Analysis of Electrically Conducting Magneto Hydrodynamic Viscous Flow between Two Moving Parallel Permeable Plates

C.K.Kirubhashankar, Dr.S.Ganesh, A.Mohamed Ismail

Abstract— **The investigation of the paper is to analyze the electrically conducting magneto hydrodynamic viscous flow between two moving parallel permeable plates. These plates are considered to be moving towards and away from each other with the constant velocity. Inverse solutions of the equations of motion are obtained assuming certain forms of the stream function. Analytical expressions for the stream function, fluid velocity components are derived. Stationary solutions in the form of jets are studied, which gives two components the first one is the motion corresponding to the potential flow and second one is the jet behavior (non-potential flow component).**

Index Terms—**Magneto hydrodynamic, viscous flow, porous media, Hermite's differential equation.**

I. Introduction

The study of flow through porous medium is of considerable interest in the field of petroleum engineering concerned with the movement of oil and gas, ground water hydrology, heat transfer in cooling systems and chemical engineering for filtration process etc.

The pioneering study of liquid motion between two parallel disks, moving towards each other or in opposite directions with a constant velocity was carried by Aristov and Gitaman[1]. Aristov and Gitaman gave a formulation of the problem that the motion of a viscous incompressible liquid between two parallel sides, moving towards each other or in opposite directions, is considered. The description of possible conditions of motion is based on the exact solution of the Navier-Stokes equations. The stability of the motion is analyzed for different initial perturbations. From the mathematical point of view, there exists a large class of process such that the motion of liquid between two parallel disks, moving towards each other or in opposite directions with a constant velocity. These include such processes as the motion of liquid through a hydraulic pump, digging through slurry and the motion of underground water can also be described with a help of the current model. It is observed for the different types of hydro dynamical problems, the mathematical descriptions are the same. So it can be explained to the water motion in a hydraulic pump (when impermeable disks are moving toward or away) similarly to the motion of underground water (when permeable disks are fixed). The second case is due to the water motion through porous media. These problems are interesting because some of their solutions, through analytically obtained. The purpose of the current work is to present details of some new two dimensional solutions of the Navier-stokes equations governing the steady-state stationary viscous flow of an incompressible Newtonian electrically conducting fluid associated with the movement of disks. The disks are supposed to be nonelectrically conducting under the influence of an external magnetic field of constant strength applied normal to the disks.

This work deals with a description of the types of possible instability of such motion. Craik and Criminale [2] described a procedure for finding classes of exact solutions of the Navier-Stokes equations. These solutions consist of a 'basic flow' with spatially uniform rates of strain and a 'disturbance' of a planar form: the disturbance is continuously distorted by the basic flow but nevertheless remains planar at all times. A somewhat similar formulation was given by Lagnado *et al*. [3], but was restricted to two-dimensional basic flows and the authors were unaware that their liberalized approximation is in fact an exact solution for single plane wave modes. Craik et al [4] described a procedure for finding classes of exact solutions of the Revilin-Erickson equation of unsteady non-Newtonian fluids. Ganesh.S and Krishnambal.S [5] studied unsteady MHD stokes flow of a viscous fluid between two parallel porous plates. They considered the fluid being withdrawn through both walls of the channel at the same rate. The flow of Newtonian and non-Newtonian fluids between parallel disks rotting about a common axis have been reviewed by Rajagopal[6]. Kempegowda, M. and Balagondar P.M[7] consider the flow of non-Newtonian fluid governed by Revilin-Erickson constitutive equation and A. Zeb, A.M.Siddiqui and M.Ahmed [8] considered flow of an incompressible Newtonian fluid produced by two parallel plates, moving towards and away from each other with constant velocity. S. Asghar, T. Hayat, A.M. Siddiqui [9] considered Moving boundary value problem in non-Newtonian fluid. Exact analytical solution for the flow of second-grade fluid for a rigid moving plate oscillating in its own plane, is obtained. The Doppler effect has been

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observed due to the motion of the plate. The shearing stress on the plate is also calculated. It is concluded that the solutions for stationary porous boundaries can be obtained from the solutions of moving rigid boundaries. Hayat T., Sohail Nadeem, Asghar S. and Siddiqui A.M. [10] discussed the flow induced by non-coaxial rotations of porous oscillating disk and a fluid at infinity.

In the present work, a weakly nonlinear magnetic field is introduced between two disks. The flow of a viscous fluid is analysed and the differential equation governing the fluid motion is based on the hydro magnetic flow induced in the fluid in the presence of a uniform magnetic field and the Darcy's law which accounts for the drag exerted by the magnetic effect. The governing nonlinear differential equations are solved analytically using separation of variables method. Furthermore, an instability analysis has been performed.

II. Formulation of the Problem

Consider the motion of viscous incompressible fluid between two parallel disks moving towards each other in the case when $h << l$ (where h is the distance between the disks and l is the length of the disks). Let us assume that the horizontal velocity does not depend on the vertical coordinate where as the vertical velocity depends linearly on the distance between the disks.

In this case, the Navier-Stokes equation have the following form: $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=2q$ (1)

Momentum equations

$$\frac{\partial q}{\partial t}+(q\cdot\nabla)q=-\frac{\nabla p}{\rho}+\frac{1}{\rho}J\times B+\nu\nabla^2 q-\frac{\nu}{k}q \quad (2)$$

Since we consider the hydro dynamic flow induced in the fluid in the presence of a uniform magnetic field B_0 normal to the plate therefore we come across the term $\frac{1}{\rho}J\times B$ in the momentum equation.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2 u}{\rho}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\nu}{k}u \quad (3)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2 v}{\rho}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\nu}{k}v \quad (4)$$

where $u=u(x,y,t), v=v(x,y,t), w=-2qz$

and $p=p(x,y,t)+(-4q^2+\frac{\nu}{k}2q)\frac{z^2}{2}$

Eliminating the pressure term and introduce the vorticity

$$\omega=\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x} \quad (5)$$

Therefore the vorticity equations is given by

$$\frac{\partial \omega}{\partial t}+u\frac{\partial \omega}{\partial x}+\omega\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}+\omega\frac{\partial v}{\partial y} =-\frac{\sigma B_0^2\omega}{\rho}+\nu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)-\frac{\nu}{k}\omega \quad (6)$$

III. Method of Analysis

For convenience of analysis let us select the potential components from the horizontal components of the velocity and introduce the flow function,

$$u(x,y)=qx+\frac{\partial\psi}{\partial y} \ \& \ v(x,y)=qy-\frac{\partial\psi}{\partial x} \quad (7)$$

where ψ is the stream function. The vorticity equation will be in the following form

$$\frac{\partial \omega}{\partial t}-\{\psi,\omega\}=-q\left(\frac{\partial(x\omega)}{\partial x}+\frac{\partial(y\omega)}{\partial y}\right)-\frac{\sigma B_0^2\omega}{\rho}+\nu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)-\frac{\nu}{k}\omega \quad (8)$$

where $\{\psi,\omega\}=\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}$

$\psi=0$ is a solution of the above equation (8) which corresponds to fluid potential motion, known as the motion near the stagnation point. Let us consider the potential one-dimensional perturbation $\delta\psi$. This perturbation is expressed by

$$\psi=\hat{\psi}+\delta\psi=k^{-2}(t)A(t)\cos(k(t)x) \quad (9)$$

To see the change of the vorticity in this duration we put the stream function $\delta\psi$ into equation (9) and comparing the coefficient of $x\sin(k(t)x)$ and $\cos(k(t)x)$ on both sides. We get,

$$k(t)=k(0)e^{-qt} \quad (10)$$

$$A(t)=A(0)e^{\left(-2qt-\frac{\sigma}{\rho}B_0^2-\frac{\nu t}{k}+(-1+e^{-2qt})\nu\frac{k^2(0)}{2q}\right)} \quad (11)$$

where $k(0), A(0)$ are free constants, determining the amplitude and wavelength at the initial point of time. Other stationary solutions are examined in the following section.

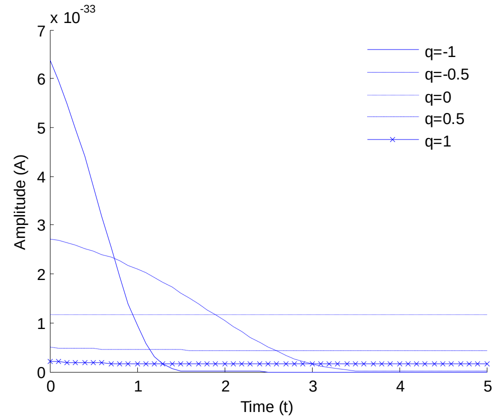

Fig 1. Amplitude versus time for different velocity of the plates

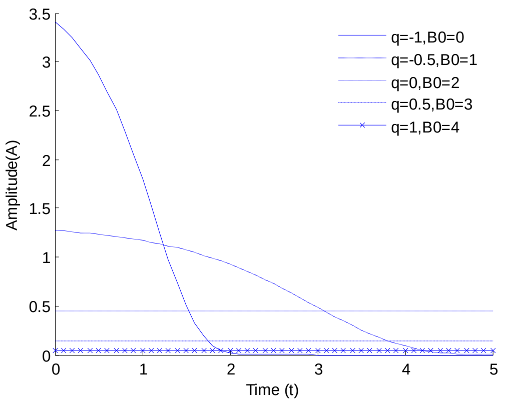

Fig 2. Amplitude versus time for different values of B_0

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IV. Solution of the Problem

Riabouchinsky type form: Here, we assume that the stream function ψ is linear in *x* and has the following form:

$$\psi(x,y) = xF(y) + G(y) \quad (12)$$

where $F(y)$ and $G(y)$ are functions in *y*.

On substituting (6) the compatibility equation (8) yields the following equations:

$$-xF(y)F'''(y) - F'(y)G'''(y) + xF'(y)F''(y) + G'(y)F''(y)$$
$$= -qF''(y) - qxyF'''(y) - qyG'''(y) - qxF''(y) - 2qG''(y)$$
$$-\frac{\sigma}{\rho}B_0^2(xF''(y) + G''(y)) + \nu(xF'^{\nu}(y) + G'^{\nu}) - \frac{\nu}{k}(xF'' + G'')$$

Comparing '*x*' powers, we get two equations

$$F(y)F'''(y) - F'(y)F''(y) = q(3F''(y) + yF'''(y)) + \left(\frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)F''(y) + \nu F'^{\nu}(y) \quad (13)$$

$$F(y)G'''(y) - G'(y)F''(y) = q(yG'''(y) + 2G''(y)) + \left(\frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)G''(y) - \nu G'^{\nu}(y) \quad (14)$$

Consider the particular case of F(*y*) = *ay* as linear function of '*y*'. From (14) we have,

$$ayG'''(y) = q(yG'''(y) + 2G''(y)) + \left(\frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)G''(y) - \nu G'^{\nu}(y)$$

i.e., $\nu G'^{\nu}(y) = (q-a)yG'''(y) + \left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)G''(y)$

Integrating twice, we obtain the following equation

$$G''(y) - \frac{(q-a)}{\nu}yG'(y) - \frac{1}{\nu}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)G(y) = 0$$

which has the form of Hermite's differential equation when two conditions are satisfied:

$\frac{q-a}{\nu} = 2$ and $\frac{1}{2\nu}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)$ is non-negative integer.

The solution of this equation has the following form:

$$\phi(y) = \frac{d^n}{dy^n}\left[A\exp\left(\frac{1}{2\nu(3+n)}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)y^2\right)\right]$$
$$= A\sum_{k=0}^{\left[\frac{n}{2}\right]}(-1)^k \frac{n!}{k!(n-2k)!}2^{n-2k}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)^{n-2k} y^{n-2k}$$

where the relation between *a* and *q* is

$a = -\frac{1+n}{3+n}q$, $n \in [0,\infty]$ and

$$[n/2] = floor\left(\frac{n}{2}\right)\begin{cases} n/2, & n\ is\ even \\ n - 1/2, & n\ is\ odd \end{cases}$$

Thus the solution of equation (11) can be written as

$$\psi(x,y) = -\frac{1+n}{3+n}qxy + A\sum_{k=0}^{\left[\frac{n}{2}\right]}(-1)^k \frac{n!}{k!(n-2k)!}2^{n-2k}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)^{n-2k} y^{n-2k} \quad (15)$$

Axial velocity potential of the fluid is given by

$$u(x,y) = qx\left(\frac{2}{3+n}\right) + A\left[\frac{2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}}{2\nu(3+n)}\right]^n \sum_{k=0}^{\left[\frac{n}{2}\right]}(-1)^k \frac{n!}{k!((n-1)-2k)!}2^{n-2k} y^{(n-1)-2k}$$

and the transverse velocity of the fluid is given by

$$v(x,y) = 2qy\left(\frac{2+n}{3+n}\right)$$

In equation (15), the first term denotes the liquid motion corresponding to the potential flow component and the second term denotes the jet behavior. Since $q < 0$, $n > 0$, $\nu > 0$, it can be seen that this second term approaches zero for $y \to \pm\infty$.

V. Conclusion

In this investigation, we recognize that, if the disks are moving apart ($q < 0$) and non-Newtonian fluid is unstable up to certain time then it is stable thereafter. This is because of the wave number $k(t)$ increases in the course of time. But in case of the disks are moving towards each other ($q > 0$) the non-Newtonian fluid is stable with both the amplitude $A(t)$ and the wave number $k(t)$ at all time.

The solution for the stream function through Hermite's differential equation is given by

$$\psi(x,y) = -\frac{1+n}{3+n}qxy + A\sum_{k=0}^{\left[\frac{n}{2}\right]}(-1)^k \frac{n!}{k!(n-2k)!}2^{n-2k}\left(2q + \frac{\sigma}{\rho}B_0^2 + \frac{\nu}{k}\right)^{n-2k} y^{n-2k}$$

In this equation the first term denotes the liquid motion corresponding to the potential flow component and the second term denotes the jet behavior.

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