points. The velocity induced by horseshoe vortex element 1 at collocation point 1 can be computed by using horseshoe routine developed before

$$(u,v,w)_{11} = HSHOE(x_1, y_1, z_1, x_{A1}, y_{A1}, z_{A1}, x_{B1}, y_{B1}, z_{B1}, x_{c1}, y_{c1}, z_{c1}, x_{D1}, y_{D1}, z_{D1}, \Gamma = 1.0)$$

The no normal flow across the wing boundary condition at this point for the first collocation point can be written as

$$[(u,v,w)_{11}\Gamma_1 + (u,v,w)_{12}\Gamma_2 + \dots\dots\dots + (u,v,w)_{1N}\Gamma_N + (U_\infty, V_\infty, W_\infty)] \cdot n_1 = 0$$

Establishing the same procedure for each of the collocation points in the discretized form of the boundary condition

$$a_{11}\Gamma_1 + a_{12}\Gamma_2 + a_{13}\Gamma_3 + ... + a_{1N}\Gamma_N = -Q_\infty \cdot n_1$$
$$a_{21}\Gamma_1 + a_{22}\Gamma_2 + a_{23}\Gamma_3 + ... + .a_{2N}\Gamma_N = -Q_\infty \cdot n_2$$
$$a_{31}\Gamma_1 + a_{32}\Gamma_2 + a_{33}\Gamma_3 + ... + a_{3N}\Gamma_N = -Q_\infty \cdot n_3$$

. . . .

. . . .

$$a_{N1}\Gamma_1 + a_{N2}\Gamma_2 + a_{N3}\Gamma_3 + ... + a_{NN}\Gamma_N = -Q_\infty \cdot n_N$$

Where the Influence co-efficient are defined as

$$a_{ij} = (u, v, w)_{ij} \cdot n_i \qquad (5)$$

The normal velocity components of the free stream flow are known and moved to the R.H.S of the equation

$$RHS = -(U_\infty, V_\infty, W_\infty) \cdot n_i \qquad (6)$$

These set of N algebraic equations with N unknown T_j that can be solved by standard solution matrix techniques. As an example for the case of planar wing with constant angle of attack α, this results in the following set of equations

$$\begin{pmatrix} a_{11}...a_{12}..........a_{1N} \\ a_{21}....a_{22}.....................a_{2N} \\ a_{31}...a_{32}..........a_{3N} \\ \\ a_{N1}...a_{N2}.........a_{NN} \end{pmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ . \\ \Gamma_N \end{bmatrix} = -Q_\infty \sin\alpha \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ 1 \end{bmatrix}$$

D. Establish R.H.S vector

The right hand side vector Equation (6) is actually the normal component of the free stream which can be computed within the influence co-efficient computations. However if an upgrade is planned to include unsteady effects or simulation of normal transpiration flows, then its recommended to do this calculation separately.

E. Solve linear set of Equations

The solution for the above described problem can be obtained by standard matrix methods. Furthermore since the influence of such an element on itself is largest, the matrix will have a dominant diagonal and the solution is stable.

F. Secondary computations: pressures, loads, velocities

The solution for the above set of equations results in a vector $(\Gamma_1, \Gamma_2 ...... \Gamma_n)$. The lift of each bound vortex segment is obtained by kutta-jowkoski condition:

$$\Delta L_j = \rho Q_\infty \Gamma_j \Delta y_j \qquad (7)$$

Where Δy_j is the panel bound vortex projection normal to the free stream. The induced drag computation is somewhat more complex. The total aerodynamic loads are the sum of contribution of individual panels. Following lifting line results Equations

$$\Delta D_j = \rho w_{ind\,j} \Gamma_j \Delta y_j \qquad (8)$$

where the induced downwash w_{indj} at each collocation point j is computed by summing up the velocity induced by all trailing vortex segments as shown in fig 6. This can be done during the phase of the influence co-efficient computation or even later by using HSHOE routine with the influence of bound vortex segment turned off.

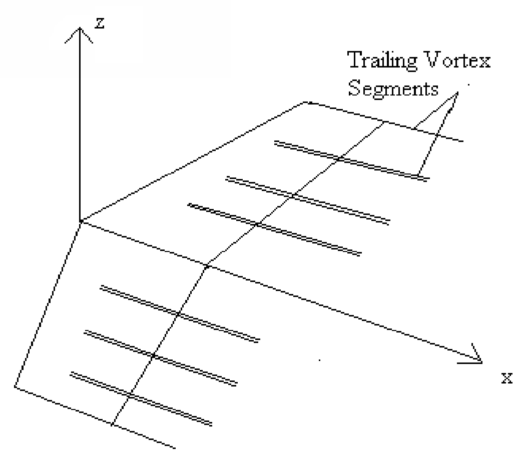

Fig.6: Array of Vortex segments responsible for induced downwash on the three-dimensional wing.

G. Ground Effect - Classical Theory

Wieselsberger presents a parameter that he refers to as an influence coefficient (σ).Since Wieselsberger attributes the formula to Prandtl, we will refer to the formula as Prandtl's formula.

$$\sigma = \frac{1-0.66\cdot(2\cdot[h/b])}{1.05+3.7\cdot(2\cdot[h/b])} \quad (9)$$

Maurice Le Sueur clarified the usage of the above equation. At a given Lift Co-efficient and the drag due to Lift Co-efficient

$$C_D = \frac{C_L^2 \cdot S}{\pi \cdot b^2}(1-\sigma) \quad (10)$$

Since the Aspect Ratio (AR) is

$$AR = \frac{b^2}{S}$$

Therefore the Drag due to Lift becomes

$$C_D = \frac{C_L^2 \cdot}{\pi \cdot AR}(1-\sigma) \quad (11)$$

This equation can be used to study the effect of Ground proximity [6] and Dihedral angle on wings of constant and varying cross-section.

V. RESULTS

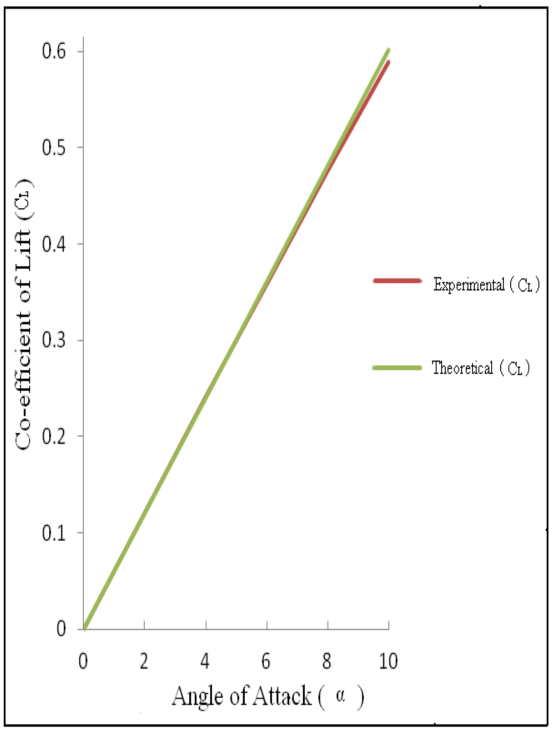

Fig. 7: Theoretical and Experimental values of co-efficient of lift with respect to Angle of Attack for a 45° swept wing.

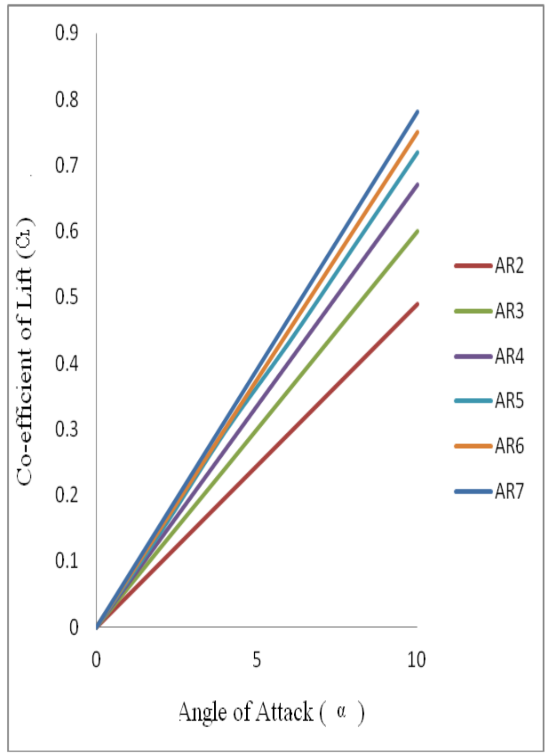

Fig.8: Variation of lift with respect to angle of attack for different aspect ratios.

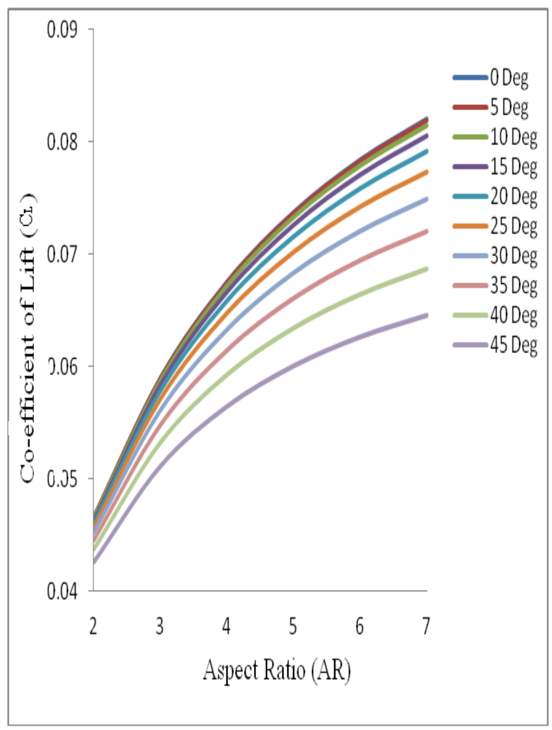

Fig. 9: Variation of Co-efficient of lift with respect to Aspect Ratio for different Sweep Back angles.

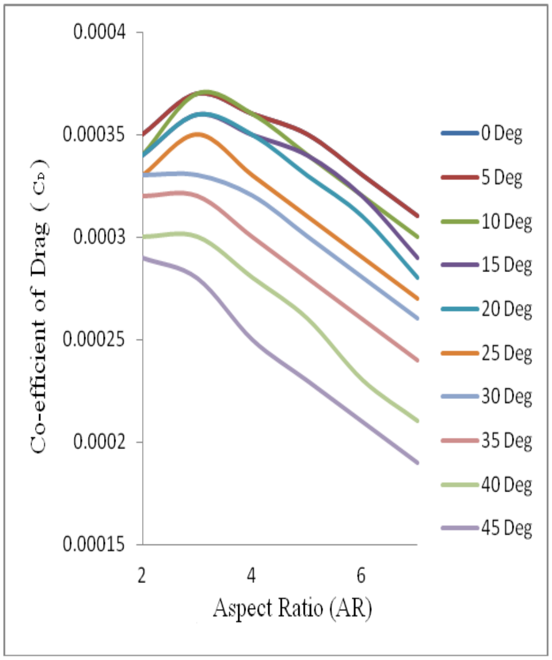

Fig.10: Variation of Co-efficient of Drag with respect to aspect ratio for different sweep angles.

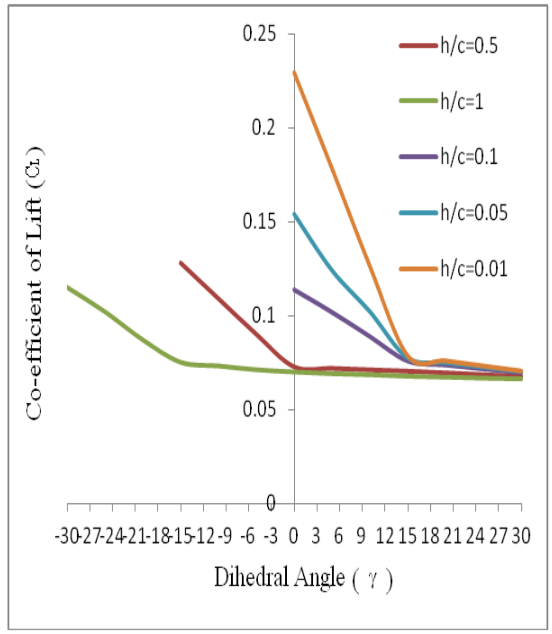

Fig.12: Variation of Co-efficient of lift with respect to Dihedral angle for different values of Ground Proximity (h/b).

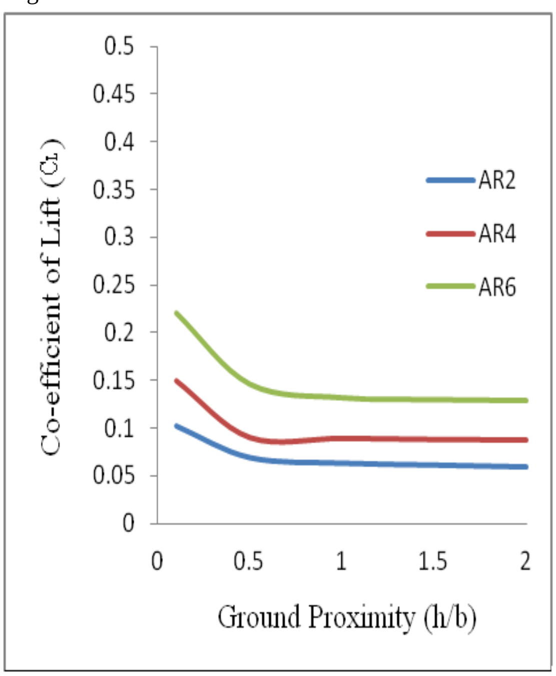

Fig. 11: Variation of Co-efficient of lift with respect to Ground proximity for different Aspect Ratios (AR).

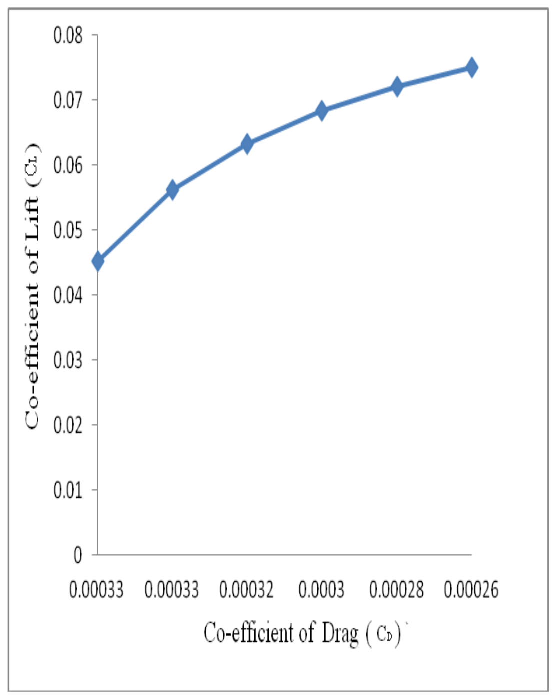

Fig. 13: Variation of Lift with respect to Drag.

V. CONCLUSION

The present work deals with the study of Lift and Drag variations of an aircraft wing. The formulation is based on finding out Lift and Drag of a wing by use of a 3-dimensional method called as Vortex Lattice Method or Panel method. The theoretical method is employed by dividing the aircraft wing into finite number of panels and determining the required parameters. Results are presented based on the analytical study of variation of Lift and Drag of an aircraft wing with respect to different parameters such as Aspect Ratio, Backward Sweep angle, Effect of ground proximity and Dihedral angle.

The most important result of Vortex Lattice Method is to establish the Lift and Drag of a wing of particular Span length and chord length.

The Lift obtained from VLM is in good agreement with experimental value of lift obtained for Brebner and weber [7] therefore validating the result. The lift of a wing decreases as the Aspect Ratio becomes smaller for a wing of general spanwise circulation. The induced Drag of a wing increases as the wing Aspect Ratio decreases for a wing with general spanwise circulation. This theory also provides valuable information about the wing's spanwise loading and about the existence of the trailing vortex wake. The theory is limited to small disturbances and large Aspect Ratio since angle of attack is considered to be very small upto a level of 10°. There are possible modifications to this theory such as addition of flapped wings. However the study of the wings with more complex geometry is difficult with this method. Using the results from this method we are able to study the variation of Lift and drag with respect to different parameters.

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