Unsteady MHD flow between two parallel plates through porous medium with One Plate Moving Uniformly and the Other Plate at Rest with Uniform Suction

A.Mohamed Ismail[1], Dr.S.Ganesh[2], C.K.Kirubhashankar [3]

Abstract— **The present study focuses on MHD flow between two parallel plates through porous medium with one in uniform motion and the other plate at rest and uniform suction at the stationary plate is discussed. The partial differential equations governing the flow are solved by similarity transformation. The axial and transverse velocity of the fluid and the pressure distribution were presented. Analytical expression is given for the velocity field and the effects of the various parameters entering into the problem are discussed with the help of graph.**

Index Terms— **Fluid flow, MHD flow, parallel plates, porous medium, similarity transformation**

I. Introduction

Magneto hydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil. Flow through porous medium has numerous Engineering and Geophysical applications. J. Hartmann and F. Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary and insulated plates. The equations which describe the MHD flow are a combination of continuity equation and Navier-Stokes equations of fluid dynamics. The governing equations are differential equations that have to be solved either analytically or numerically. A. S. Berman [2] studied the laminar flow in channel with porous walls. J. Hartmann [3] considered the magnetic field in the laminar flow of an electrically conducting liquid. I. A. Hassaninen, M. A. Mansour [4] has investigated the magnetic flow through the porous medium between two infinite plates. E. A. Hamza [5] has studied the suction and injection effects of flow between parallel plates. V. Soundalgekar, A. Uplekar [6] studied the effect of heat transfer considering constant temperature. C. B. Sing, P. C. Ram [7] considered laminar flow of an electrically conducting fluid through a channel in the presence of transverse magnetic field under the influence of periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. The necessities of modern machinery have motivated the interest in fluid flow studies, which involve the interaction of several phenomena. One such study is presented, when a viscous fluid flows over a porous surface has its significance in many engineering problems such as flow of liquid in a porous bearing D. D. Joseph and L. N. Tao [8], in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purifications process. R. E. Cunningham and R. J. Williams [9] also reported several geophysical applications of flow in porous medium, viz. porous rollers and its natural occurrence in the flow of rivers through porous banks and beds and the flow of oil through underground porous rocks. The mathematical theory of the flow of fluid through a porous medium was initiated by H. Darcy [10]. For the steady flow, he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces. Later on H. C. Brinkman [11] proposed modification of the Darcy's law for porous medium. In the most of the examples, the fluid flows through porous medium, have two regions. In region I, the fluid is free to flow and in region II, the fluid flows through the porous medium. S. M. Cox [12] considered the two dimensional flow of a viscous fluid in a channel with porous walls. Many research works concerning the Hartmann flow has been obtained under different physical effects [13-15]. S.Ganesh, S.Krishnambal [16] studied unsteady MHD stokes flow of a viscous fluid between two parallel porous plates. They considered the fluid being withdrawn through both walls of the channel at the same rate.

In this paper, the unsteady fluid flow through the parallel plate channel under the influence of magnetic field and assess the effect to velocity through porous medium and the solution are expressed in terms of Hartmann number.

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II. MODEL FORMULATION

The flow of an incompressible viscous fluid between two parallel porous plates $y = -h$ and $y = h$ in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates.

Let u and v be the velocity components in the x and y directions respectively in the flow field at time t.

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The equation of continuity is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

Equations of momentum are:

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu u}{K\rho} \quad (2)$$

$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

III. ASSUMPTION

1. Flow between non conducting two parallel plates.
2. Viscosity of the fluid is considered as constant.
3. One plate is in uniform motion and the other plate (stationary plate) at rest with uniform suction.
4. *u* and *v* are velocity components in the direction of *x* and *y* respectively.

IV. NOTATIONS

ρ -	Density of the fluid
μ -	Coefficient of viscosity
K -	Porous medium
ψ -	Stream function
η -	Dimensionless distance
σ -	Electrical conductivity of the fluid
ν -	Kinematic viscosity $(\nu=\mu/\rho)$
B_0 -	Electromagnetic induction
H_0 -	Transverse magnetic field
M -	Hartmann number $\left(M=\sqrt{\frac{\sigma B_0^2 h^2}{\rho\nu}}=B_0 h\sqrt{\frac{\sigma}{\mu}}\right)$

V. GENERAL SOLUTIONS TO THE PROBLEM

With the help of discussions in the previous sections, Let us choose the solutions of the equations (1)-(3) respectively as

$$u=u(x,y)e^{-nt}$$
$$v=v(x,y)e^{-nt} \quad (4)$$
$$p=p(x,y)e^{-nt}$$

With the boundary conditions

$$u(x,h)=U, \qquad u(x,-h)=0$$
$$v(x,h)=0 \qquad v(x,-h)=-v_0 \quad (5)$$

Let $\eta=\frac{y}{h}$ be the dimensionless distance and let $\nu=\frac{\mu}{\rho}$ be the kinematic viscosity and the equations (1), (2) and (3) become

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial u}{\partial\eta}=0 \quad (6)$$

$$-nu=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial\eta^2}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu u}{K\rho} \quad (7)$$

$$-nv=-\frac{1}{\rho h}\frac{\partial p}{\partial\eta}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial\eta^2}\right) \quad (8)$$

The boundary conditions are converted into

$$u(x,1)=U, \qquad u(x,-1)=0$$
$$v(x,1)=0 \qquad v(x,-1)=-v_0$$

(9)

The stream function $\psi(x,y)$ is defined as

$$\psi(x,\eta)=\left(hu(0)-v_0 x\right)f(\eta) \quad (10)$$

and $u(x,y)=\frac{\partial\psi}{\partial y}$ $\quad v(x,y)=-\frac{\partial\psi}{\partial x}$

From equations (2), (3), (9) and (10), we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=\left(h\,u(0)-v_0 x\right)\left[-nf'-\frac{\nu}{h^2}f'''(\eta)+\frac{\sigma B_0^2}{\rho}f'+\frac{\mu}{K\rho}f'\right] \quad (11)$$

$$-\frac{1}{\rho h}\frac{\partial p}{\partial\eta}=-nv_0 f(\eta)-\frac{\nu v_0}{h^2}f''(\eta)$$

(12)

Partially differentiating the equations (11) & (12) with respect to 'η' & 'x' respectively

$$\frac{\partial^2 p}{\partial\eta\partial x}=\left(hu(0)-v_0 x\right)\frac{\partial}{\partial\eta}\left(-nf'-\frac{\nu}{h^2}f'''(\eta)+\frac{\sigma B_0^2}{\rho}f'+\frac{\nu}{K}f'\right) \quad (13)$$

$$\frac{\partial^2 p}{\partial x\partial\eta}=0 \quad (14)$$

From equations (13) & (14), we get

$$\frac{d}{d\eta}\left(-nf'-\frac{\nu}{h^2}f'''(\eta)+\frac{\sigma B_0^2}{\rho}f'+\frac{\nu}{K}f'\right)=0 \quad (15)$$

Integrating equation (15), we have

$$f'''(\eta)+\frac{nh^2}{\nu}f'(\eta)-\frac{\sigma B_0^2 h^2}{\rho\nu}f'(\eta)-\frac{1}{K}f'(\eta)=C$$

$$f'''(\eta)-(\alpha^2 h^2+M^2)f'(\eta)=C$$

(16)

where M is the Hartmann number and $\alpha^2=\frac{-n}{\nu}+\frac{1}{K}$

Equation (16) reduces to the form

$$(D^3-(\alpha^2 h^2+M^2)D)f(\eta)=C$$

(17)

with the boundary conditions

$$f(1)=0, \qquad f(-1)=-1$$
$$f'(1)=0, \qquad f'(-1)=0 \quad (18)$$

Hence the solution of (17) subject to the boundary condition (18) is

$$f(\eta)=\frac{\left(\begin{array}{l}(1-\eta)\sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2}\\ +\sinh\left(\sqrt{\alpha^2h^2+M^2}\,\eta\right)-\sinh\sqrt{\alpha^2h^2+M^2}\end{array}\right)}{2\left(\sinh\sqrt{\alpha^2h^2+M^2}-\sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2}\right)} \quad (19)$$

Substituting the value of $f(\eta)$ in the stream function

Hence the Axial Velocity becomes

$$u=u(x,y)e^{-nt}$$
$$=\frac{1}{h}\frac{\partial\psi}{\partial\eta}e^{-nt}$$
$$=\frac{1}{h}(hu(0)-v_0 x)f'(\eta)e^{-nt}$$
$$=\left(u(0)-\frac{v_0 x}{h}\right)e^{-nt}\frac{\left(\begin{array}{l}\cosh\left(\left(\sqrt{\alpha^2h^2+M^2}\right)y/h\right)-\\ \sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2}\end{array}\right)}{2(\sinh\sqrt{\alpha^2h^2+M^2}-\sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2})}$$

Transverse Velocity becomes

$$
\begin{aligned}
v &= v(x,y)e^{-nt} \\
&= -\frac{\partial \psi}{\partial x}e^{-nt} \\
&= v_0 e^{-nt}\,\frac{\begin{pmatrix}(1-(y/h))\sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2}+ \\ \sinh\left(\left(\sqrt{\alpha^2h^2+M^2}\right)y/h\right)-\sinh\sqrt{\alpha^2h^2+M^2}\end{pmatrix}}{2(\sinh\sqrt{\alpha^2h^2+M^2}-\sqrt{\alpha^2h^2+M^2}\cosh\sqrt{\alpha^2h^2+M^2})}
\end{aligned}
$$

VI. PRESSURE DISTRIBUTION

The Pressure Drop can be obtained from (7), (8) and (19)

$$
p(x,\eta)-p(0,0)=\left(u_0x-\frac{v_2x^2}{2h}\right)\times\left(\frac{\mu}{h^2}f'''(\eta)-(\frac{\mu}{K}+\sigma B_0^2-n\rho)f'(\eta)\right)
$$

$$
+v_0h\left[\left(n\rho\int_0^{\eta}f(\eta)d\eta\right)+\left(\frac{\mu}{h^2}\int_0^{\eta}f''(\eta)d\eta\right)\right]
$$

VII. RESULTS AND DISCUSSIONS

Analytical solutions of this problem are obtained and the outcome is illustrated graphically. Figs.1–9 shows the features of important physical parameters on the average entrance velocity, density, time, Hartmann number and pressure distributions. Throughout the computations u0=0.5,v0=0.5,x=1,y=-1to1,h=2,a=1;K=5,ρ=1,μ=0.5,σ=0.5 ,n=0.5,t=0.2,B_0=0.5,M=1 are considered as input value for the graph. Figs. 1-9 present the effect of axial velocity of fluid, transverse velocity of fluid and pressure distribution respectively.

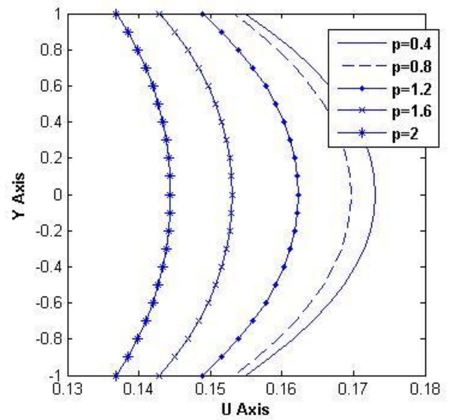

Figure1.Axial Velocity when ρ increases

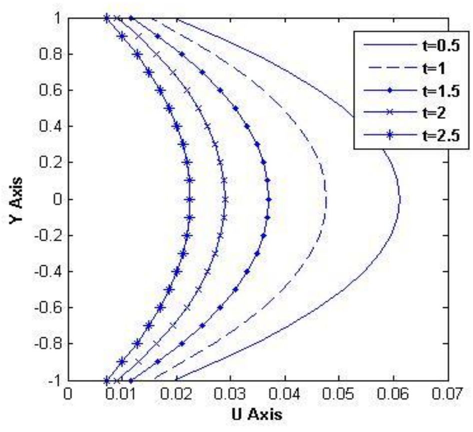

Figure2. Axial Velocity when time increases

Figs. 1, 2 and 4 shows that the velocity of the fluid decreases when ρ, t, M increases. Figure 3 shows that velocity of fluid increases as u_0 increases. Figs. 1-4 shows that the magnitude of the upper plate and lower plate are same. Figs.5–7 shows that the variation of transverse velocity with respect to the variations of the parameter ρ, v_0 and M. Figs. 5–7 shows the magnitude of the upper and lower plates are not same. Figure 5 shows that transverse velocity of the fluid increases as ρ, v_0 and M increases.

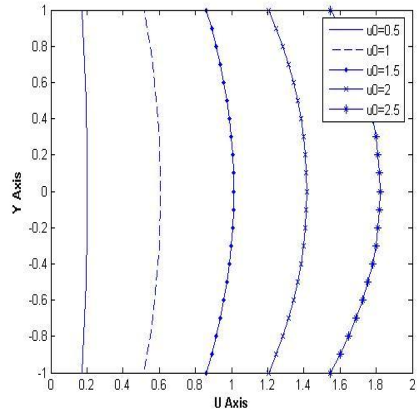

Figure3. Axial Velocity when u_0 increases

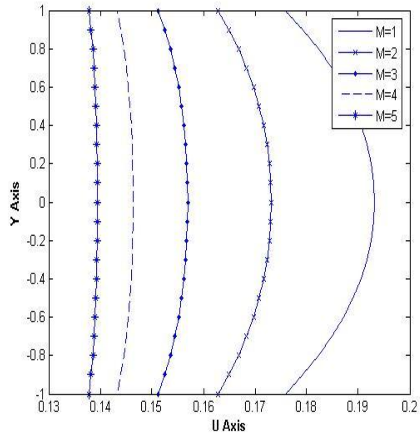

Figure4. Axial Velocity when M increases

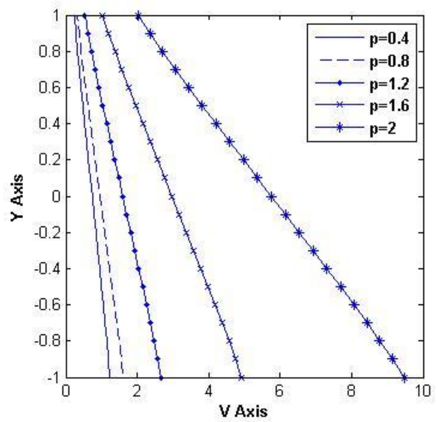

Figure5. Transverse Velocity when ρ increases

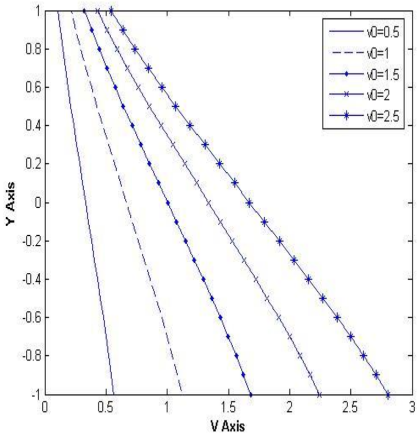

Figure6. Transverse Velocity when v_0 increases

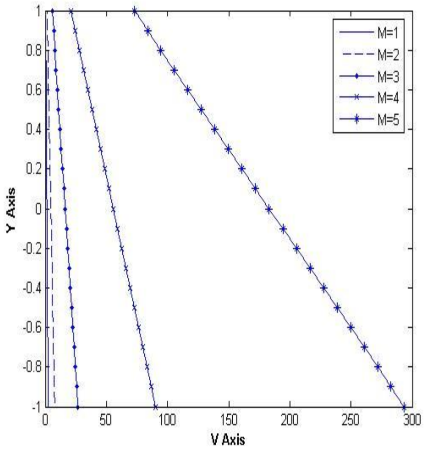

Figure7. Transverse Velocity when M increases

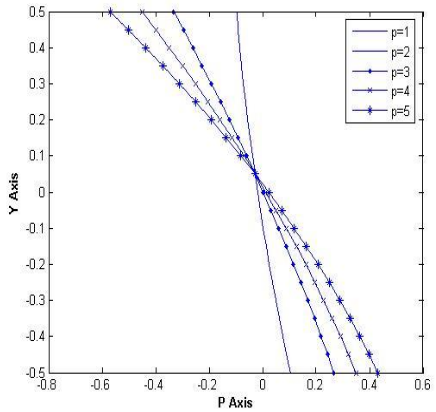

Figure 8.Pressure when ρ increases

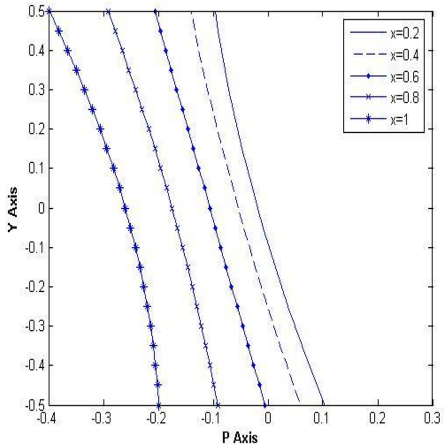

Figure9.Pressure when x increases

Figure 8 and 9 shows that variations of the pressure of the fluid with respect to the parameter ρ,x. Figure 7 shows that the pressure of the fluid increases when x <0 and decreases when x>0. Figure 9 shows that pressure of the fluid decreases as x increases. Figure 8 and 9 shows that the magnitudes of the upper and lower plates are same. The above results reduce to the results of [16] when K-infinity.

VIII. CONCLUSION

Analytical solutions are obtained for the Unsteady MHD flow between two parallel plates through porous medium with One Plate Moving Uniformly and the Other Plate at Rest with Uniform Suction. The Similarity transformation method is used to solve the problem and the results are evaluated analytically and displayed graphically. In the light of the present investigation, following conclusions are drawn:

- The Axial velocity of the fluid decreases as density (ρ), time, Hartmann number (M) increases.
- The Axial velocity of the fluid increases as average entrance velocity (u_0) increases
- Transverse velocity of fluid increases as density, Hartmann number (M) and suction (v_0) increases.
- Pressure of the fluid decreases as x increases.

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