

Convergence Performance of LMS & Combined LMS-LMS Beamforming Algorithm

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Abstract - A Beamforming is a beam steering technique in wireless communication to steer the beam in the desired direction. Desired signal can be estimate in the presence of noise or interfering signal with the help beamforming. LMS is one of the beamforming algorithm which is very popularly used. LMS algorithm is well known because of its simplicity and ease of computation. The LMS incorporates an iterative procedure that makes successive corrections to the weight vector. LMS Algorithm has a drawback, which required large no. of iteration for convergence. Many alternative approaches being developed to increase the speed of convergence. This paper presents the convergence performance of LMS and new approach which is the combination of two LMS stages.

Index Terms— Beamforming, Least Mean Square (LMS)

I. INTRODUCTION

Now a days, adaptive smart antenna system is very popular in the field of wireless communication which are adapted to the demanding high bit rate or high quality in broadband commercial wireless communication such as mobile internet or multi-media services. A smart antenna system consist of multiple antenna element where each antenna element receive the signals and it combines the signal to improve the overall performance of wireless communication system. Beamforming is a technique which simultaneously place a beam maximum towards the signal of interest (SOI) and ideally nulls toward directions of interfering signals or signals not of interest (SNOIs). The LMS algorithm is a popular solution used in beamforming technique. It incorporates an iterative procedure that makes successive corrections to the weight vector which eventually leads to the minimum mean square error. With an LMS algorithm it is not possible to enhance both the convergence speed and lower the steady state error simultaneously. Also its convergence is dependent on the eigenvalue spread variation of the input signal. When an LMS algorithm is adopted in the implementation of an N-element array, the computation cost is in the order of $O(2N+1)$.

II. REVIEW

The LMS algorithm has become one of the most popular adaptive signal processing techniques adopted in many applications, including antenna array beamforming. the least mean square (LMS) based algorithms offer a relatively simple adaptive array beamforming solution[1]. However, the performance of these algorithms often depends on the actual step size adaptation process. Also, since these algorithms make use of LMS processing, their operations are influenced by the characteristics of the input signals[3]. For the LMS algorithm family, there is always a trade off between the speed and the residual error floor when a given adaptation step size is used.

III. CLASSICAL LMS BEAMFORMING

Beamforming is the method used to create the radiation pattern of the antenna array by adding constructively the phases of the signals in the direction of the user and nulling the interfering signal. With this technology, each user's signal is transmitted and received by the base station only in the direction of particular user. This drastically reduces the overall interference in the system. A smart antenna system consists of an array of antennas that together direct different transmission/reception beams toward each user in the system.

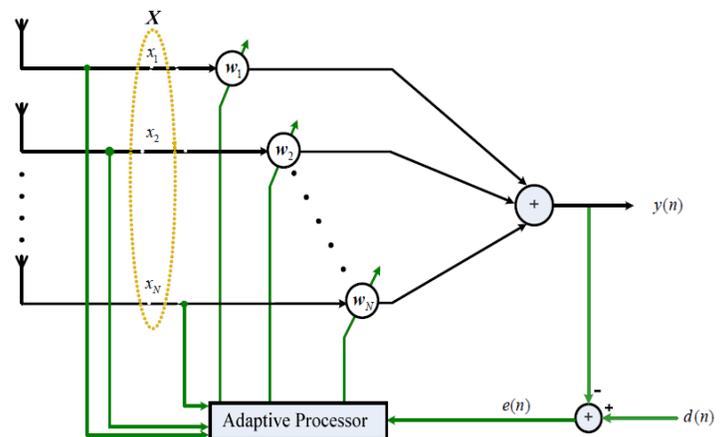


Fig. 1 – LMS Adaptive antenna array [1]

This method of transmission and reception is called beamforming and is made possible through smart (advanced) signal processing at the baseband. In beamforming, each user's signal is multiplied with complex weights that adjust the magnitude and phase of the signal to and from each antenna. This causes the output from the array of antennas to form a transmit/receive beam in the desired direction and minimizes the output in other directions.

The LMS algorithm was first proposed by Widrow and Hoff as an implementation of the steepest-descent based approach to estimate the gradient of the error signal. It is computationally efficient, but is a bit slow in convergence. Also its convergence is dependent on the eigenvalue spread variation of the input signal. When an LMS algorithm is adopted in the implementation of an N-element array, the computation cost is in the order of $O(2N + 1)$ multiplications. Consider the LMS adaptive array as shown in Fig.1. According to the method of steepest-descent, the updating of the weight vector is carried out in such a way that minimizes the error signal given by –

$$e(n) = d(n) - y(n) \quad (1)$$

where $d(n)$ is the zero-mean reference signal, and $y(n)$ is the output signal of the beamformer, such that

$$y(n) = W^H(n)X(n) \quad (2)$$

where $X(n)$ is a $N \times 1$ complex vector of the received signal, which is assumed to have zero mean; $W(n)$ is the complex weight vector having the same size as $X(n)$, and $(\cdot)^H$ denotes the Hermitian (i.e., transpose and conjugate) matrix of (\cdot) .

The LMS algorithm may be defined as

$$y(n) = W^H(n)X(n) \quad (3)$$

$$e(n) = d(n) - y(n) \quad (4)$$

$$W(n+1) = W(n) + \mu e^*(n) X(n) \quad (5)$$

The low computation complexity and robustness of the LMS algorithm have made it very popular in various applications, including adaptive antenna arrays. However, with an LMS algorithm, it is not possible to enhance both the convergence speed and lower the steady state error floor simultaneously. This is due to the fact that when a larger step size is chosen, the algorithm converges quicker but with a larger residual error floor. On the other hand, the use of a smaller step will lead to slower convergence and lower steady state error floor. Since then, many modifications have been proposed in the literature to try to overcome the compromise between convergence speed and error floor of the conventional LMS algorithm. Most of these modified LMS algorithms make use of some criteria to regulate the step size value. For example, an initial large adaptation step size could be used to speed up the convergence. When close to the steady state, smaller step sizes are then introduced to decrease the level of adjustment, hence maintaining a low error floor.

IV. COMBINED LMS-LMS BEAMFORMING

This algorithm has a very different approach to achieve fast convergence with an LMS based algorithm. The least mean square-least mean square (LLMS) algorithm involves the use of two LMS algorithm sections, LMS_1 and LMS_2 separated by an array image factor F_L as shown in Fig.2. Such an arrangement maintains the low complexity generally associated with LMS algorithm. It can be shown that an N -element antenna array employing the LLMS algorithm involves $4N+1$ complex multiplications and $3N$ complex additions, i.e. slightly doubling the computational requirements of a conventional LMS algorithm scheme. With this novel approach of LLMS algorithm scheme, as shown in Fig. 2, the intermediate output y_{LMS1} , F_L yielded from the first LMS or LMS_1 stage, is multiplied by the array image factor F_L of the desired signal. The resultant "filtered" signal is further processed by the second LMS algorithm or algorithm section. For the adaptation process, the error signal of algorithm LMS_2 e_2 is fed back to combine with that of LMS_1 algorithm, to form the overall error signal e_{LLMS} , for updating the tap weights of LMS_1 algorithm. As shown in Fig.2, a common external reference signal $d(n)$ is used for both the two LMS algorithm sections, i.e. d_1 and d_2 . Moreover, this external reference signal may be replaced after a few initial iterations by y_{LLMS1} in place of d_2 , and y_{LLMS} for d_1 to produce a self-referenced version of the LLMS algorithm scheme

Estimation of Array Image Factor \mathcal{F}

For self-adaptive beamforming, it requires that the array image factor \mathcal{F} be adjusted automatically to always tracking the AOA of the desired signal. A simple method for estimating \mathcal{F} is now described.

Consider the array inputs to the algorithm section as –

$$\begin{aligned} X_1(t) &= [x_{1,1}(t), x_{1,2}(t), \dots, x_{1,N}(t)]^T \\ &= A_d S_d(t) + A_i S_i(t) + n(t) \end{aligned}$$

where $S_d(t)$, $S_i(t)$ and $n(t)$ are the desired signal, interference signal and noise, respectively, A_d and A_i are complex $N \times 1$ array factors for the desired signal and the co channel interference, respectively. By referencing with respect to the first antenna element, then A_d and A_i are given by

$$\begin{aligned} A_d &= [1, e^{-j\psi_d}, e^{-2j\psi_d}, \dots, e^{-(N-1)j\psi_d}]^T \\ A_i &= [1, e^{-j\psi_i}, e^{-2j\psi_i}, \dots, e^{-(N-1)j\psi_i}]^T \end{aligned}$$

where according to the far-field plane wave model, $\psi_d = 2\pi (d \sin(\theta_d)/\lambda)$ and $\psi_i = 2\pi (d \sin(\theta_i)/\lambda)$, d is the spacing between adjacent antenna elements, and λ is the carrier wavelength.

$$x_{1,k}(t) = A_{d,k} S_d(t) + A_{i,k} S_i(t) + n_k(t) \quad (6)$$

where $A_{d,k}$ is the k^{th} element of A_d with $k = 1, 2, \dots, N$. The output of the individual LMS_1 algorithm tap weights (w_1) are then given by

$$x'_{1,k}(t) = w_{1,k} x_{1,k}(t) \quad (7)$$

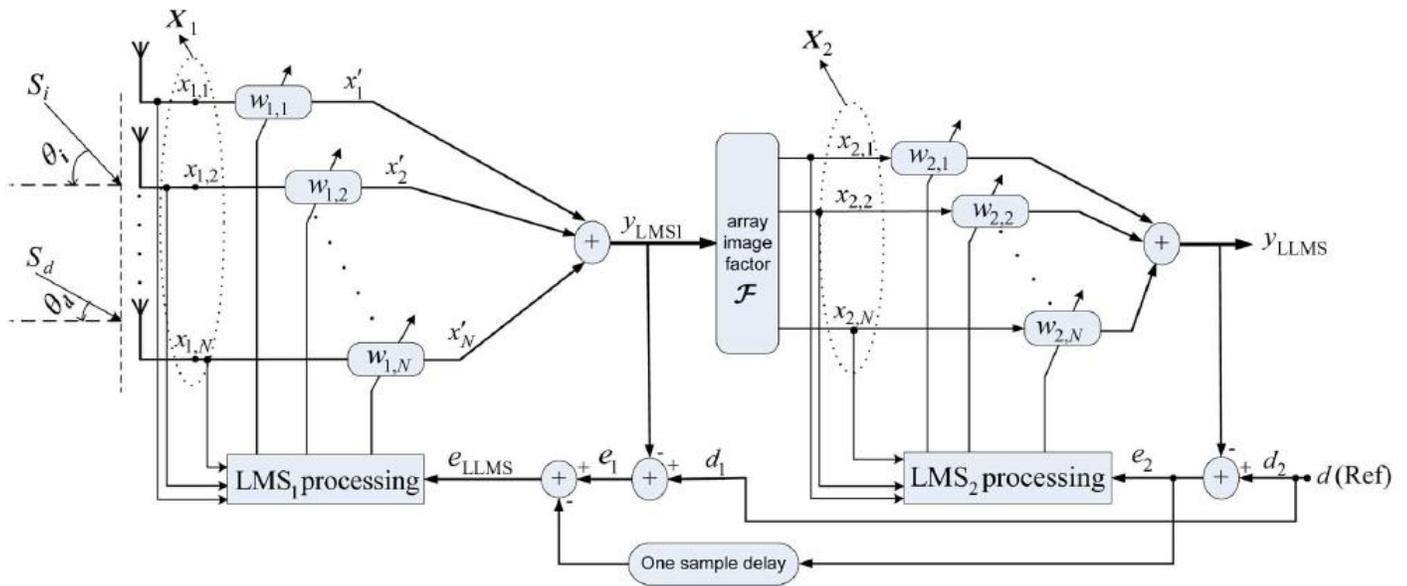


Fig 2. LLMS Beamforming algorithm with reference signal[11]

the instantaneous values of the elements of A_d can be expressed as

$$A_{d,k}(t) = \frac{x'_{1,k}(t)}{w_{1,k}(t)y_{LMS1}(t) + c} \quad (8)$$

Convergence of the LLMS Algorithm:

We consider the case when an external reference signal is used. From Fig.2, the overall error signal for updating the LLMS algorithm at the n th iteration is given by -

$$e_{LLMS}(n) = e_1(n) - e_2(n-1) \quad (9)$$

with the individual error signals

$$e_i(n) = d_i(n) - W_i^H X_i(n) \quad (10)$$

where the subscript i takes on the value of 1 and 2 for the

LMS₁ and the LMS₂ algorithm stages respectively, $X_i(.)$ and $W_i(.)$ represent the input signal and weight vectors respectively, and $(.)^H$ denotes the Hermitian matrix of $(.)$. The input signal of the LMS₂ algorithm stage is derived from the LMS₁ algorithm, such that

$$X_2(n) = \mathcal{F}_L y_{LMS1} W_1^H(n) X_1(n) \quad (11)$$

Where \mathcal{F}_L is the image of the array vector of the desired and is assumed fixed for this analysis.

V. PERFORMANCE RESULT

Environment used for Simulation :

CASE I:

- Uniform Linear Array with spacing distance of 0.5
- No. Of Antenna Element = 8
- Desired Angle = 10 , Interfering Angle = 20

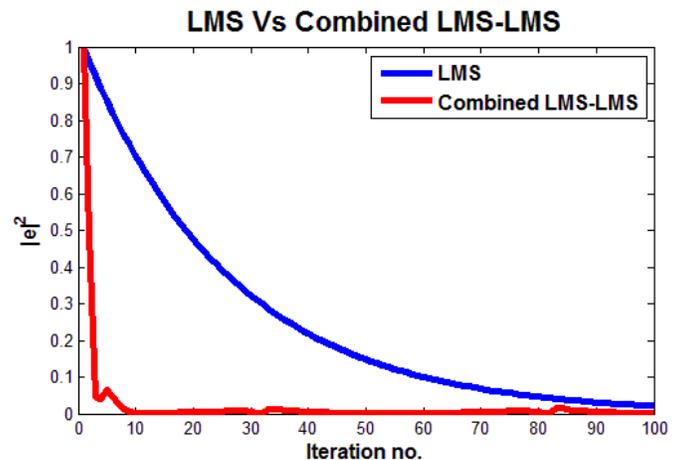


Fig.3 Convergence Performance with N = 8

CASE II:

- Uniform Linear Array with spacing distance of 0.5
- No. Of Antenna Element = 6
- Desired Angle = 10 , Interfering Angle = 20

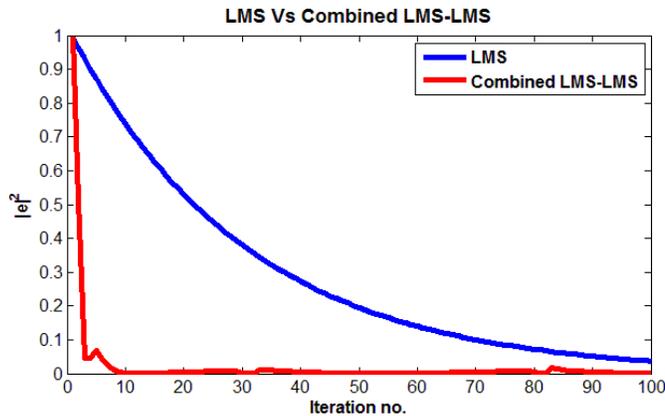


Fig.4 Convergence Performance with N = 6

VII. CONCLUSION

LMS Beamforming algorithms is well known for its simplicity and robustness but Combined LMS-LMS Beamforming Algorithm performs very well and speed of convergence is very high in Combined LMS-LMS as seen from the convergence performance.

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