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DETERIORATING ITEMS STOCHASTIC INVENTORY MODEL FOR OLIGOPOLY MARKET

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ABSTRACT

This paper develops a model to determine an optimal ordering policy for deteriorating items and allowable shortage for future supply uncertainty for two suppliers and multiple suppliers. In case of two suppliers, spectral theory is used to derive explicit expression for the transition probabilities of a four state continuous time Markov chain representing the status of the systems. These probabilities are used to compute the exact form of the average cost expression. We use concepts from renewal reward processes to develop average cost objective function. Optimal solution is obtained using Newton Rapson method in R programming. Finally sensitivity analysis of the varying parameter on the optimal solution is done. We have extended the case of two suppliers to multiple suppliers and for the multiple supplier problem, assuming that all the suppliers have similar availability characteristics, we develop a simple model and show that as the suppliers become large, the model reduces to classical EOQ model.

Keywords: Future supply uncertainty, two suppliers, Deteriorating items, Oligopoly market.

1. INTRODUCTION

Inventory can be considered as an accumulation of physical commodity that can be used to satisfy some future demand for that commodity. The main and foremost reason for maintaining inventory level is to shorten the gap between demand and supply for the commodity under consideration. Any inventory system consists of an input process and output process. The input process refers to supply either by means of production or purchase while the output process refers to demand due to which depletion of inventory occurs. Thus, supply is a replenishment process, whereas demand is a depletion process.

Though the inventories are essential and provide an alternative to production or purchase in future, they also mean lock up capital of an enterprise. Maintenance of inventories also costs money by way of expenses on stores, equipment, personnel, insurance etc. Thus excess of inventories are undesirable. This calls for controlling the inventories in the most profitable way. Hence inventory theory deals with the determination of the optimal level of such ideal resources. An excellent survey on research in inventory management in a single product, single location inventory environment is provided by Lee and Nahmias (1993).

Perishable inventory forms a large portion of total inventory and include virtually all foodstuffs, pharmaceuticals, fashion goods, electronic items, periodicals (magazines/ Newspapers), digital goods (computer software, video games, DVD) and many more as they lose value with time due to deterioration and/ or obsolescence. Perishable goods can be broadly classified into two main categories based on: (i) Deterioration (ii) Obsolescence .Deterioration refers to damage, spoilage, vaporization, depletion, decay (e.g. radioactive substances), degradation (e.g. electronic components) and loss of potency (e.g. chemicals and pharmaceuticals) of goods. Obsolescence is loss of value of a product due arrival of new and better product. Perishable goods have continuous or discrete loss of utility and therefore can have either fixed life or random life. Fixed life perishable products have a deterministic, known and definite shelf life and examples of such goods are pharmaceuticals, consumer packed goods and photographic films. On the other hand, random life perishable products have a shelf life that is not known in advance and variable depending on variety factors including storage atmosphere. Items are discarded when they spoil and the time to spoilage is uncertain. For example, fruits, vegetables, dairy products, bakery products etc., have random life.

A large number of researchers developed the models in the area of deteriorating inventories. At first Whitin (1957) considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Various types of inventory models for items deteriorating at a constant rate were discussed by Roy Chowdhury and Choudhuri (1983). A complete survey of the published literature in mathematical modeling of deteriorating inventory systems is given by Raafat (1991). Goyal and Giri (2001) developed recent trends of inventory models for deteriorating items. Teng and Chang (2005) determined economic production quantity in an inventory model for deteriorating items.

Supply uncertainty can have a drastic impact on firms who fail to protect against it. Supply uncertainty has become a major topic in the field of inventory management in recent years. Supply disruptions can be caused by factors other than major catastrophes. More common incidents such as snow storms, customs delays, fires, strikes, slow shipments, etc. can halt production and/or transportation capability, causing lead time delays that disrupt material flow. Silver (1981) appears to be first author to discuss the need for models that deal with supplier uncertainty. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem, for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Parry (1996) generalized the formulation of Parlar and Berkin (1991) by first assuming that the reorder point r is a non negative decision variable instead of being equal to zero. Kandpal and Tinani (2009) developed inventory model for deteriorating items with future supply uncertainty under inflation and permissible delay in payment for single supplier.

In this paper it is assumed that the inventory manager may place his order with any one of two suppliers who are randomly available. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 whereas state 3 denotes the non-availability of either of them. State 0 indicates that supplier 1 and supplier 2 both are available. Here it is assumed that one may place order to either one of the two suppliers or partly to both. State 1 represents that supplier 1 is available but supplier 2 is not available. State 2 represents that supplier 1 is not available but supplier 2 is available.

2. NOTATIONS, ASSUMPTIONS AND MODEL

The inventory model here is developed on the basis of following assumptions.

(a) Demand rate d is deterministic and it is d>1.

(b) We define *Xi* and *Yi* be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where i=1, 2. We specifically assume that $X_i \sim exp\ (\lambda_i)$ and $Y_i \sim exp\ (\mu_i)$. Further *Xi* and *Yi* are independently distributed.

(c) Ordering cost is Rs. *k*/order (d) Holding cost is Rs. *h*/unit/unit time.

(e) Shortage cost is Rs. π/unit.

(f) q_i= order upto level i=0, 1, 2 (g) r=reorder upto level; *qi* and *r* are decision variables.

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(h) Time dependent part of the backorder cost is Rs. $\hat{\pi}$ /unit/time.

(i) Purchase cost is Rs. c/unit

(j) T_{00} is the expected cycle time. T_{00} is a decision variable.

The policy we have chosen is denoted by (q_0, q_1, q_2, r). An order is placed for q_i units i=0, 1, 2, whenever inventory drops to the reorder point r and the state found is i=0, 1, 2.When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of r units is reduced. If the process stays in state 3 for longer time then the shortages start accumulating at rate of d units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to q_i +r units where i=0, 1, 2. The cycle of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length.

i.e. Ac (q_0, q_1, q_2, r) $=\dfrac{C_{00}}{T_{00}}$ where, C_{00}=E (cost per cycle) and T_{00}=E (length of a cycle). Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The solution is provided in the lemma (refer Parlar and Perry [1996]).

$A(q_i, r, \theta)$ = cost of ordering+ cost of holding inventory+ cost of items that deteriorate during a single interval that starts with an inventory of q_i units and ends with r units.

$$A(q_i, r, \theta) = k + \frac{1}{2}\frac{hq_i^2}{(d+\theta)} + \frac{hrq_i}{(d+\theta)} + \frac{\theta c q_i}{(d+\theta)}$$
$$i = 0,1,2$$

Lemma 3.1:

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right]$$

i=0,1,2 and $C_{30} = \overline{C} + \sum_{i=1}^{2} \rho_i C_{i0}$

where $\rho_i = \dfrac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r - (d+\theta)\right) + \left(\pi\delta d + h(d+\theta) + \hat{\pi}\right) - \theta c\delta\right] + \frac{\theta c}{\delta}$$

(refer Parlar and Perry [1996]).

Theorem 3.2: The Average cost objective function for deteriorating items in case of two suppliers is given by Ac= $\frac{C_{00}}{T_{00}}$,

$$AC = \frac{C_{00}}{T_{00}} = \frac{A(q_0, r, \theta) + P_{01}C_{10} + P_{02}C_{20} + P_{03}(\bar{C} + \rho_1 C_{10} + \rho_2 C_{20})}{\frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\bar{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

4. NUMERICAL ANALYSIS

In this section we verify the results by a numerical example. We assume that

(a) k =Rs. 5/order, c=Rs.1/unit, d=20/units , θ=4, h=Rs. 5/unit/time, π=Rs. 250/unit, $\hat{\pi}$ =Rs.25/unit/time, λ_1=0.25, λ_2=1, μ_1=2.5, μ_2=0.5

With these parameters the long run probabilities are obtained as p_0=0.303, p_1=0.606, p_2 =0.030 and p_3=0.061. The optimal solution is obtained as

q_0=1.86448, q_1=10.490, q_2=15.44333, r=20.4988 and

Ac= $\frac{C_{00}}{T_{00}}$ = 197.81

5. SENSITIVITY ANALYSIS

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed. We resolve the problem to find optimal values of q_0, q_1, q_2, r and Ac. The optimal values of q_0, q_1, q_2 and Ac are shown in table 5.1.

Table 5.1

Sensitivity Analysis Table by varying the parameter values of μ_1

μ_1	q_0	q_1	q_2	r	Ac
2.4	0.7827	10.6468	15.3941	20.4989	234.57
2.5	1.86448	10.4905	15.4433	20.4988	197.81
2.6	3.2545	10.3081	15.497	20.4987	186.73
2.7	5.03811	10.0839	15.5592	20.4986	181.83
2.8	7.3501	9.7962	15.6347	20.4984	179.52

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases the value of q_0, q_1, q_2 and r decreases which result in decrease in average cost.

6. MULTIPLE SUPPLIERS

We generalize the model and consider the case where there are M suppliers, and at any time suppliers may be available or not available which we represent as ON or OFF state. The stochastic process representing the supplier availabilities would have 2^M states: 0,1,2,------- 2^M-1. State 0 would correspond to the situation where all the suppliers are ON , state 1 would correspond to only the M^{th} supplier being OFF etc. and finally 2^M-1 would correspond to all being OFF.

The system of equations for C_{i0} is obtained as

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)A(q_i, r, \theta) + \sum_{j=1}^{2^M-1} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right]$$

i=0,1,2,-------2^M-2

and $$C_{2^M-1,0} = \overline{C} + \sum_{i=1}^{M} \rho_i C_{i0} \quad \text{Where } \rho_i = \frac{\mu_i}{\delta}$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^{2}}\left[he^{\frac{\delta r}{(d+\theta)}}\left(\delta r-(d+\theta)\right)+\left(\pi\delta d+h(d+\theta)+\hat{\pi}\right)-\theta c\delta \right]+\frac{\theta c}{\delta}\ ,$$

with $\delta = \sum_{j=i}^{M} \mu_j$

Equation T_{i0} are written in a similar way.

Solving the above equations require the exact solution for the transient probabilities $P_{ij}(t)$ of the CTMC with the 2^M states which appears to be a formidable task.

As the number of suppliers is very large, that is we have a situation approximating a free market, we can develop a much simpler model by assuming that if an order needs to be placed and at least one supplier is available, then the order quantity will be q units regardless of which supplier is available. We combine the first 2^M-1 states where at least one supplier is available and define a super state o. The last state denoted by $\bar{\imath}$, is the state where all the suppliers are OFF.

With these assumptions the expected cost and the expected length of a cycle are obtained as

$C\,o\,o = A(q,r,\theta) + P\,o\,\bar{\imath}\,(q/(d+\theta))\,C\,\bar{\imath}\,o(r)$,

$T\,o\,o = q/(d+\theta) + P\,o\,\bar{\imath}\,(q/(d+\theta))/M_{\mu}$

$AC = C\,o\,o\,/\,T\,o\,o$

Conclusion: When the number of suppliers become large the objective function of multiple suppliers problem reduces to that of classical EOQ model. This can be shown by arguing that as the length of stay in state $\bar{\imath}$ is exponential with parameter M_{μ} it becomes a degenerate random variable with mass at 0; that is the process never visits or stays in state $\bar{\imath}$.

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