COMPARATIVE STUDY OF SNR ESTIMATION TECHNIQUES FOR RAYLEIGH AND RICIAN CHANNELS

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Abstract: The Orthogonal Frequency Division Multiplexing (OFDM) is a promising approach to achieve higher data rates with sufficient performance in frequency selective channels. Hence it is the proposed multiplexing scheme for future Wireless communication and Wi-Max, and these schemes uses Adaptive Modulation & Coding (AMC) to select the appropriate modulation & coding scheme. The major input parameter for AMC is Signal to Noise ratio (SNR). So SNR estimation is required to operate the system with higher data rates. In this paper we compare various SNR estimators for different channel models based on preamble, and the suitable estimators are suggested for different channel models.

Keywords: OFDM, MMSE estimator, Boumard’s estimator, Ren’s estimator.

1. INTRODUCTION

Orthogonal Frequency Divisional Multiplexing (OFDM) scheme has been adopted in many broadband systems. Compared to the conventional Single Carrier (SC) modulation schemes, OFDM enables the efficient use of the available channel bandwidth and easy control of the signal spectrum mask in the Transmitter and the Receiver side. OFDM support high data rates, allows simple equalization, and is robust to constant timing offset due to the adoption of Cyclic Prefix (CP) for which it is suggested as a multiple access scheme for future wireless communication. OFDM provides strong robustness against Inter symbol interference (ISI) by dividing the broadband channel into many narrowband sub channels in such a way that the attenuation across each sub channel stays flat. In OFDM the sub carrier frequencies are chosen as orthogonal to each other meaning that cross talk between the sub channels is eliminated and inter carrier guard bands are not required. Orthogonalization of sub channels is performed with low complexity by using FFT. A major advantage of OFDM systems is its ability to divide the input high rate data stream into many low-rate streams [11]-[12] that are transmitted in parallel, thereby increasing the symbol duration and reducing the inter symbol interference over frequency-selective fading channels.

To optimize the system performance it is required to estimate SNR at the receiver. In OFDM average SNR is estimated. There are two general categories of average SNR estimators. Data-aided (DA) estimators are based on either perfectly or estimated knowledge of the transmitted data. However, a certain portion of data is needed for estimation purposes, which reduces bandwidth efficiency. Blind or in-service estimators derive SNR estimate from an unknown information-bearing portion of the received signal preserving efficiency at the cost of decreased performance. For packet based communications, block of information data is usually preceded by several training symbols (preambles) of known data used for synchronization and equalization purposes.

Therefore, DA SNR estimators can utilize preambles without additional throughput reduction. Most of the SNR estimators proposed in the literature so far are related to single carrier transmission. In [10], a detailed comparison of various algorithms is presented, together with the derivation of the Cramer-Rao bound (CRB)[9]. Most of these algorithms can be directly applied to OFDM systems in Additive White Gaussian Noise (AWGN) [2], while the SNR estimation in frequency selective channels additionally requires efficient estimation of Channel State information (CSI). In this paper, we compare various algorithms for the average SNR estimation in wireless OFDM systems. The SNR per subcarrier can be additionally estimated using channel estimates and the estimated average SNR. The proposed estimator utilizes preamble structure, proposed by Morelli and Mengali in [3]. Compared to Schmidl and Cox synchronization method [4], it allows synchronization over a wider frequency offset range with only one preamble, hence reducing the training symbol overhead. Since the proposed estimation algorithm relies on the signal samples at the output of the FFT, its performance depends strongly on the given preamble structure. The remainder of this paper is organized as follows. Section 2 provides the system model and specifies the SNR estimation problem. The Section 3 describes various channel models like AWGN, Rayleigh, and Rician, In Section 4 various SNR estimators are described such as Minimum Mean Square Error (MMSE), Boumard’s, Ren’s estimators are briefly described and according SNR estimates are given. The performance is analyzed by computer simulations in Section 5. Finally, some concluding remarks are given in Section 6.

2. SYSTEM MODEL

The OFDM system design exploits the frequency selective channels by adaptive transmission. In this type of transmission particular range of frequencies suffer from the interference or attenuation, the carriers within that range can be disabled or made to run slower. Here the adaptable transmission parameters are bandwidth, data rate and power. In many wireless OFDM systems the transmission is normally organized in frames. The frame structure is given in Fig.1.

![Frame structure](image)

Fig. 1. Frame structure

From the Fig.1the sequence of data symbols is preceded by several preambles of known data for synchronization and channel estimation purposes. We consider general model of frame structure composed of K preambles where each preamble contains N modulated subcarriers. The SNR estimation is performed in frequency domain given model which contains only frequency domain characteristic of received signal in frequency selective AWGN channel.

Let \( C(k, n) \) denote the complex data symbol on \( n \)th subcarrier in \( k \)th preamble, where \( k = 0, \ldots, K - 1 \) and \( n = 0, \ldots, N - 1 \). It is assumed that modulated subcarrier has unit magnitude, i.e.

\[
|C(k, n)|^2 = 1
\]  

(1)

Then at the receiver assuming perfect synchronization after FFT. Then the received signal on \( n \)th subcarrier in \( k \)th preamble can be given as

\[
Y(k, n) = \sqrt{S}C(k, n)H(k, n) + \sqrt{\eta} \eta(k, n)|C(k, n)|^2
\]  

(2)

where \( \eta(k, n) \) is sampled complex zero-mean AWGN of unit variance, \( S \) and \( \sqrt{\eta} \) are transmitted signal power and noise power on each subcarrier, respectively, and

\( H(k, n) \) is the channel frequency response given by

\[ H(k, n) = H_{k, n} \]

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\[ H(k, n) = \sum_{i=1}^{L} h_i(kT_s) e^{-j2\pi n_i \nu / \lambda} \]  

Where \( h_i(kT_s) \) and \( t_i \) denote the channel \( i^{th} \) path gain and delay during the \( k^{th} \) preamble, respectively, \( T_s \) is the duration of the OFDM preamble and \( L \) is the length of the channel memory.

Our initial assumption is that channel is constant during the whole frame, since we consider SNR estimation algorithms for the purposes of adaptive transmission. Therefore, time index \( k \) is omitted during the estimation procedure, i.e. \( H(k, n) \) is replaced by \( H(n) \). Then the average SNR of the \( k^{th} \) received OFDM preamble can be expressed as

\[
\rho_{av}(k) = \frac{E\left[\left\{\sqrt{\mathcal{C}}(k, n)H(n)\right\}^2\right]}{E\left[\left\{\sqrt{\mathcal{W}}(k, n)\right\}^2\right]} = \frac{S}{W} \tag{4}
\]

Where

\[
\sum_{n=0}^{N-1} |H(n)|^2 = N \tag{5}
\]

is satisfied, while the SNR of the \( n^{th} \) subcarrier is given by

\[
\rho(k, n) = \frac{E\left[\left\{\sqrt{\mathcal{C}}(k, n)H(n)\right\}^2\right]}{E\left[\left\{\sqrt{\mathcal{W}}(k, n)\right\}^2\right]} = \frac{S|H(n)|^2}{W} = \rho_{av} |H(n)|^2 \tag{6}
\]

3. VARIOUS WIRELESS CHANNEL MODELS

The performance of any wireless system is affected by the medium of propagation, namely the characteristics of the channel. The received signal consists of a combination of attenuated, reflected, refracted, and diffracted replicas of the transmitted signal. On top of all this, the channel adds noise to the signal and can cause a shift in the carrier frequency, if the transmitter or receiver is moving (Doppler Effect). Understanding these effects on the signal is important because the performance of a radio system is dependent on the radio channel characteristics.

3.1 AWGN channel

For the Additive White Gaussian Noise (AWGN) channel the received signal is equal to the transmitted signal with some portion of white Gaussian noise added [10]. This channel is particularly important for discrete models operating on a restricted number space, because this allows optimizing the circuits in terms of their noise performance.

\[ s(t) = s(t) + n(t) \tag{7} \]

Where \( n(t) \) is a sample of a Gaussian random process. This represents white Gaussian noise.

3.2 Rayleigh Fading

Rayleigh fading is caused by multipath reception. The mobile antenna receives a large number, say \( N \) reflected and scattered waves. Because of wave cancellation effects, the instantaneous received power seen by a moving antenna becomes a random variable, dependent on the location of the antenna. The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received signal power [15]. It describes the probability of the signal level being received due to fading. The relative phase of multiple reflected signals can cause constructive or destructive interference at the Receiver. This is experienced over very short distances (typically at half wavelength distances), thus is given the term fast fading. These variations can vary from 10-30dB over a short distance.

\[ \rho_{av, MIMO} = \frac{\hat{s}_{MIMO}}{\hat{W}_{MIMO}} \tag{8} \]

Where

\[ \hat{S}_{MIMO} = \left| \sum_{n=0}^{N-1} \mathcal{Y}(n) \mathcal{C}(n) \right|^2 \tag{9} \]

And

\[ \hat{W}_{MIMO} = \frac{1}{N} \sum_{n=0}^{N-1} |\mathcal{Y}(n)|^2 - \hat{S}_{MIMO} \tag{10} \]

are the MMSE estimates of \( S \) and \( W \), respectively.

4.4. Boccardo’s Estimator

The main advantage of this estimator is that it exists both in time and frequency domain. It is the second-order moment-based SNR estimator for MIMO OFDM system in slow varying channel in both time and frequency domain.

Assume that the channel is time-invariant and that two identical preambles are used for SNR estimation, i.e. \( k = 0, 1 \) and \( C(0, n) = C(1, n) = C(n) \), for \( n = 1, \ldots, N \). Average SNR estimate can be expressed as

\[ \hat{\rho}_{av, Boc} = \frac{\hat{s}_{boc}}{\hat{W}_{boc}} \tag{11} \]
Where
\[ S_{\text{Ren}} = \frac{1}{N} \sum_{n=0}^{N-1} \left| \tilde{H}(n) \right|^2 \] \hspace{1cm} (12)
\[ \tilde{W}_{\text{Ren}} = \frac{1}{4N} \sum_{n=0}^{N-1} \left| \frac{C(n-1)(Y(0,n) + Y(1,n)) - C(n)Y(0,n-1) + Y(1,n-1))}{2} \right|^2 \] \hspace{1cm} (13)

These are the estimates of S and W, respectively, and
\[ \tilde{H}(n) = \frac{C\ast(n)}{2} (Y(0,n) + Y(1,n)) \] \hspace{1cm} (14)

Is the least square estimate of \( H(n) \) averaged over two preamble symbols, Using \( \tilde{H}(n) \) SNR on \( n^{th} \) subcarrier is estimated as
\[ \hat{\rho}(n) = \frac{\left| \tilde{H}(n) \right|^2}{\tilde{W}_{\text{Ren}}} \] \hspace{1cm} (15)

4.3 Ren’s Estimator

It is the estimator which is very robust against frequency selectivity [5]. This also based on second-order moments and more accurate compared with Boumard’s estimator, and it requires two identical preambles for the estimation of average SNR. The average SNR is given as
\[ \hat{\rho}_{\text{av, Ren}} = \frac{S_{\text{Ren}}}{\tilde{W}_{\text{Ren}}} \] \hspace{1cm} (16)

Where
\[ \tilde{W}_{\text{Ren}} = \frac{4}{N} \sum_{n=0}^{N-1} \left| \text{Im}[Y(0,n)C\ast(0,n)\tilde{H}(n)/|\tilde{H}(n)|] \right|^2 \] \hspace{1cm} (17)

And
\[ S_{\text{Ren}} = \frac{1}{N} \sum_{n=0}^{N-1} \left| Y(0,n) \right|^2 - \tilde{W}_{\text{Ren}} \] \hspace{1cm} (18)

The \( \tilde{H}(n) \) defined in (14) will be used to estimate W and S. It is shown that the performance is independent of the channel frequency response estimation although the estimated channel states are used in average SNR estimation. Additionally, SNR on \( n^{th} \) subcarrier is estimated as in (15) using the noise power estimate from (16).

5. SIMULATION RESULTS

The performance of various SNR estimators is compared under various channel conditions using Monte-Carlo simulations. The evaluation is measured in terms Normalized mean squared error (NMSE) of average SNR and SNR per subcarrier, and these are given as
\[ \text{NMSE}_{\text{av}} = \frac{1}{N_t} \sum_{t=1}^{N_t} \left( \frac{\hat{\rho}_{\text{av},t} - \rho_{\text{av}}}{\rho_{\text{av}}} \right)^2 \] \hspace{1cm} (19)

Where \( \hat{\rho}_{\text{av},t} \) is the estimated average SNR in the \( t^{th} \) trial, and \( \rho_{\text{av}} \) is actual average value. Similarly
\[ \text{NMSE}_{\text{ac}} = \frac{1}{N_t} \sum_{t=1}^{N_t} \left( \frac{\hat{\rho}_{\text{ac},t} - \rho_{\text{ac}}}{\rho_{\text{ac}}} \right)^2 \] \hspace{1cm} (20)

The \( \hat{\rho}(n) \) is the \( i^{th} \) trial estimated value of \( \rho(n) \).

These simulations are performed for OFDM systems parameters under Wi-Max specifications[8] which are given as

<table>
<thead>
<tr>
<th>Name of the parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of sub carriers</td>
<td>256</td>
</tr>
<tr>
<td>Cyclic prefix</td>
<td>32</td>
</tr>
<tr>
<td>Channel models</td>
<td>a)AWGN, b)Rayleigh, c) Rician</td>
</tr>
<tr>
<td>No. of channel Taps</td>
<td>3</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>10</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>Qpsk</td>
</tr>
<tr>
<td>Estimators</td>
<td>Ren’s, Boumard’s, MMSE for Q=2,4,8</td>
</tr>
</tbody>
</table>

Fig.4. shows the NMSE_{av} for AWGN channel and this is compared with Normalized Cramer Rao Bound (NCRB) to assess the performance where the NCRB is given as \( NCRB = \frac{1}{N} \left( \frac{1}{\rho_{\text{av}}} + 1 \right) \)

Fig. 5. NMSE vs Average SNR in Rayleigh and Rician channels

From Fig.5, frequency selective channels like Rayleigh and Rician the Boumard’s estimator is highly sensitive to frequency selectivity. For Channel b and c due to multipath fading more impact on Boumard’s estimator due to which it is very sensitive to frequency selective channels. Fig.6. shows the average SNR per subcarrier for channel b and c. For lower SNR values all estimators perform bad and also it is observed that as the SNR
values are increasing the NMSE\textsubscript{av} is approaching to NMSE\textsubscript{pr} are similar.

Fig. 6. NMSE vs Average SNR per sub carrier in Rayleigh and Rician channels

6. CONCLUSIONS

In this paper, we compare various SNR estimators for OFDM systems using various channel models. As the Boumard's estimator is not performed well for low values of SNR, Similarly the Ren's and Boumard's estimator are very immune to frequency selectivity. From the derivation point of view the complexity of PS estimator is lower than others where it requires N multiplications. The performance of these estimators can be further improved by combining estimated average noise power with more sophisticated channel estimation algorithms using pilot subcarriers where the PS estimator uses only one preamble and also it does not require any knowledge of transmitted sub carrier.

References