ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 8, August 2013

DYNAMIC AND STABILITY ANALYSES OF DELAMINATED RECTANGULAR COMPOSITE PANELS

[1]Suhas, [2]Dr D L Prabhakara

ABSTRACT **– The composite materials have significant applications over metallic materials in different fields of structural engineering. Structural elements subjected to in-plane periodic forces may lead to parametric or dynamic instability under certain combination of load parameters which caused resonant transverse vibrations. The study of dynamic stability itself requires a special investigation of basic problems of vibration and static stability. The present investigation deals with the study of vibration, static and dynamic stability of delaminated composite plates using finite element method. The effects of various geometrical parameters like aspect ratios, delamination size, number of layers on the free vibration and stability characteristics of delaminated composite plates have been analysed numerically. The numerical values are compared with the reference values and the results are concluded. Some case studies have been considered and the results of these studies are presented.**
Keywords: Laminated Composites, Delamination, Free Vibration, Static Stability, Dynamic stability

I. INTRODUCTION

Laminated composite plates are extensively used in the construction of aerospace, civil, marine, automotive and other high performance structures due to their high stiffness to weight and strength to weight ratios, excellent fatigue resistance, long durability and many other superior properties compared to the conventional metallic materials [1]. Since the strength or stiffness to weight ratios of composites is very high compared to the conventional isotropic materials, the components made out of these materials are quite slender. This creates a serious problem of stability under the action of in-plane static or dynamic loads. As the stiffness of the composites is very large, naturally one can expect them to vibrate at higher frequencies. Very thin composite panels are used as skin of aircraft structures which are susceptible to follower aerodynamic forces which in turn cause flutter vibrations. These problems necessitate the study of buckling, free vibration and flutter characteristics of thin machine or structural elements [2]. Classical solution to these kinds of problems is quite tedious, if not impossible. Hence one has to resort to some numerical technique. Finite element method is one of the most powerful numerical technique adapted by many investigators to conduct the numerical Experimentation [3].

Suhas *[1], Dept. of Mechanical Engineering, VTU/SCEM, Mangalor , India Mob No: +91 8970046230*

Dr. D. L. Prabhakara *[2], Dept. of Civil Engineering, VTU/SCEM Mangalore, India, Mob No:+91 9449845966*

II. OBJECTIVE

Composites are used where one expects high strength to weight ratio or high stiffness to weight ratio. Generally this situation arises in the design of Aircraft/Aerospace vehicles. Looking at the disadvantages, composites are susceptible to delamination and subsequent degradation of the thin structural components. Vibration and stability analyses are compliment to each other [4]. The structure looses its stability under certain conditions i.e when the frequencies of free vibration become real and different.

The objectives of the present work are to:
(i) Formulate the vibration and stability problems using Hamilton's principle, (ii) Analyse free vibration and stability behavior of the composite plates with delamination and then to draw conclusions

II. MATHEMATICAL FORMULATION

The mathematical formulation of bending, vibration, static buckling and dynamic buckling problems of layered composite plate has been presented using Hamilton's principle. The primary ingredients for applying this principle are (i) linear and non-linear strain energies, (ii) kinetic energy, work done by the external forces and also the stress field in the plate. To workout the various quantities mentioned above, basic formulations of strain displacement and stress strain relations for a composite element are to be established [5].

A. Strain-Displacement Relation

Consider an infinitely small element of size dx/dy in a composite plate of thickness 't' as shown in Fig.1. The mid-plane displacement of this element be u_0, v_0 and w_0 along x, y and z axes respectively.

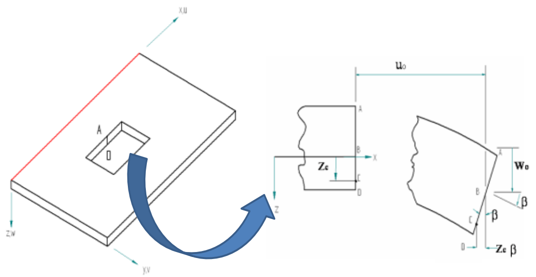

Fig.1: Geometry of Delamination in x-z Plane

ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 8, August 2013

The strains on any general layer can be by

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2}$$
$$\varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} \quad (1)$$

Eq. (1) can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2)$$

In which

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}$$ are the mid-surface strains

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}$$ are the mid-surface curvatures

B. Constitutive Relation For A Laminate

Laminate can be formed by sandwiching a number of laminae whose principal axes need not be in a particular direction. For determining the stress strain relation of a composite laminate, one has to start from the stress-strain relation of an individual lamina. Consider the lamina with local and global coordinates as shown in Fig. 2.

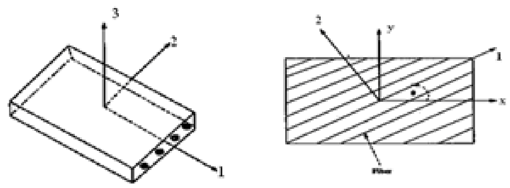

Fig. 2: Lamina with principal coordinates and global coordinates.

Using the Generalized Hooke's law for orthotropic material, the stress strain relation corresponding to the 1-2 axes can be written as [1]

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

or in the abbreviated way $[\sigma] = [Q][\varepsilon]$ (3)

in which $[Q]$ is called reduced stiffness matrix and it is expressed in terms of elastic constants as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}}{1-\nu_{12}\nu_{21}} & \frac{}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

C. STRESS RESULTANTS

Consider a laminate made out of N layers of lamina as shown in Fig.3. The thickness of a general k^{th} layer be t_k. The distances from the negative face to positive face of this k^{th} layer be z_{k-1} and z_k. The stress resultants are shown in Fig. 4.

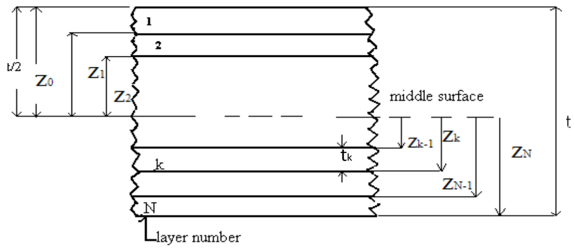

Fig. 3: Geometry of N-layered Laminate

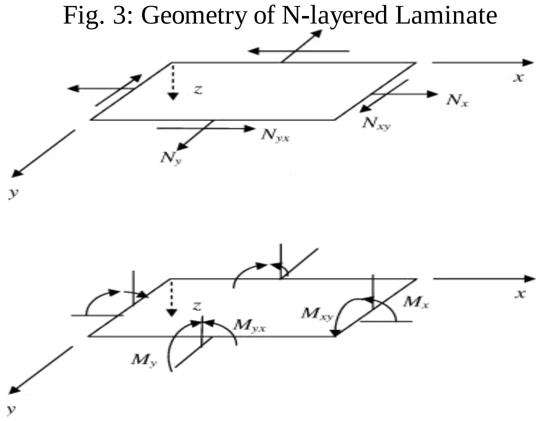

Fig. 4: Force and moment resultants

The in-plane stress resultants can be obtained using the following integration.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz \quad (4)$$

Performing layer-wise integration and summing of the resultant forces we get

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{i=1}^{N} [\bar{Q}](z_k - z_{k-1}) \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} + \sum_{i-1}^{N} [\bar{Q}] \left(\frac{z_k^2 - z_{k-1}^2}{2} \right) \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

The Moment resultants are given by (5)

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdz \tag{5}$$

Performing layer-wise integration and summing of the resultant moments we get

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{i=1}^{N} [\bar{Q}] \left(\frac{z_k^2 - z_{k-1}^2}{2} \right) \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} + \sum_{i=1}^{N} [\bar{Q}] \left(\frac{z_k^3 - z_{k-1}^3}{3} \right) \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Similarly, the shear stress resultants are given by

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \tau_{xy} \\ \tau_{yz} \end{Bmatrix} dz \tag{6}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{bmatrix} G_{23} & 0 \\ 0 & G_{23} \end{bmatrix} dz$$

Performing layer-wise integration and summing of the resultant moments we get

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \sum_{1}^{N} (z_k - z_{k-1}) \begin{bmatrix} G_{23} & 0 \\ 0 & G_{23} \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \alpha h \begin{bmatrix} G_{23} & 0 \\ 0 & G_{23} \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} \tag{7}$$

Where α is called warping factor

Expressing the stress resultant and strain-curvature relation for the composite plate in a matrix form, we get

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} & 0 & 0 \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & H_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix}$$

In abbreviated way the above equation can be re-written as

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & H \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \\ \phi \end{Bmatrix}$$

The above equation can be further abbreviated as

$\{N\}=[C]\ \{\varepsilon\}$

in which $[D]= \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & H \end{bmatrix}$ is called stiffness matrix

D. EXPRESSIONS FOR ENERGIES

In the present formulation, there will be participation of three kinds of energies, namely, strain energy due to linear strains, strain energy due to non-linear strains and kinetic energy.

(i) The linear strains have been expressed in terms of mid-plane displacements and mid-plane curvatures. If the stress field is known on the differential element then the strain energy due to linear strains can be expressed as

$$U_1 = \frac{1}{2} \int_v \{\varepsilon_l\}^T \{\sigma\} dv \tag{8}$$

Substituting the expression for stresses in terms of strains, we get

$$U_1 = \frac{1}{2} \int_A \{\varepsilon_l\}^T [C] \{\varepsilon_l\} dA \tag{9}$$

(ii) The strain energy U_2 due to non-linear strains in the entire plate panel can be expressed in terms of initial in-plane stresses and non-linear strains as given below.

$$U_2 = \int_v \{\sigma\}^T \{\varepsilon_{nl}\} dv \tag{10}$$

Performing integration over the thickness, we get the expression for strain energy interms of stress resultants.

$$U_2 = \int_A \{N\}^T \{\varepsilon_{nl}\} dA \tag{11}$$

(iii) By knowing the velocities, one can derive the kinetic energy of the plate. If $\dot{u}, \dot{v}$ and $\dot{w}$ are the translational velocities and $\dot{\theta}_x$ and $\dot{\theta}_y$ are the rotational velocities of a differential element considered in the kth layer of the lamina, then the kinetic energy 'T' of a laminate consisting of L number of layers can be written as,

$$T = \frac{1}{2} \rho \int_A \sum_{k=1}^{L} \int_{z_{k-1}}^{z_k} \left[(\dot{u} + z\dot{\theta}_x)^2 + (\dot{v} + z\dot{\theta}_y)^2 + \dot{w}^2 + z^2 \dot{\theta}_x^2 + z^2 \dot{\theta}_y^2 \right] dz\, dA \tag{12}$$

in which ρ is the mass per unit volume.

Performing integration over the thickness of the laminae,

$$T = \frac{1}{2} \rho \int_A \sum_{k=1}^{L} \Bigg[(z_k - z_{k-1}) \dot{u}^2 + \frac{2(z_k^2 - z_{k-1}^2)}{2} \dot{u}\dot{\theta}_x + (z_k - z_{k-1}) \dot{v}^2 + \frac{2(z_k^2 - z_{k-1}^2)}{2} \dot{v}\dot{\theta}_y + (z_k - z_{k-1}) \dot{w}^2 + \frac{2(z_k^3 - z_{k-1}^3)}{3} \dot{\theta}_x^2 + \frac{2(z_k^3 - z_{k-1}^3)}{3} \dot{\theta}_y^2 \Bigg] dA \tag{13}$$

After summing up for all the layers, the above equation can be expressed as

$$T = \int_A \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\theta}_x \\ \dot{\theta}_y \end{Bmatrix}^T \begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\theta}_x \\ \dot{\theta}_y \end{Bmatrix} dA$$

in which,

$$P_1 = \sum_{k=1}^{L} \rho_k (z_k - z_{k-1}), \quad P_2 = \sum_{k=1}^{L} \rho_k \frac{(z_k^2 - z_{k-1}^2)}{2} \quad \text{and} \quad P_3 = \sum_{k=1}^{L} \rho_k \frac{2(z_k^3 - z_{k-1}^3)}{3}$$

The expression for kinetic energy is written in abbreviated form as

$$T = \frac{1}{2} \int_A \{\dot{g}\}^T [P]\ \{\dot{g}\}\ dA \tag{14}$$

in which,

$$[P]=\begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix} \qquad \{\dot{g}\}=\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\theta}_x \\ \dot{\theta}_y \end{Bmatrix}$$

To get the governing differential equations for the problems under investigation, Hamilton's principle can be applied. The Hamilton's principle is expressed as

$$\int_{t1}^{t2}[\delta(T-U_1-PU_2)]dt=0 \qquad (15)$$

After performing the integration the above equation reduces to

$$[M]\{\ddot{d}\}+[[K_e]-P[K_G]]\{d\}=0 \qquad (16)$$

E. FINITE ELEMENT FORMULATION

For the problems involving complexities in loading and boundary conditions, classical methods are not easily adaptable and numerical solution techniques like finite element method are well preferred. The present formulation is developed hereby for the analysis of laminated composite plate type of structures using a first order shear deformable theory [6].

An eight node rectangular element shown in Fig. 5 is being used in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. The in-plane deformations u, v is considered for in-plane stress analysis. Element shall be oriented in the natural coordinate systems and shall be transferred into the Cartesian coordinate system using Jacobian matrix. In the analysis of thin plates where the element is assumed to have mid-surface nodes, the interpolation polynomials [7] are used to derive the element shape function. The interpolation polynomial is written as follows.

$$u=\alpha_1+\alpha_2 r+\alpha_3 s+\alpha_4 r^2+\alpha_5 rs+\alpha_6 s^2+\alpha_7 r^2 s+\alpha_8 rs^2$$

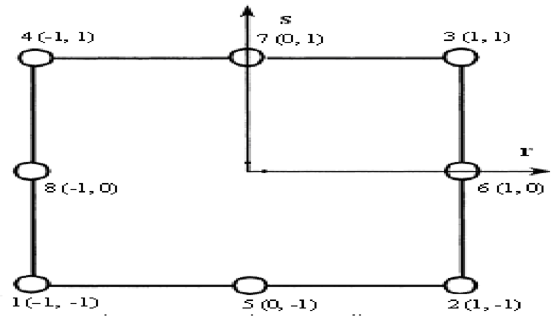

Fig. 5: 8 noded isoparametric element

Shape functions in natural co-ordinates for an eight nodded element can be expressed as

$$\{N\}^T=\{N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6 \quad N_7 \quad N_8\}$$

a. Linear strain-nodal displacement matrix:

The strain field at any point in the eight noded isoparametric element can be expressed in terms of the nodal displacement vector. The variation of the displacement and rotations can be expressed by the shape function .

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix}=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & 0 & 0 & -\frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & 0 & -\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \\ 0 & 0 & \frac{\partial N_i}{\partial x} & N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & -N_i \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{x_i} \\ \theta_{y_i} \end{Bmatrix} \qquad i=1,8$$

The above relation is rearranged and written in a concise matrix form as follows. The strain- nodal displacement vector for j^{th} element is written in abbreviated form as $\{\varepsilon\}_j=[B]_j\{d\}_j$,Where $[B]$ is called strain displacement matrix.

The total strain energy, in the entire plate is obtained by summing the energies of all the individual elements,

$$U_1=\sum_{j=1}^{m}\frac{1}{2}\int_A\{d\}_j^T[B]_j^T[C]_j[B]_j\{d\}_j\,dA$$

Alternatively the above equation is expressed as

$$U_1=\frac{1}{2}\sum_{j=1}^{m}\{d\}_j^T[k_e]_j\{d\}_j \qquad (17)$$

in which $[k_e]=\int_A[B]_j^T[C]_j[B]_j\,dA$ is the stiffness matrix of j^{th} element.

The above summations are made in the sense of finite element assemblage, taking the global displacement vector to be {d}, which results in to the following relation.

$$U_1=\frac{1}{2}\{d\}^T[K_e]\{d\} \qquad (18)$$

b. Geometric stiffness matrix:

The expression for strain energy U_2 due to non-linear strains in the entire plate panel is given in eq. 10. Using shape functions, the non-linear strain vector {f} can be expressed in terms of nodal displacement vector

ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 8, August 2013

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_y}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \end{Bmatrix} = \sum_{i=1}^{8} \begin{Bmatrix} \frac{\partial N_i}{\partial x} u_i \\ \frac{\partial N_i}{\partial y} u_i \\ \frac{\partial N_i}{\partial x} v_i \\ \frac{\partial N_i}{\partial y} v_i \\ \frac{\partial N_i}{\partial x} w_i \\ \frac{\partial N_i}{\partial y} w_i \\ \frac{\partial N_i}{\partial x} \theta_{x_i} \\ \frac{\partial N_i}{\partial y} \theta_{x_i} \\ \frac{\partial N_i}{\partial x} \theta_{y_i} \\ \frac{\partial N_i}{\partial x} \theta_{y_i} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{x_i} \\ \theta_{y_i} \end{Bmatrix}$$

i=1,8

Summing up the strain energies due to non linear strains of all the elements of the discretised structure, the total energy can be written as

$$U_2 = \sum_{j=1}^{m} \frac{1}{2} \int_A \{d\}_j^T [G]_j^T [S]_j [G]_j \{d\}_j dA \qquad (19)$$

In an abbreviated form of the above equation is written as

$$U_2 = \frac{1}{2} \sum_{j=1}^{m} \{d\}_j^T [k_G]_j \{d\}_j \qquad (20)$$

in which $[k_G]_j$ is called the geometric or stress stiffness matrix of j^{th} element and is expressed as

$$[k_G]_j = \int_A [G]_j^T [S]_j [G]_j dA$$

The summations given in above Eq are made in the sense of finite element assemblage, taking the global displacement vector to be $\{d\}$, which results into the following relation.

$$U_2 = \frac{1}{2} \{d\}^T [K_G] \{d\} \qquad (21)$$

In which $[K_G]$ is called the geometric stiffness matrix of the entire plate panel.

c. Mass matrix:

Considering the j^{th} element in the discretized plate panel the velocity vector can be expressed in terms shape functions and nodal velocity vectors of that element as

$$\dot{u} = \sum_{i=1}^{8} N_i \dot{u}_i,\ \dot{v} = \sum_{i=1}^{8} N_i \dot{v}_i,\ \dot{w} = \sum_{i=1}^{8} N_i \dot{w}_i, \qquad (22)$$
$$\dot{\theta}_x = \sum_{i=1}^{8} N_i \dot{\theta}_{xi} \text{ and } \dot{\theta}_y = \sum_{i=1}^{8} N_i \dot{\theta}_{yi}$$

The total kinetic energy of the entire plate/shell panel is

$$T = \sum_{j=1}^{m} \frac{1}{2} \int_A \{\dot{d}\}_j^T [m]_j \{\dot{d}\}_j dA \qquad (23)$$

The summations given in Eq. 23 are made in the sense of finite element assemblage, taking the global velocity vector to be {d}, which results into the following relation.

$$T = \frac{1}{2} \{\dot{d}\}^T [M] \{\dot{d}\}$$

(24)

Substituting the above expressions of the energies, we get

$$[M]\{\ddot{d}\} + [[K_e] - P[K_G]]\{d\} = 0$$

(25)

The above equation is the most general governing differential equation of motion for free vibration, static and dynamic instability problems of composite plates

d. Formulation of static and dynamic problems:

From the governing differential equation given in Eq. 25, formulations can be made for a variety of static and dynamic problems.

(i) Vibration without in-plane load:
Setting P=0 in Eq. 25, yields the governing equations for free vibration problem

$$[M]\{\ddot{d}\} + [K_e]\{d\} = \{0\} \qquad (26)$$

(ii) Vibration with in-plane load

Setting P=P (a static non-follower in-plane load), Eq. 25, yields the governing equations for vibration problem with static in-plane non-follower load. (+ indicates tensile load and – indicates compressive load)

$$[M]\{\ddot{d}\} + [[K_e] \pm P[K_G]]\{d\} = \{0\} \qquad (27)$$

The eigenvalues of Eqs. 26 and 27 give the squares of natural frequencies for different modes. The lowest value of frequency is termed as the fundamental natural frequency.

(iii) Static stability or buckling:

Setting P=P (a static non-follower in-plane load), acceleration $\{\ddot{d}\}$=0, Eq. 25, yields the governing equations for a static stability problem with static in-plane non-follower load.

$$[K_e]\{d\} - P[K_G]\{d\} = \{0\} \qquad (28)$$

The eigenvalues of the above equations Eq. 28 are the buckling loads for different modes. The lowest buckling load is termed as the fundamental critical load of the structure

III. RESULTS AND DISCUSSION

A typical delaminated plate is shown in fig.1. Here the plate is considered to be simply supported and the boundary conditions of the entire composite plate are given in Table 1. Delamination in the composite plates, greatly effect the dynamic behavior of structures. So in the present investigation, natural frequency of delaminated Graphite-epoxy composite plates with $\frac{E_{11}}{E_{12}} = 40$, $\frac{G_{12}}{E_{12}} = 0.5$ and $\nu_{12} = 0.25$ are determined numerically [1]. 20x20 mesh is considered for the entire plate and the meshing in the delaminated area is m_dx n_d.

ISSN: 2278 – 7798
International Journal of Science, Engineering and Technology Research (IJSETR)
Volume 2, Issue 8, August 2013

The effects of various parameters like (i) aspect ratio, (ii) delamination area and (iii) number of layers are studied critically. Numerical results are presented for free vibration of antisymmetric cross-ply ($0/90^0$) delaminated composite plates [9].

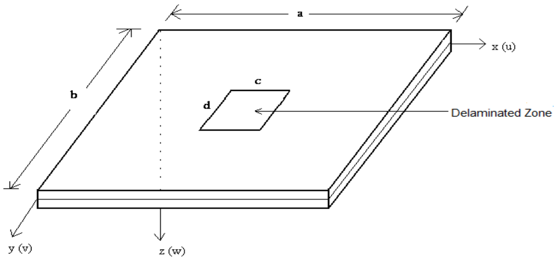

Fig. 6: Delaminated Plate

Table 1: Geometric boundary conditions [8]

Boundary condition	Position of the edge	
Simply supported (S)	$u = w = \theta_x = 0$ at x=0 and $w = \theta_x = 0$ at x=a	$v = w = \theta_y = 0$ at y=0 and $w = \theta_y = 0$ at y=b

Nomenclatue:

a, b – Dimensions of the plate along x and y directions.
α - Aspect ratio of the plate, $\frac{b}{a}$

λ =Non dimensional buckling load, $\lambda = \frac{N_x b^2}{\pi^2 D_{22}}$

$\bar{\omega}$ - Non dimensional natural frequency, $\bar{\omega} = \frac{\omega b^2}{\pi^2}\sqrt{\frac{\rho}{D_{22}}}$

D_{22} is the 2,2 element of [D] matrix in stiffness matrix [C] of the laminate.
m_d, n_d - delamination mesh size.

A. Vibration Analysis of Laminated Composite Plates with Delamination:

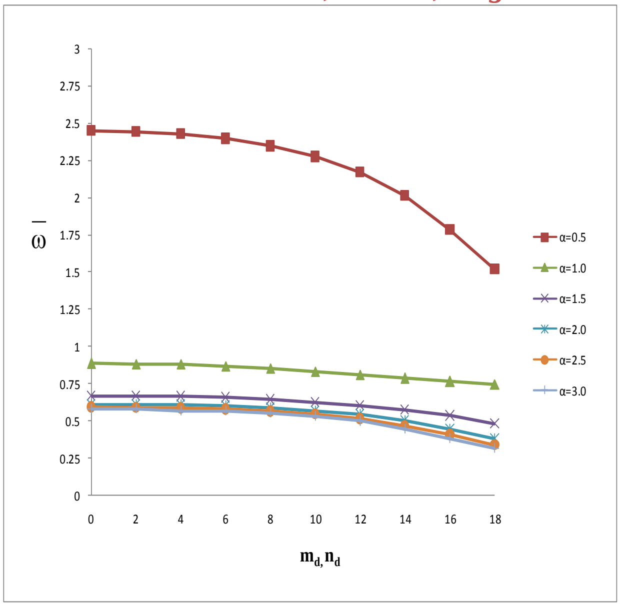

Fig. 7: Non-dimensional vibration frequencies for 2 layered antisymmetric crosss-ply delaminated plate.

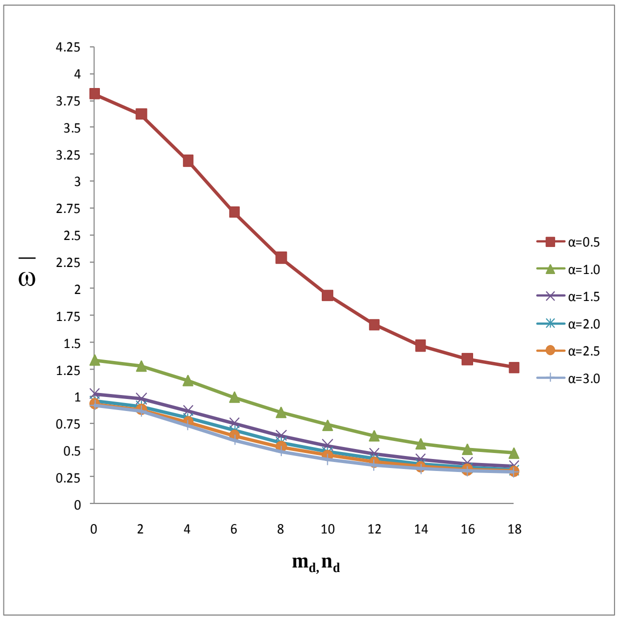

Fig. 8 : Non-dimensional vibration frequencies for 4 layered antisymmetric crosss-ply delaminated plate.

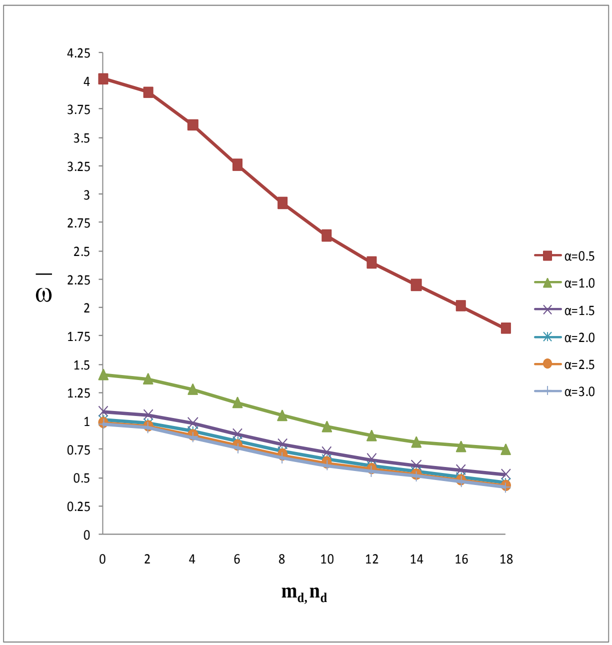

Fig. 9 : Non-dimensional vibration frequencies for 6 layered antisymmetric crosss-ply delaminated plate.

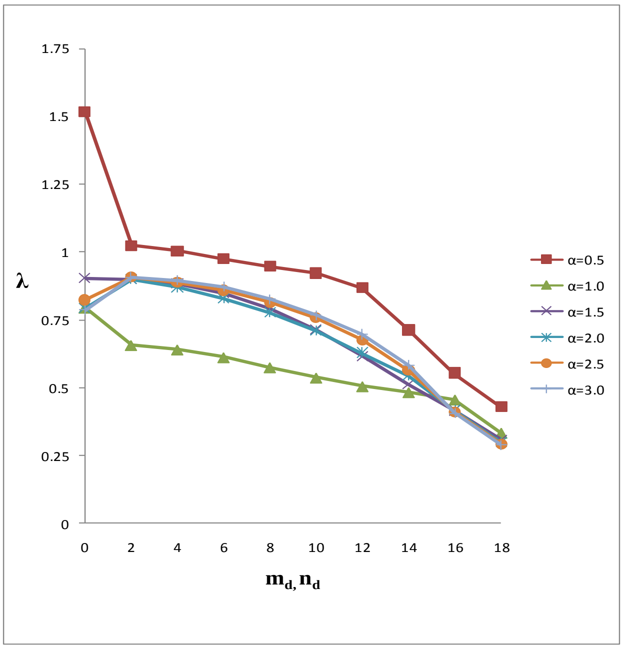

Fig. 10 : Non-dimensional Buckling Loads for 2 layered antisymmetric cross-ply delaminated plates under uniform uniaxial compression

B. Buckling/static Stability Analysis of Laminated Composite Plates with Delamination under Unidirectional In-Plane Load:

The buckling behavior of the laminated plates is studied. The laminated plate is subjected to uniform uni-axial compressive loads.

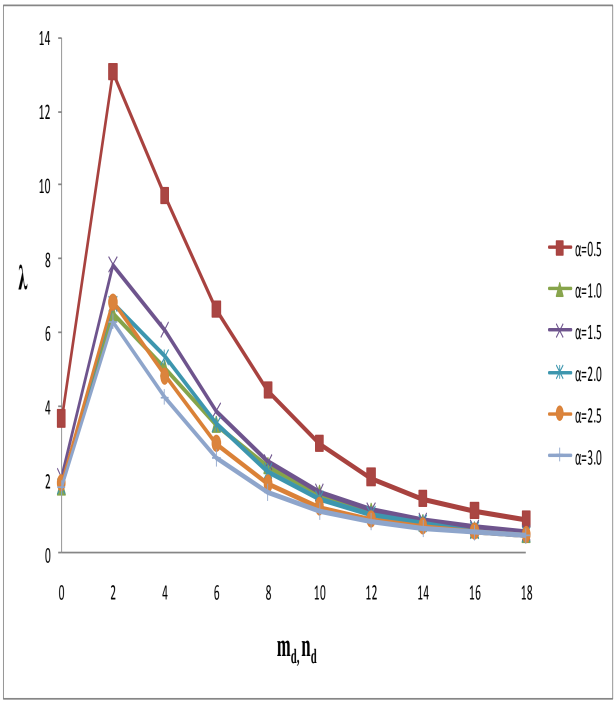

Fig. 11 : Non-dimensional Buckling Loads for 4 layered Antisymmetric cross-ply delaminated plates under uniform uniaxial compression

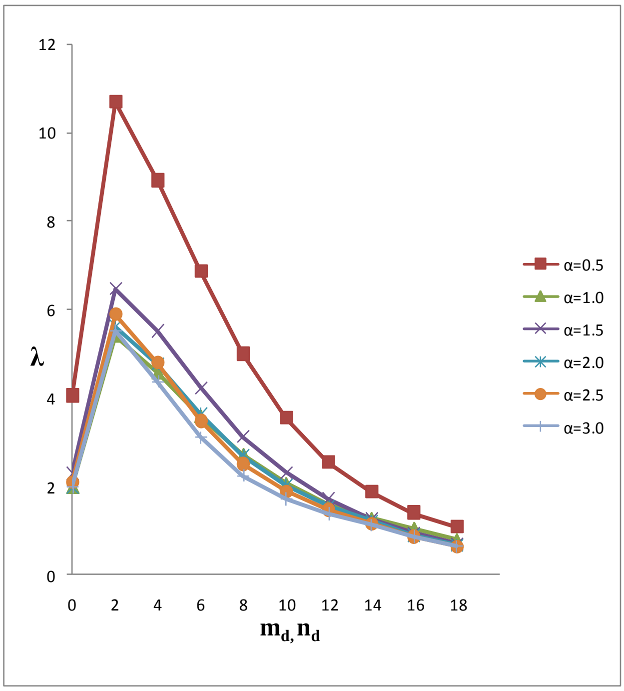

Fig. 12 : Non dimensional buckling loads for 6 layeerd antisymmetric cross-ply delaminated plates under uniform uniaxial compression.

IV. CONCLUSION

The present work deals with the study of the vibration and buckling characteristics of delaminated composite plates. Results are presented for the free vibration, buckling characteristics of delaminated composite plates. The effects of various geometrical parameters like delamination area, number of layers and aspect ratios on the free vibration and stability characteristics of delaminated composite plates have been analysed. The conclusions drawn in respect of different studies are presented below.

A. Vibration analysis:

The main thrust of the present study is the modal analysis of delaminated composite plates, because of its inherent link with stability analysis. The effect of different parameters on the frequencies of vibration is observed.

(i) There is a good agreement between the numerical and the reference results. (ii) The frequencies of vibration decreases with introduction and further increase of size of delamination in composite plates. (iii) It is observed that natural frequency increases with increase in number of layers and decreases with increase in aspect ratios for delaminated plate. (iv) The vibration frequency reduces with increasing delamination sizes. Thus increase in the sizes of delamination has a deteriorating effect on the plate dynamic stiffnesses.

B. Buckling analysis:

The present study includes numerical study of buckling analysis of delaminated composite plates. The conclusions are summarized as given below.

(i) For delaminated composite plate, with the increase of percentage of delamination area, the non-dimensional buckling load initially increases and then gradually decreases. This may be attributed to a fact that there will be local buckling near the zone of delamination. (ii) The rate of change of buckling load is not uniform for increase in percentage of delamination. (iii) For delaminated composite plate, with the increase of number of layers, the non-dimensional buckling load also increases. (iv) With the increase in aspect ratios of delaminated composite plates the non-dimensional buckling loads decreases.

V. REFERENCES

[1]. Robert M Jones, "Mechanics of Composite Materials," Taylor and Francis Group, LLC, 2012.

[2]. Ramachandra L.S. and Panda Sarat Kumar. "Dynamic instability of composite plates subjected to non-uniform in-plane loads," Journal of Sound and Vibration, Vol. 331, No. 1, 2012, pp. 53-65.

[3]. Grover Neeraj, Singh B.N. and Maiti D.K., "Analytical and finite element modeling of laminated composite and sandwich plates: An assessment of a new shear deformation theory for free vibration response," International Journal of Mechanical Sciences, Vol. 67, 2012, pp. 89-99.

[4]. Ravi Kumar, L. Datta, P.K. and Prabhakara, D.L., "Dynamic instability characteristics of laminated composite plates subjected to partial follower edge load with damping," International Journal of Mechanical Sciences, Vol. 45, 2003, pp. 1429–1448.

[5]. Ravi Kumar, L., Datta, P.K. and Prabhakara. D.L., "Dynamic instability characteristics of laminated composite doubly curved panels subjected to partially distributed follower edge loading," International Journal of Solids and Structures, Vol. 42, 2005, pp. 2243–2264.

[6]. Chen Chun-Sheng, Chen Wei-Ren and Chien Rean-Der. "Stability of parametric vibrations of hybrid composite plates," European Journal of Mechanics -Solids, Vol. 28, No. 2, 2009, pp. 329-337.

[7]. Sahu, S.K., "Stability of Laminated Composite Pretwisted Cantilever Panels" Journal of Reinforced Plastics and Composites, Vol. 24, No. 12, 2005, pp. 1327-1334.

[8]. Timoshenko S. and Krieger S.W., "Theory of plates and shells," Mc-Graw-Hill, New York, 2010.

[9]. Tracy J.J. and Pardoen, G.C., "Effect of delamination on the natural frequencies of composite laminates," Journal of Composite Materials, Vol. 23, No. 12, 1989, pp. 1200–1215.

AUTHOR'S PROFILE

SUHAS is working in Sahyadri College of Engineering and Management as an Assistant Professor in the department of Mechanical Engineering. He received the M.Tech degree in Machine Design from Sahyadri college of Engineering and Management(SCEM), Mangalore affiliated to Visvesvaraya Technological University (VTU), in 2013 and B.E degree from Adichunchanagiri Institute of Technology, Chikmagalur in 2011. His areas of interest are Mechanical Vibrations and Mechanics of Materials.

Dr. D. L. Prabhakara, an eminent scholar and a renowned academician. Dr Prabhakara did his M.Tech (Applied Mechanics) at IIT Madras and his Ph.D (Aerospace Engg.) at IIT Kharaghpur. He has put in 37 years of service in the teaching profession and has published about 60research papers in National and International journals and conferences. Dr Prabhakara is a reviewer of reputed national and international journals. Currently 5 research scholars are pursuing Ph.D program under his guidance. He is also an author of the book titled 'Graphic Statics'. He has served in almost all the academic and statutory boards of almost all the universities including VTU.