Hotelling's T^2 charts with variable sample size, sampling interval, and warning limits

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Abstract— **The idea of variable warning limit is applied for *variable sample size and sampling interval* (VSSI) Hotelling's T^2 charts in order to lessen the frequency of switches between the pairs of values of the sample sizes and sampling interval lengths of those charts during their implementations. Expressions for performance measures for the *variable sample size, sampling interval, and warning limits* (VSSIWL) T^2 charts are developed. The performances of these charts are compared numerically with that of VSSI and VSSI (1, 3) T^2 charts, where VSSI (1, 3) T^2 charts are the VSSI T^2 charts with runs rule (1, 3) for switching between the pairs of values of sample sizes and sampling interval lengths. It is observed that VSSIWL T^2 charts can be set to yield exactly similar performances as that of VSSI (1, 3) T^2 chart, to yield the performances in-between that of VSSI (1, 3) and VSSI T^2 charts, or to yield significantly lower switching rate than even that of VSSI (1, 3) T^2 charts and the statistical performance almost similar to that of VSSI and VSSI (1, 3) T^2 charts.**

Index Terms—**Adaptive control chart, average number of samples to signal, average number of observations to signal, average number of switches to signal, Multivariate Statistical process control, Steady-state average time to signal.**

I. INTRODUCTION

Control chart developed by W. A. Shewhart in 1920s is an effective on-line statistical process control technique used worldwide for monitoring quality of manufacturing processes and products. It detects shifts in parameters of quality characteristics of the processes/products from their target values. Nowdays it is also extensively being used for monitorings nonmamufacturing peocesses. It detects large shifts very efficiently, however, is relatively less efficient in detecting small shifts. A number of modifications have been proposed to the original Shewhart control chart in order to improve its efficiency in detecting small shifts. The well-known modifications include use of warning limits [1], runs rules for signaling [2], integration of two charts [3], and adaptive (variable) design parameters [4].

Shewhart Control chart has three design parameters, viz, sampling interval, sample size, and control limit(s). The chart is termed to be static if all these parameters are fixed throughout the period of process monitoring, whereas it is termed as adaptive if at least one of the design parameters is variable and takes a value for a trial according to the location(s) of the control statistic for the previous trial(s). The variable parameters are called adaptive paramerers. The general principal of choosing values of the adaptive parameters for a trial is as follows. If the last plotted point(s) indicate possibility of shift, choose short sampling interval and/or large sample size and/or narrow control limits for the next trial. On the other hand, if that indicate possibility of safe or in-control process, choose long sampling interval and/or small sample size and/or wide control limits for the next trial.

Reynolds et al. [4] were the first to propose an adaptive control chart. They proposed *variable sampling interval* (VSI) $\overline{X}$ charts. Later on, Prabhu, et al. [5] and Costa [6] independently proposed variable sample size $\overline{X}$ charts. Prabhu, et al. [7] proposed *variable sample size and sampling interval* (VSSI) $\overline{X}$ charts. Costa [8] proposed the adaptive $\overline{X}$ charts in which all the three design parameters are variable. Tagaras [9] reported an extensive survey of the research on adaptive control charts until 1997.

The idea has also been applied to other univariate and multivariate Shewhart charts as well as to cumulative sum and exponentially weighted moving average control charts. Further references include, for example, Zimmer et al. [10], Aparasi and Haro [11], Costa and Rahim [12], Epprecht et al. [13], Reynolds and Stoumbos [14], Wu et al. [15], Yu and Hou [16], Chen [17], Wu et al.[18], Yang and Su. [19], Mahadik and Shirke [20], Jiang et al. [21], Jensen et al. [22], Luo et al. [23], Wu et al.], Shi et al. [25], Celano [26], Faraz and Moghadam [27], Mahadik and Shirke [28, 29], Li and Wang [30], Epprecht et al. [31], Shu et al. [32], Dai et al.[33], Mahadik [34-40], Faraz and Saniga [41], Nenes [42], Kooli and Limam [43], and Lee [44].

The general conclusion of the research on adaptive control charts is that adapting one or more design parameters of a control chart increases its statistical as well as economic performances significantly.

II. THE RESEARCH PROBLEM

The weakness of any adaptive control chart is the inconvenience in its administration due to frequent switches between the values of its adaptive design parameters.

In order to reduce the frequency of switches between sampling interval lengths of VSI charts, Amin and Letsinger [45] proposed the use runs rules for switching between these lengths. Mahadik [34-35] studied the exact properties of VSI charts with such runs rules. Further, Mahadik [37-38] applied the runs rules for switching between the pairs of values of sample sizes and sampling interval lengths of VSSI

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$\overline{X}$ and VSSI T^2 charts and have shown that switching runs rule (1, 3), described later in this paper, is the best runs rule for reducing the frequency of switches.

Alternatively, Mahadik [39-40] suggested the use of variable warning limits to reduce frequency of switches between the values of adaptive design parameters of VSI and VSSI $\overline{X}$ charts.

In the present paper, the notion of variable warning limits is applied to VSSI T^2 charts. This provides a comparable option to switching runs rule (1, 3) for reducing the frequency of switches between the pairs of values of sample sizes and sampling interval lengths of VSSI T^2 charts.

The subsequent sections provide the general description of a *variable sample size, sampling interval, and warning limits* (VSSIWL) T^2 chart. Expressions for performance measures for this chart are derived. The performances of VSSIWL T^2 charts are numerically compared with that of VSSI T^2 charts with and without switching runs rule (1, 3). An example illustrates implementation of VSSIWL T^2 charts. This is followed by the Conclusions.

III. A VSSIWL T^2 Chart

Suppose the $p > 1$ related quality characteristics $\boldsymbol{X}$ = $(X_1, X_2, ..., X_p)'$ to be monitored jointly, follow p-variate joint normal distribution with mean vector $\boldsymbol{\mu}$ and known variance covariance matrix Σ. Let $\boldsymbol{\mu_0}$ be the target mean vector. An occurrence of an assignable cause shifts $\boldsymbol{\mu}$ from $\boldsymbol{\mu_0}$ to $\boldsymbol{\mu_1} \neq \boldsymbol{\mu_0}$. A VSSIWL T^2 chart to monitor $\boldsymbol{\mu}$ is as described below.

The control statistic is T_i^2 = $n(i)\,(\overline{\boldsymbol{X}}_{\boldsymbol{i}} - \boldsymbol{\mu_0})'\Sigma^{-1}(\overline{\boldsymbol{X}}_{\boldsymbol{i}} - \boldsymbol{\mu_0})$, where $\overline{\boldsymbol{X}}_{\boldsymbol{i}}$, i = 1, 2, …, is the mean vector of the i^{th} sample of size $n(i)$ drawn on $\boldsymbol{X}$. Note that when $\boldsymbol{\mu} = \boldsymbol{\mu_0}$, T_i^2 follows central chi-square distribution with p degrees of freedom, and when $\boldsymbol{\mu} = \boldsymbol{\mu_1}$, for given $n(i) = n$, it follows non-central chi-square distribution with p degrees of freedom and non-centrality parameter $n(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\Sigma^{-1}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0}) = nd^2$, where d = $\sqrt{(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\Sigma^{-1}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})}$ is the Mahalanobis distance used to measure a change in the process mean vector. Let L be the control limit of the chart, which is equal to $\chi^2_{p,\alpha}$, the upper α^{th} quantile of central chi-square distribution with p degrees of freedom. The chart signals an out-of-control state when T_i^2 falls above L. Let $t(i)$ be the length of sampling interval between $(i-1)^{\text{st}}$ and i^{th} trials and $w(i)$ be the warning limit of the chart for the i^{th} trial, i = 1, 2, …. The values of $(n(i), t(i), w(i))$ can be either (n_1, t_1, w_1) or (n_2, t_2, w_2), where n_1, n_2, t_1, t_2, w_1, and w_2 are such that $n_{min} \le n_1 \le n_2 \le n_{max}$, $t_{max} \ge t_1 \ge t_2 \ge t_{min}$, and $L > w_1 \ge w_2 > 0$, where n_{min} and n_{max} are the smallest and largest possible sample sizes, respectively, while t_{min} and t_{max} are the shortest and longest possible sampling intervals, respectively.

For the $(i-1)^{\text{st}}$ trial, the warning limit $w(i-1)$ partitions the in-control area $(0, L)$ of the chart into two regions $I_1 = [0, w(i-1)]$ and $I_2 = (w(i-1), L)$, i = 2, 3, … as shown in figure 1.

When T_{i-1}^2 falls below L, the triplet of values of adaptive design parameters for the i^{th} trial is chosen according to the following rule.

$$(n(i), t(i), w(i)) = \begin{cases} (n_1, t_1, w_1), & \text{if } Z_{i-1} \in I_1 \\ (n_2, t_2, w_2), & \text{if } Z_{i-1} \in I_2 \end{cases} \quad ; i = 2,3,...$$

Note that choosing the values of the adaptive parameters using this rule is quite rational. Falling of T_{i-1}^2 in region I_1 may reasonably be interpreted as the process is safe and hence in such situation it is reasonable to relax the control by using long sampling interval, small sample size, and wide warning limit for the next trial. On the other hand, if T_{i-1}^2 falls in region I_2, one may suspect a shift in the process mean vector and under such circumstances to confirm or deny the suspicion, it is reasonable to to tighten the control by using short sampling interval, large sample size, and narrow warning limit for the next trial.

The values of $(n(1), t(1), w(1))$ can be chosen using an arbitrary probability distribution. In practice, it is recommended to choose the triplet (n_2, t_2, w_2) for that to provide additional protection against the problems that may exist initially. The trial following an out-of-control signal is again treated to be the first trial.

In practice, in order to avoid the complexity in the administration, only one warning limit may be shown anywhere on the chart in between the control limit and X-axis to represent the two. Suppose this warning limit is w. When $w(i) = w_j$, j = 1, 2, plot T_i^2 anywhere within $[0, w]$ and (w, L) when it is within $[0, w_j]$ and (w_j, L), respectively.

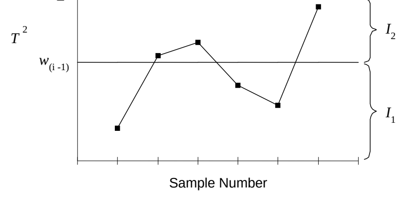

Figure 1: A VSSIWL T^2 Chart

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In the next section, expressions for performance measures for a VSISWL T^2 chart are derived.

IV. Performance Measures

The appropriate measures of statistical performance of a VSSIWL T^2 chart are *steady-state average time to signal* (SSATS), *average number of samples to signal* (ANSS), and *average number of observations to signal* (ANOS). SSATS is the expected value of the time between a shift that occurs at some random time after the process starts and the time the chart signals. ANSS and ANOS are the expected values of the number of samples and the number of observations, respectively taken from the time of a shift to the time the chart signals. The administrative performance is measured through *average number of switches to signal* (ANSW), which is the expected value of the number of switches between the triplets of values of sample size, sampling interval length, and warning limit from a shift to the signal.

Let SSATS_d, ANSS_d, ANOS_d, and ANSW_d be the SSATS, ANSS, ANOS, and ANSW, respectively of a control chart when the process mean has shifted from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$ in d units. First, the expressions for SSATS_d, ANSS_d, and ANOS_d are derived below using a Markov chain approach.

Henceforth, the i^{th} trial refers to the i^{th} trial after a shift when $i > 0$ and the last trial before the $(i + 1)^{\text{st}}$ trial when $i \leq 0$. Also, T_i^2 refers to the sample point corresponding to the i^{th} trial.

Define the three states 1, 2, and 3 of the Markov Chain corresponding to whether a sample point is plotted in I_1, I_2, and I_3 = $[L, \infty)$, respectively. State 3 is the absorbing state, as the control charting process is restarted when a sample point falls in region I_3. The transition probability matrix is given by

$$\mathbf{P^d} = \begin{bmatrix} p_{11}^d & p_{12}^d & p_{13}^d \\ p_{21}^d & p_{22}^d & p_{23}^d \\ 0 & 0 & 1 \end{bmatrix},$$

where p_{jk}^d is the transition probability that j is the prior state and k is the current state, when the process mean vector has shifted by d units. For example,

$$p_{12}^d = \Pr_d [T_i^2 \in I_2 \mid T_{i-1}^2 \in I_1]$$

$$= \Pr_d [T_i^2 \in I_2 \mid n(i) = n_1,\ w(i) = w_1]$$

$$= \text{F}_{\lambda_1}(L) - \text{F}_{\lambda_1}(w_1)$$

where $\text{F}_{\lambda_i}(\cdot)$ is the cumulative distribution function of non-central chi-square distribution with p degrees of freedom and non-centrality parameter $\lambda_i = n_i d^2$, i = 1, 2.

Then, SSATS_d and ANSS_d are given by

$$\text{SSATS}_d = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{t} - \text{E}(U), \quad (1)$$

$$\text{ANSS}_d = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\mathbf{1},$$

and

$$\text{ANOS}_d = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{n},$$

where $\mathbf{I}$ is the identity matrix of order 2, $\mathbf{P_1^d}$ is the sub matrix of $\mathbf{P^d}$ that contains the probabilities associated with the transient states only, $\boldsymbol{t}' = (t_1, t_2)$, $\mathbf{1}' = (1, 1)$, $\boldsymbol{n}' = (n_1, n_2)$, and $\boldsymbol{b}' = (b_1, b_2)$, b_j being the conditional probability that T_0^2 falls in I_j given that it falls below L, j = 1, 2. We note that $b_2 = 1 - b_1$. The Expression for b_1 is derived in appendix A.

$\text{E}(U)$ in equation (1) is the expected value of the time U between the 0^{th} trial and the shift. Assuming that an assignable cause of a process shift occurs according to a Poisson process, it can be shown that $\text{E}(U) = \text{E}[t(1)]/2$. Hence,

$$\text{SSATS}_d = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{t} - \text{E}[t(1)]/2.$$

Now, to derive the expression for ANSW_d, let

$$Y_i = \begin{cases} 1, \text{ if } (T_{i-1}^2 \in I_1, T_i^2 \in I_2) \\ 2, \text{ if } (T_{i-1}^2 \in I_2, T_i^2 \in I_1) \\ 3, \text{ if } (T_{i-1}^2 \in I_1, T_i^2 \in I_1) \\ 4, \text{ if } (T_{i-1}^2 \in I_2, T_i^2 \in I_2) \\ 5, \text{ if } T_i^2 > L \end{cases}, \quad i = 1, 2, \ldots$$

It is easy to see that $\{Y_i, i = 1, 2, \ldots\}$ is a Markov Chain with transition probability matrix

$$\mathbf{Q^d} = \begin{bmatrix} 0 & p_{21}^d & 0 & p_{22}^d & p_{23}^d \\ p_{12}^d & 0 & p_{11}^d & 0 & p_{13}^d \\ p_{12}^d & 0 & p_{11}^d & 0 & p_{13}^d \\ 0 & p_{21}^d & 0 & p_{22}^d & p_{23}^d \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, the expression for ANSW_δ is given by

$$\text{ANSW}_d = \boldsymbol{a}'(\mathbf{I_1} - \mathbf{Q_1^d})^{-1}\boldsymbol{e},$$

where, $\mathbf{I_1}$ is the identity matrix of order 4, $\mathbf{Q_1^d}$ is the sub matrix of $\mathbf{Q^d}$ that contains the probabilities associated with the transient states only, $\boldsymbol{e} = (1, 1, 0, 0)'$, and $\boldsymbol{a} = (a_1, a_2, a_3, a_4)'$, a_j being the initial probability of state j, j = 1, 2, 3, 4, given by

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$$a_j = \Pr_d[Y_1 = j] = \begin{cases} b_1 p_{12}^d \ , \ j = 1 \\ b_2 p_{21}^d \ , \ j = 2 \\ b_1 p_{11}^d \ , \ j = 3 \\ b_2 p_{22}^d \ , \ j = 4 \end{cases}.$$

The following section evaluates the performances of VSSIWL T^2 charts.

V. Performance Evaluation of VSSIWL T^2 Charts

In this section, the performances of VSSIWL T^2 charts are evaluated by comparing that with that of static, VSSI, and VSSI (1, 3) T^2 charts. VSSI (1, 3) T^2 charts refer to the VSSI T^2 charts with runs rule (1, 3) for switching between the pairs of values of sample sizes and sampling interval lengths. When the successive three sample points before the i^{th} trial fall below the control limit, runs rule (1, 3) chooses the pair (n_2, t_2) for the i^{th} trial if at least one of them falls in the warning region, otherwise it chooses the pair (n_1, t_1). See Mahadik [38] for the details of VSSI T^2 charts with switching runs rules. The reason of choosing VSSI (1, 3) T^2 charts for comparison is that among various switching runs rules considered by Mahadik [38], runs rule (1, 3) is the best choice for reducing ANSW values of VSSI T^2 charts.

The four charts mentioned above are designed such that their in-control statistical performances are matched. This is done by keeping design parameter L of all the four charts the same, keeping the design parameters n_1, n_2, and t_2 of VSSIWL, VSSI, and VSSI (1, 3) T^2 charts the same and choosing their t_1 and warning limits such that E[n(1)] = n_0 and E[t(1)] = t_0 hold, where n_0 and t_0 are the sample size and sampling interval length of the static T^2 charts. These constraints uniquely determine t_1 and warning limits of the VSSI, and VSSI (1, 3) T^2 charts. As a VSIWL T^2 charts has two warning limits, by fixing one of then the constraints uniquely determine the other and t_1.

The $SSATS_d$, $ANSS_d$, $ANOS_d$, and $ANSW_d$ values of such statistically matched charts are then computed for various values of d. Tables 1, 2, 3, and 4, respectively, show these values for five different sets of matched charts. The computations indicate the following facts in general.

There is no significant difference among the statistical performances of VSSI, VSSI (1, 3), and VSIWL T^2 charts. The administrative performance of a VSSI (1, 3) T^2 chart is substantially superior to that of the matched VSSI T^2 chart. If the wider warning limit, that is, w_1 of a VSSIWL T^2 chart, is set to be equal to the warning limit of the matched VSSI (1, 3) T^2 chart, the two charts yield almost similar administrative performances. If w_1 is set in between the warning limits of the VSSI and VSSI (1, 3) T^2 charts, the VSSIWL T^2 chart yield the administrative performance that is superior to that of VSSI T^2 chart and inferior to that of VSSI (1, 3) T^2 chart. On the other hand, if w_1 is set to be slightly larger than the warning limit of the VSSI (1, 3) T^2 chart, it yields the administrative performances that is substantially better even than that of VSSI (1, 3) T^2 chart. In this case, the in-control ANSW of the VSSIWL T^2 chart is about 32% of that of VSSI T^2 chart and about 80% of that of VSSI (1, 3) T^2 chart. However if w_1 is set to be noticeably larger than the warning limit of VSSI (1, 3) T^2 chart then it adversely affects the statistical performance of VSSIWL T^2 chart for detecting the large shifts.

The following example demonstrates the implementation of VSSI, VSSI (1, 3), and VSSIWL T^2 charts.

VI. Example

The statistically matched VSSI, VSSI (1, 3), and three VSSIWL T^2 charts with the design parameters, viz, p = 4, n_0 = 7, n_1 = 3, n_2 = 12, t_1 = 1.64 , t_2 = 0.2, and L = 14.8603 are implemented simultaneously and independently for a process. The process is initially in control when the implementation of the charts is started and a shift of size 0.75 unit occurs at 10 hours after that. Table 5 shows the sample means taken for the VSSI and VSSI (1, 3) T^2 charts along with the corresponding times and the pairs ($n(i)$, $t(i)$). Similarly, table 6 shows the sample means taken for the three VSSIWL T^2 charts along with the corresponding times and the triplets ($n(i)$, $t(i)$, $w(i)$). The pair (n_2, t_2) is used for the first sample for the VSSI chart and for the first three samples for the VSSI (1, 3) chart. The pairs for the subsequent samples are chosen according to the rules of the respective charts. Similarly, the triplet (n_2, t_2, w_2) is used for the first sample for the three VSSIWL charts and the triplets for the subsequent samples are chosen according to the rules of the respective charts. Table 7 shows the performances of the five charts.

VII. Conclusions

The notion of variable warning limit is applied to VSSI T^2 charts. The purpose is to reduce the frequency of switches between the pairs of values of sample sizes and sampling interval lengths of those charts which cause administrative inconvenience. Expressions for the performance measures, viz, SSATS, ANSS, ANOS, and ANSW for VSSIWL T^2 charts are developed. The effects of variable warning limits on the performances of the VSSI T^2 charts are evaluated by comparing the performances of VSSIWL T^2 charts with that of VSSI and VSSI (1, 3) T^2 charts. It is observed that there are no significant differences among the statistical performances of the three charts. A VSSIWL T^2 chart can be set to yield administration

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performance dramatically better than that of VSSI T^2 charts and even significantly better than that of a VSSI (1, 3) T^2 chart when its statistical performances are almost similar to that of VSSI and VSSI (1, 3) T^2 charts.

Thus, variable warning limit is very much efficient in reducing the inconvenience associated with the implementation of a VSSI T^2 chart caused by the frequent switches between the values of its adaptive design parameters without affecting its statistical performance.

In future, the idea of variable warning limits can be evaluated for attribute control charts as well as for EWMA and CUSUM charts.

Appendix A: Expression for b_1

We have

$$b_1 = \Pr[T_0^2 \in I_1 | T_0^2 < L]$$

$$= \Pr[T_0^{2'} \in I_1], \text{ where } T_0^{2'} \text{ is } T_0^2 \text{ truncated on } [0, L).$$

$$= \Pr[T_0^{2'} \in I_1 \mid T_{-1}^{2'} \in I_1] \Pr[T_{-1}^{2'} \in I_1] + \Pr[T_0^{2'} \in I_1 \mid T_{-1}^{2'} \in I_2] \Pr[T_{-1}^{2'} \in I_2]$$

$$= \Pr[T_0^{2'} \in I_1 \mid n(0) = n_1, w(0) = w_1] b_1 + \Pr[T_0^{2'} \in I_1 \mid n(0) = n_2, w(0) = w_2] b_2$$

$$= \frac{F_0(w_1)}{F_0(L)} b_1 + \frac{F_0(w_2)}{F_0(L)} (1 - b_1)$$

Solving this for b_1, we get

$$b_1 = \frac{F_0(w_2)}{F_0(L) - F_0(w_1) + F_0(w_2)},$$

where $F_0(\cdot)$ is the cumulative distribution function of central chi-square distribution with p degrees of freedom.

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Table 1 The SSATS values of static, VSSIWL, VSSI, and VSSI (1, 3) T^2 charts

Chart	w_1	w_2	SSATS for a shift of size									
			0	0.25	0.5	0.75	1	1.5	2	2.5	3	4
Case 1: p = 5, n_0 = 2, n_1 = 1, n_2 = 4, t_1 =1.45, t_2 = 0.1, L = 16.7496												
Static			199.50	168.45	108.70	60.93	32.61	9.78	3.49	1.54	0.86	0.52
VSSIWL	7.00	3.87	199.50	162.43	87.10	32.63	10.63	2.28	1.27	0.90	0.70	0.55
VSSIWL	8.50	2.68	199.50	161.15	83.50	29.85	10.11	2.95	1.70	1.12	0.81	0.57
VSSIWL	9.00	2.40	199.50	160.81	82.66	29.43	10.31	3.29	1.88	1.21	0.86	0.58
VSSI	5.70		199.50	164.01	91.88	37.41	12.91	2.17	1.04	0.76	0.63	0.53
VSSI (1, 3)	8.50		199.50	163.13	89.01	34.33	11.67	3.03	1.71	1.12	0.81	0.57
Case 2: p = 3, n_0 = 3, n_1 = 1, n_2 = 7, t_1 =1.4, t_2 = 0.2, L = 12.8382												
Static			199.50	142.18	67.22	29.08	13.08	3.39	1.26	0.69	0.53	0.50
VSSIWL	4.50	1.94	199.50	129.89	38.14	8.91	3.46	1.68	1.12	0.83	0.68	0.54
VSSIWL	5.64	1.22	199.50	128.27	36.35	9.35	4.42	2.26	1.41	0.98	0.74	0.55
VSSIWL	6.00	1.06	199.50	127.94	36.21	9.76	4.86	2.49	1.52	1.03	0.77	0.56
VSSI	3.38		199.50	132.64	42.59	10.17	3.21	1.28	0.91	0.73	0.63	0.54
VSSI (1, 3)	5.63		199.50	131.29	40.14	10.08	4.50	2.26	1.40	0.97	0.74	0.55
Case 3: p = 2, n_0 = 4, n_1 = 2, n_2 = 8, t_1 =1.35, t_2 = 0.3, L = 10.5966												
Static			199.50	115.03	41.42	15.28	6.38	1.66	0.73	0.53	0.50	0.50
VSSIWL	3.00	1.15	199.50	101.47	21.22	4.80	2.08	1.00	0.70	0.58	0.53	0.50
VSSIWL	4.07	0.58	199.50	99.45	20.11	5.23	2.57	1.19	0.77	0.60	0.53	0.50
VSSIWL	4.50	0.45	199.50	98.94	20.13	5.59	2.84	1.29	0.80	0.61	0.53	0.50
VSSI	2.18		199.50	104.24	23.87	5.20	1.91	0.89	0.67	0.58	0.53	0.50
VSSI (1, 3)	4.07		199.50	101.59	21.37	5.31	2.59	1.19	0.77	0.60	0.53	0.50
Case 4: p = 7, n_0 = 5, n_1 = 3, n_2 = 10, t_1 =1.36, t_2 = 0.1, L = 20.2777												
Static			199.50	143.62	65.80	26.03	10.53	2.32	0.85	0.55	0.50	0.50
VSSIWL	10.00	5.99	199.50	131.68	37.46	7.21	2.39	1.01	0.66	0.54	0.51	0.50
VSSIWL	11.71	4.37	199.50	129.01	33.89	7.14	3.04	1.29	0.75	0.57	0.51	0.50
VSSIWL	12.50	3.83	199.50	128.07	33.11	7.61	3.52	1.45	0.80	0.58	0.52	0.50
VSSI	8.52		199.50	134.92	43.21	9.01	2.38	0.85	0.61	0.53	0.51	0.50
VSSI (1, 3)	11.71		199.50	133.34	40.14	8.42	3.15	1.29	0.75	0.57	0.51	0.50
Case 5: p = 4, n_0 = 7, n_1 = 3, n_2 = 12, t_1 =1.64, t_2 = 0.2, L = 14.8603												
Static			199.50	108.46	33.35	10.39	3.83	0.95	0.54	0.50	0.50	0.50
VSSIWL	5.00	2.49	199.50	89.66	13.52	2.80	1.39	0.76	0.59	0.53	0.51	0.50
VSSIWL	6.24	1.77	199.50	87.23	12.87	3.24	1.74	0.88	0.62	0.53	0.51	0.50
VSSIWL	6.75	1.54	199.50	86.49	12.90	3.53	1.93	0.94	0.63	0.53	0.51	0.50
VSSI	3.71		199.50	93.29	15.48	2.78	1.16	0.68	0.57	0.52	0.51	0.50
VSSI (1, 3)	6.24		199.50	89.86	13.48	3.26	1.76	0.88	0.62	0.53	0.51	0.50

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Table 2 The ANSS values of static, VSSIWL, VSSI, and VSSI (1, 3) T^2 charts

Chart	w_1	w_2	ANSS for a shift of size									
			0	0.25	0.5	0.75	1	1.5	2	2.5	3	4
Case 1: p = 5, n_0 = 2, n_1 = 1, n_2 = 4, t_1 =1.45 , t_2 = 0.1, L = 16.7496												
Static			200.00	168.95	109.20	61.43	33.11	10.28	3.99	2.04	1.36	1.02
VSSIWL	7.00	3.87	200.00	167.54	98.59	43.87	18.05	4.82	2.51	1.83	1.55	1.23
VSSIWL	8.50	2.68	200.00	167.34	97.23	42.46	17.75	5.23	2.77	1.96	1.60	1.24
VSSIWL	9.00	2.40	200.00	167.29	96.91	42.25	17.87	5.43	2.88	2.01	1.63	1.24
VSSI	5.70		200.00	167.79	100.39	46.30	19.35	4.74	2.37	1.75	1.52	1.23
VSSI (1, 3)	8.50		200.00	167.65	99.31	44.73	18.64	5.27	2.77	1.96	1.60	1.24
Case 2: p = 3, n_0 = 3, n_1 = 1, n_2 = 7, t_1 =1.4 , t_2 = 0.2, L = 12.8382												
Static			200.00	142.68	67.72	29.58	13.58	3.89	1.76	1.19	1.03	1.00
VSSIWL	4.50	1.94	200.00	136.79	47.28	13.94	5.84	2.63	1.93	1.65	1.45	1.17
VSSIWL	5.64	1.22	200.00	136.23	46.25	14.22	6.49	3.02	2.12	1.73	1.48	1.17
VSSIWL	6.00	1.06	200.00	136.12	46.17	14.49	6.79	3.17	2.19	1.76	1.49	1.17
VSSI	3.38		200.00	137.74	49.84	14.75	5.68	2.36	1.80	1.59	1.43	1.17
VSSI (1, 3)	5.63		200.00	137.27	48.43	14.70	6.54	3.02	2.12	1.73	1.48	1.17
Case 3: p = 2, n_0 = 4, n_1 = 2, n_2 = 8, t_1 =1.35 , t_2 = 0.3, L = 10.5966												
Static			200.00	115.53	41.92	15.78	6.88	2.16	1.23	1.03	1.00	1.00
VSSIWL	3.00	1.15	200.00	108.69	28.10	8.08	3.72	1.87	1.45	1.23	1.09	1.00
VSSIWL	4.07	0.58	200.00	107.87	27.44	8.35	4.04	1.98	1.48	1.23	1.09	1.00
VSSIWL	4.50	0.45	200.00	107.67	27.45	8.59	4.22	2.04	1.49	1.24	1.09	1.00
VSSI	2.18		200.00	109.81	29.68	8.33	3.61	1.80	1.43	1.23	1.09	1.00
VSSI (1, 3)	4.07		200.00	108.74	28.19	8.41	4.05	1.99	1.48	1.23	1.09	1.00
Case 4: p = 7, n_0 = 5, n_1 = 3, n_2 = 10, t_1 =1.36 , t_2 = 0.1, L = 20.2777												
Static			200.00	144.12	66.30	26.53	11.03	2.82	1.35	1.05	1.00	1.00
VSSIWL	10.00	5.99	200.00	139.86	48.93	13.20	5.04	2.07	1.50	1.22	1.07	1.00
VSSIWL	11.71	4.37	200.00	139.16	47.10	13.16	5.45	2.24	1.53	1.23	1.07	1.00
VSSIWL	12.50	3.83	200.00	138.91	46.70	13.45	5.76	2.34	1.56	1.23	1.07	1.00
VSSI	8.52		200.00	140.70	51.87	14.30	5.04	1.98	1.47	1.22	1.07	1.00
VSSI (1, 3)	11.71		200.00	140.29	50.30	13.94	5.52	2.24	1.53	1.23	1.07	1.00
Case 5: p = 4, n_0 = 7, n_1 = 3, n_2 = 12, t_1 =1.64 , t_2 = 0.2, L = 14.8603												
Static			200.00	108.96	33.85	10.89	4.33	1.45	1.04	1.00	1.00	1.00
VSSIWL	5.00	2.49	200.00	100.98	21.74	5.87	2.78	1.56	1.28	1.11	1.03	1.00
VSSIWL	6.24	1.77	200.00	100.22	21.42	6.11	2.97	1.61	1.29	1.11	1.03	1.00
VSSIWL	6.75	1.54	200.00	99.98	21.44	6.27	3.07	1.64	1.30	1.11	1.03	1.00
VSSI	3.71		200.00	102.13	22.71	5.86	2.65	1.52	1.28	1.11	1.03	1.00
VSSI (1, 3)	6.24		200.00	101.05	21.72	6.12	2.98	1.61	1.29	1.11	1.03	1.00

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Table 3 The ANOS values of static, VSSIWL, VSSI, and VSSI (1, 3) T^2 charts

Chart	w_1	w_2	ANOS for a shift of size									
			0	0.25	0.5	0.75	1	1.5	2	2.5	3	4
Case 1: p = 5, n_0 = 2, n_1 = 1, n_2 = 4, t_1 =1.45 , t_2 = 0.1, L = 16.7496												
Static			400.00	337.90	218.40	122.85	66.22	20.56	7.99	4.09	2.71	2.05
VSSIWL	7.00	3.87	400.00	345.33	221.59	111.61	51.48	14.15	6.66	4.63	3.87	2.89
VSSIWL	8.50	2.68	400.00	347.35	223.87	111.83	51.38	14.40	6.81	4.67	3.85	2.86
VSSIWL	9.00	2.40	400.00	347.87	224.40	111.87	51.42	14.52	6.87	4.69	3.85	2.85
VSSI	5.70		400.00	342.85	218.58	111.23	51.91	14.10	6.57	4.60	3.88	2.90
VSSI (1, 3)	8.50		400.00	344.24	220.39	111.47	51.68	14.43	6.81	4.67	3.85	2.86
Case 2: p = 3, n_0 = 3, n_1 = 1, n_2 = 7, t_1 =1.4 , t_2 = 0.2, L = 12.8382												
Static			600.00	428.04	203.17	88.74	40.75	11.68	5.29	3.56	3.10	3.00
VSSIWL	4.50	1.94	600.00	442.37	185.05	64.47	26.96	10.16	7.38	6.50	5.71	4.11
VSSIWL	5.64	1.22	600.00	446.00	185.75	64.55	27.32	10.38	7.41	6.44	5.62	4.08
VSSIWL	6.00	1.06	600.00	446.74	185.80	64.62	27.48	10.47	7.43	6.42	5.59	4.06
VSSI	3.38		600.00	436.17	183.30	64.70	26.87	10.00	7.35	6.55	5.77	4.14
VSSI (1, 3)	5.63		600.00	439.21	184.26	64.68	27.35	10.38	7.41	6.44	5.62	4.08
Case 3: p = 2, n_0 = 4, n_1 = 2, n_2 = 8, t_1 =1.35 , t_2 = 0.3, L = 10.5966												
Static			800.00	462.12	167.66	63.10	27.50	8.64	4.93	4.13	4.01	4.00
VSSIWL	3.00	1.15	800.00	473.17	148.84	48.18	21.38	9.56	7.17	5.73	4.69	4.03
VSSIWL	4.07	0.58	800.00	476.81	148.77	48.42	21.72	9.62	7.10	5.68	4.67	4.03
VSSIWL	4.50	0.45	800.00	477.71	148.77	48.64	21.91	9.65	7.07	5.65	4.66	4.03
VSSI	2.18		800.00	468.20	149.00	48.41	21.26	9.53	7.20	5.75	4.69	4.03
VSSI (1, 3)	4.07		800.00	472.95	148.85	48.47	21.74	9.62	7.10	5.68	4.67	4.03
Case 4: p = 7, n_0 = 5, n_1 = 3, n_2 = 10, t_1 =1.36 , t_2 = 0.1, L = 20.2777												
Static			1000.00	720.61	331.48	132.67	55.15	14.12	6.76	5.24	5.02	5.00
VSSIWL	10.00	5.99	1000.00	741.93	305.57	96.52	37.14	13.47	9.33	7.11	5.66	5.01
VSSIWL	11.71	4.37	1000.00	749.39	306.09	96.46	37.91	13.71	9.26	7.04	5.64	5.01
VSSIWL	12.50	3.83	1000.00	752.00	306.20	96.90	38.50	13.90	9.20	7.00	5.60	5.00
VSSI	8.52		1000.00	732.86	304.72	98.09	37.13	13.34	9.36	7.14	5.67	5.01
VSSI (1, 3)	11.71		1000.00	737.29	305.17	97.58	38.04	13.71	9.26	7.04	5.64	5.01
Case 5: p = 4, n_0 = 7, n_1 = 3, n_2 = 12, t_1 =1.64 , t_2 = 0.2, L = 14.8603												
Static			1400.00	762.74	236.96	76.26	30.31	10.12	7.28	7.01	7.00	7.00
VSSIWL	5.00	2.49	1400.00	774.60	200.50	57.20	25.00	12.80	10.20	8.30	7.30	7.00
VSSIWL	6.24	1.77	1400.00	779.60	200.30	57.60	25.30	12.80	10.10	8.30	7.30	7.00
VSSIWL	6.75	1.54	1400.00	781.10	200.30	57.90	25.50	12.80	10.10	8.30	7.30	7.00
VSSI	3.71		1400.00	767.10	201.00	57.20	24.80	12.70	10.20	8.30	7.30	7.00
VSSI (1, 3)	6.24		1400.00	774.10	200.50	57.60	25.40	12.80	10.10	8.30	7.30	7.00

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Table 4 The ANSW values of static, VSSIWL, VSSI, and VSSI (1, 3) T^2 charts

Chart	w_1	w_2	ANSW for a shift of size									
			0	0.25	0.5	0.75	1	1.5	2	2.5	3	4
Case 1: p = 5, n_0 = 2, n_1 = 1, n_2 = 4, t_1 =1.45 , t_2 = 0.1, L = 16.7496												
VSSIWL	7.00	3.87	57.50	47.93	27.34	11.01	3.71	0.80	0.58	0.52	0.43	0.22
VSSIWL	8.50	2.68	33.54	27.90	15.57	5.99	2.04	0.66	0.55	0.49	0.41	0.21
VSSIWL	9.00	2.40	27.75	23.08	12.81	4.90	1.71	0.63	0.54	0.48	0.40	0.20
VSSI	5.70		88.44	74.05	43.58	18.91	6.86	1.14	0.62	0.53	0.45	0.22
VSSI (1, 3)	8.50		33.54	28.29	16.73	7.07	2.49	0.69	0.56	0.49	0.41	0.21
Case 2: p = 3, n_0 = 3, n_1 = 1, n_2 = 7, t_1 =1.4 , t_2 = 0.2, L = 12.8382												
VSSIWL	4.50	1.94	55.28	36.74	11.09	2.43	0.88	0.60	0.55	0.48	0.38	0.16
VSSIWL	5.64	1.22	29.22	19.24	5.55	1.34	0.69	0.57	0.52	0.44	0.35	0.15
VSSIWL	6.00	1.06	28.43	18.71	5.39	1.32	0.69	0.57	0.52	0.44	0.35	0.15
VSSI	3.38		88.44	59.76	19.77	4.64	1.34	0.63	0.57	0.49	0.39	0.16
VSSI (1, 3)	5.63		33.54	22.74	7.26	1.70	0.74	0.58	0.53	0.45	0.36	0.15
Case 3: p = 2, n_0 = 4, n_1 = 2, n_2 = 8, t_1 =1.35 , t_2 = 0.3, L = 10.5966												
VSSIWL	3.00	1.15	58.17	31.11	6.87	1.44	0.68	0.51	0.37	0.21	0.09	0.00
VSSIWL	4.07	0.58	33.54	17.88	3.83	0.96	0.61	0.49	0.35	0.20	0.08	0.00
VSSIWL	4.50	0.45	26.77	14.28	3.08	0.86	0.58	0.47	0.34	0.20	0.08	0.00
VSSI	2.18		88.44	48.00	11.61	2.40	0.85	0.53	0.39	0.22	0.09	0.00
VSSI (1, 3)	4.07		33.54	18.40	4.21	1.00	0.62	0.49	0.35	0.20	0.08	0.00
Case 4: p = 7, n_0 = 5, n_1 = 3, n_2 = 10, t_1 =1.36 , t_2 = 0.1, L = 20.2777												
VSSIWL	10.00	5.99	52.45	36.95	12.15	2.51	0.86	0.56	0.40	0.21	0.07	0.00
VSSIWL	11.71	4.37	30.15	21.09	6.47	1.41	0.69	0.53	0.37	0.19	0.06	0.00
VSSIWL	12.50	3.83	22.93	16.01	4.83	1.14	0.65	0.51	0.36	0.19	0.06	0.00
VSSI	8.52		81.22	57.91	21.04	4.88	1.31	0.59	0.42	0.21	0.07	0.00
VSSI (1, 3)	11.71		30.16	21.89	7.91	1.82	0.74	0.53	0.37	0.19	0.06	0.00
Case 5: p = 4, n_0 = 7, n_1 = 3, n_2 = 12, t_1 =1.64 , t_2 = 0.2, L = 14.8603												
VSSIWL	5.00	2.49	62.73	29.36	4.46	0.84	0.52	0.41	0.26	0.11	0.03	0.00
VSSIWL	6.24	1.77	39.32	18.13	2.68	0.66	0.49	0.40	0.25	0.11	0.03	0.00
VSSIWL	6.75	1.54	32.16	14.78	2.20	0.61	0.48	0.39	0.25	0.10	0.03	0.00
VSSI	3.71		98.27	47.20	7.94	1.27	0.57	0.42	0.27	0.11	0.03	0.00
VSSI (1, 3)	6.24		39.34	18.82	2.87	0.67	0.50	0.40	0.25	0.11	0.03	0.00

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Table 5 The details of the VSSI and VSSI (1, 3) T^2 charts in the example

VSSI with w = 3.71			VSSI (1, 3) with w = 6.24		
Time in hours	$(n(i), t(i))$	T^2	Time in hours	$(n(i), t(i))$	T^2
0.2	(12, 0.2)	11.48	0.2	(12, 0.2)	0.77
0.4	(12, 0.2)	1.97	0.4	(12, 0.2)	3.37
2.04	(3, 1.64)	3.00	0.6	(12, 0.2)	7.86
3.68	(3, 1.64)	1.87	0.8	(12, 0.2)	0.23
5.32	(3, 1.64)	7.76	1	(12, 0.2)	2.70
5.52	(12, 0.2)	4.82	1.2	(12, 0.2)	3.56
5.72	(12, 0.2)	1.60	2.84	(3, 1.64)	4.33
7.36	(3, 1.64)	8.10	4.48	(3, 1.64)	1.72
7.56	(12, 0.2)	5.85	6.12	(3, 1.64)	1.18
7.76	(12, 0.2)	1.49	7.76	(3, 1.64)	4.57
9.4	(3, 1.64)	3.09	9.4	(3, 1.64)	2.77
11.04	(3, 1.64)	2.56	11.04	(3, 1.64)	2.42
12.68	(3, 1.64)	7.39	12.68	(3, 1.64)	5.79
12.88	(12, 0.2)	5.52	14.32	(3, 1.64)	6.81
13.08	(12, 0.2)	8.57	14.52	(12, 0.2)	13.33
13.28	(12, 0.2)	1.95	14.72	(12, 0.2)	16.03
14.62	(3, 1.64)	3.74	14.92	(12, 0.2)	11.61
14.82	(12, 0.2)	15.86	15.12	(12, 0.2)	14.81
			15.32	(12, 0.2)	13.89
			15.52	(12, 0.2)	15.20

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Table 6 The details of the VSSIWL T^2 charts in the example

VSSIWL with w_1 =5, w_2 =2.49			VSSIWL with w_1 = 6.24, w_2 = 1.77			VSSIWL with w_1 =6.75, w_2 = 1.54		
Time in hours	$(n(i), t(i))$	T^2	Time in hours	$(n(i), t(i))$	T^2	Time in hours	$(n(i), t(i))$	T^2
0.2	(12, 0.2)	1.97	0.2	(12, 0.2)	1.97	0.2	(12, 0.2)	1.64
1.84	(3, 1.64)	2.77	0.4	(12, 0.2)	2.77	0.4	(12, 0.2)	2.63
3.48	(3, 1.64)	5.85	0.6	(12, 0.2)	5.85	0.6	(12, 0.2)	5.33
3.68	(12, 0.2)	3.39	0.8	(12, 0.2)	3.39	0.8	(12, 0.2)	0.74
3.88	(12, 0.2)	5.78	1	(12, 0.2)	5.78	2.44	(3, 1.64)	2.96
4.08	(12, 0.2)	1.29	1.2	(12, 0.2)	1.29	4.08	(3, 1.64)	0.91
5.72	(3, 1.64)	0.47	2.84	(3, 1.64)	0.47	5.72	(3, 1.64)	1.21
7.36	(3, 1.64)	2.14	4.48	(3, 1.64)	2.14	7.36	(3, 1.64)	3.80
9	(3, 1.64)	5.15	6.12	(3, 1.64)	5.15	9	(3, 1.64)	7.18
9.2	(12, 0.2)	2.31	7.76	(3, 1.64)	2.31	9.2	(12, 0.2)	3.25
			9.4	(3, 1.64)	4.34	9.4	(12, 0.2)	4.91
10.84	(3, 1.64)	5.41				9.6	(12, 0.2)	14.82
11.04	(12, 0.2)	7.99	11.04	(3, 1.64)	3.79	9.8	(12, 0.2)	4.22
11.24	(12, 0.2)	5.73	12.68	(3, 1.64)	8.39			
11.44	(12, 0.2)	10.89	12.88	(12, 0.2)	6.37	10	(12, 0.2)	7.44
11.64	(12, 0.2)	2.84	13.08	(12, 0.2)	14.78	10.2	(12, 0.2)	7.69
11.84	(12, 0.2)	9.93	13.28	(12, 0.2)	24.79	10.4	(12, 0.2)	6.17
12.04	(12, 0.2)	14.11				10.6	(12, 0.2)	8.88
12.24	(12, 0.2)	15.43				10.8	(12, 0.2)	9.77
						11	(12, 0.2)	6.49
						11.2	(12, 0.2)	10.87
						11.4	(12, 0.2)	8.72
						11.6	(12, 0.2)	11.69
						11.8	(12, 0.2)	22.62

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Table 7 Performances of the T^2 charts in the example

Chart	Number of switches during in-control period	Total number of switches until the signal	Time in hours from the shift to the signal	Number of samples during the in-control period	Number of samples during the out-of-control period
VSSI	5	8	4.82	11	7
VSSI (1, 3)	1	2	5.52	11	9
VSIWL with w_1 = 5, w_2 = 2.49	4	6	2.24	10	8
VSIWL with w_1 = 6.24, w_2 = 1.77	1	2	3.28	11	5
VSIWL with w_1 = 6.75, w_2 = 1.54	2	2	1.8	13	10

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