State Space Modeling, Simulation and Comparative Analysis of a conceptualised Electrical Control Signal Transmission Cable for ROVs

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***Abstract*— Electrical signal transmission cables can be mathematically modeled either by lumped parameters approach or distributed parameters approach depending on the frequency of the signal to be transmitted. Unlike high frequency signal transmission systems, Low frequency electrical transmission cables such as Remote Operated Vehicle (ROV) control signal cables (tethers) can be modeled using lumped parameter approach. In this paper, we model, simulate and analyse the behaviour of a conceptualised electrical cable that is used to transmit low frequency electrical control signals. We conceptualise a cable as if resistance, capacitance and inductance are placed at particular distinct points along the cable. The models are analysed and compared by increasing the magnitude of RLC (Resistor, Inductance and Capacitance) components for a fixed length versus increase in length by replicating the same RLC components of the cable along transmission line by the same factor. Analysis of responses from these models is made subject to step and square input signals. Comparison was based on most popular parameters that are used in transient signal responses. Simulation results confirmed the signal dumping effect of increasing the cable length.**

***Index Terms*— Lumped parameters, Signal Transmission Cables, ROV Cables, State Space Model, Simulation**

I. INTRODUCTION

Modelling electrical signal transmission cable requires prior knowledge about the frequency of the signal to be transmitted [3] and this dictates the modelling method. Unlike signals in high frequency bands such as radio frequency bands where transmission systems are modelled by distributed parameter approach using partial differential equations[1],[2], lower frequency transmission cables such as control signal transmission systems can modelled by lumped parameter approach using Ordinary Differentials Equation [1],[2]. These low frequency control signals are practically applicable in Remote Operated Vehicles (ROV) deployed in subsea activities which can be monitored onshore. In ROVs, control signal cables are aligned and shielded together with data transmission cables such as optical cables to transmit signals such as row video data required for real-time subsea Monitoring [5].

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In this paper, we base on well known behaviour of RLC circuitry subject to AC signal source [2] and model a low frequency electrical based transmission cable using Ordinary Differential Equations (ODEs). Our aim is not to propose a novel technique but to make a conceptualised comparative analysis. We first model arbitrarily an "L" length cable (1L Model) with specified magnitude of RLC components as depicted in (fig. 1). We further model "2L" length cable (2L model) by duplicating the first model along the transmission length. Based on the first model in (fig.1), we also develop another model by using a cable of fixed length "L" but doubling the magnitude of RLC components (2Va model) as depicted in section IV.

These models are simulated subject to both step and square input signals. Output signals are analysed and the behaviour of the system models are compared based on rise time, peak amplitude, settling time and final state value of the output signal.

II. RLC COMPONENTS STATE SPACE (SSM) MODEL

The relation between the voltage V_R, across the resistor, R and the current through it, I_R caused by potential difference across it, is ohmic in nature [6], [7] and is described in (1)

$$I_R \propto V_R ; I_R \propto \frac{1}{R} \Leftrightarrow V_R = RI_R \tag{1}$$

Unlike in resistors, the voltage across the capacitor changes with respect to time.The rate at which the voltage across the capacitor changes with respect to time is proportional to the charging current through it and inversely proportional to the magnitude of its capacitance[6],[7] as shown in (2)

$$\frac{dV_c(t)}{dt} = \frac{1}{c} i_c(t) \Leftrightarrow i_c(t) = C\frac{dV_c(t)}{dt} \qquad (2)$$

For an inductor, the charging current changes with time and is proportional the voltage applied across it and inversely proportional to the magnitude of its inductance [6], [7] as shown from equation (3)

$$\frac{di_L(t)}{dt} = \frac{1}{L} V_L(t) \Leftrightarrow V_L(t) = L\frac{di_L(t)}{dt} \qquad (3)$$

Having already described the behavior of each RLC component in ODE perspective in section II, we model entire cable in sections III and IV based on these RLC fundamentals.

III. INCREASED LENGTH CABLE MODEL

Based on individual RLC component state space representation in (1), (2) and (3), the circuit in (fig.1b) can be represented in state space in expressions (4).

$$\left.\begin{array}{l}
\begin{bmatrix} \frac{dV_c(t)}{dt} \\ \frac{di_{L1}(t)}{dt} \\ \frac{di_{L2}(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -(\frac{R_2+R_L}{L_2}) \end{bmatrix} \begin{bmatrix} V_c(t) \\ i_{L2}(t) \\ i_{L2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ 0 \end{bmatrix} V_i(t) \quad (4) \\
V_o(t) = \begin{bmatrix} 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} V_c(t) \\ i_{L1}(t) \\ i_{L2}(t) \end{bmatrix} + 0V_i(t)
\end{array}\right\} \qquad (4)$$

It is a common practice to express such models in a standard state space model in (7) and we do so by assumptions in (5) to get expression (6) according our circuit in fig.1b.

$$\left.\begin{array}{l}
V_c(t) = x_1(t), i_{L1}(t) = x_2(t), i_{L2}(t) = x_3(t), \\
V_o(t) = y(t), and V_i(t) = u(t)
\end{array}\right\} \qquad (5)$$

$$\left.\begin{array}{l}
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R_1/L_1 & 0 \\ 1/L_2 & 0 & -(R_2+R_L)/L_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_3(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} u(t) \\
y(t) = \begin{bmatrix} 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + 0u(t) \\
y(t) = \begin{bmatrix} 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + 0u(t)
\end{array}\right\} \qquad (6)$$

$$\left.\begin{array}{l}
\dot{X}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t) = Du(t)
\end{array}\right\} \qquad (7)$$

$$\left.\begin{array}{l}
R_1 = R/2, R_2 = R/2, \\
L_1 = L/2 \Rightarrow 1/L_1 = 2/L \\
L_2 = L/2 \Rightarrow 1/L_2 = 2/L \\
R = 20\Omega/L, C = 7.5\mu F/L, \\
R_L = 100\Omega, and, l = 0.01H/L
\end{array}\right\} \qquad (8)$$

For simplicity, based on assumptions (8), respective SSM matrices are depicted in (9)

$$\left.\begin{array}{l}
A = \begin{bmatrix} 0 & 1/C & -1/C \\ -2/L & -R/L & 0 \\ 2/L & 0 & -(R+2R_L)/L \end{bmatrix} = \begin{bmatrix} 0 & 133333.33 & -133333.33 \\ -200 & -2000 & 0 \\ 200 & 0 & -22000 \end{bmatrix} \\
B = \begin{bmatrix} 0 \\ 2/L \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 200 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & R_L \end{bmatrix} = \begin{bmatrix} 0 & 0 & 100 \end{bmatrix}, and, D = 0
\end{array}\right\} \qquad (9)$$

A "2L" LENGTH CABLE MODEL

In this model, we duplicate the RLC components in fig (1b) along the transmission line to make it double length of the first model. The double length model (2L) is depicted in fig (2). By considering respective loops, we get expression in (10).

$$\left.\begin{array}{l}
\begin{bmatrix} \frac{dV_{c1}(t)}{dt} \\ \frac{dV_{c2}(t)}{dt} \\ \frac{di_{L1}(t)}{dt} \\ \frac{di_{LT}(t)}{dt} \\ \frac{di_{L2}(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/C_1 & -1/C_2 & 0 \\ 0 & 0 & 0 & 1/C_1 & -1/C_2 \\ -1/L_1 & 0 & -R_1/L_1 & 0 & 0 \\ 1/L_T & -1/L_T & 0 & R_T/L_T & 0 \\ 0 & 1/L_2 & 0 & 0 & -(R_2+R_L)/L_2 \end{bmatrix} \begin{bmatrix} V_{c1}(t) \\ V_{c2}(t) \\ i_{L1}(t) \\ i_{LT}(t) \\ i_{L2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L_1 \\ 0 \\ 0 \end{bmatrix} V_i(t) \\
V_o(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} V_{c1}(t) \\ V_{c2}(t) \\ i_{L1}(t) \\ i_{LT}(t) \\ i_{L2}(2) \end{bmatrix} + 0V_i(t)
\end{array}\right\} \qquad (10)$$

We again represent expressions (10) in form of (7) to get (11) that makes it easier for simulations subject to input signals.

$$\left.\begin{array}{l}
\begin{bmatrix} \dot{X}1(t) \\ \dot{X}2(t) \\ \dot{X}3(t) \\ \dot{X}4(t) \\ \dot{X}5(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/C1 & -1/C2 & 0 \\ 0 & 0 & 0 & 1/C1 & -1/C2 \\ -1/L1 & 0 & -R1/L1 & 0 & 0 \\ 1/LT & -1/LT & 0 & RT/LT & 0 \\ 0 & 1/L2 & 0 & 0 & -(R2+RL)/L2 \end{bmatrix} \begin{bmatrix} x1(t) \\ x2(t) \\ x3(t) \\ x4(t) \\ x5(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L1 \\ 0 \\ 0 \end{bmatrix} u(t) \\
y(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & RL \end{bmatrix} \begin{bmatrix} x1(t) \\ x2(t) \\ x3(t) \\ x4(t) \\ x5(t) \end{bmatrix} + 0u(t)
\end{array}\right\} \qquad (11)$$

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Taking assumptions in (8) and (12), SSM matrices are depicted in (13)

$$\left.\begin{aligned} & L_1 = L/2 \Rightarrow 1/L_1 = 2/L, \\ & L_2 = L/2 \Rightarrow 1/L_2 = 2/L_2 \\ & L_T = L_1 + L_2 \Rightarrow 1/L_T = 1/L \\ & R_2 = R_1 = R/2 \Rightarrow R_T/L_T = R/L \\ & (R_2 + R_L)/L_2 = (R + 2R_L)/L \end{aligned}\right\} \quad (12)$$

$$\left.\begin{aligned} & A = \begin{bmatrix} 0 & 0 & 1/C_1 & -1/C_1 & 0 \\ 0 & 0 & 0 & 1/C_2 & -1/C_2 \\ -2/L & 0 & -R/L & 0 & 0 \\ 1/L & -1/L & 0 & R/L & 0 \\ 0 & 2/L & 0 & 0 & -(2R+R_L)/L \end{bmatrix} \\ & A = \begin{bmatrix} 0 & 0 & 133333.33 & -133333.33 & 0 \\ 0 & 0 & 0 & 133333.33 & 133333.3 \\ -200 & 0 & -2000 & 0 & 0 \\ 100 & -100 & 0 & 2000 & 0 \\ 0 & 200 & 0 & 0 & -22000 \end{bmatrix} \\ & B = \begin{bmatrix} 0 \\ 0 \\ 2/L \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 200 \\ 0 \\ 0 \end{bmatrix}; C = \begin{bmatrix} 0 & 0 & 0 & 0 & RL \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 100 \end{bmatrix}, \\ & and, D = 0 \end{aligned}\right\} \quad (13)$$

IV. Increased RLC Values Model

While in section III, we duplicated lumped parameters in "1L" model along transmission length to make "2L" model, in this section we conceptualise the cable as if the RLC magnitude were increased but keeping its length "L" to develop model "2Va" model.

Based on expressions (6), (7) and (8), we double the values of RLC components associated with circuitry depicted in fig (1b) to obtain model (14)

$$\left.\begin{aligned} & A = \begin{bmatrix} 0 & 1/2C & -1/2C \\ -1/L & -R/L & 0 \\ 1/L & 0 & -(R+R_L)/L \end{bmatrix} = \begin{bmatrix} 0 & 66666.67 & -66666.67 \\ -100 & -2000 & 0 \\ 100 & 0 & -12000 \end{bmatrix}, \\ & B = \begin{bmatrix} 0 \\ 1/L \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & RL \end{bmatrix} = \begin{bmatrix} 0 & 0 & 200 \end{bmatrix}, D = 0 \end{aligned}\right\} \quad (14)$$

V. Circuit Model Representation

L is a Unit length of the cable in the model
R is the resistance of the cable per unit length in Ω/L
L is inductance of the cable per unit length in H/L
C is capacitance of the cable per unit length in F/L

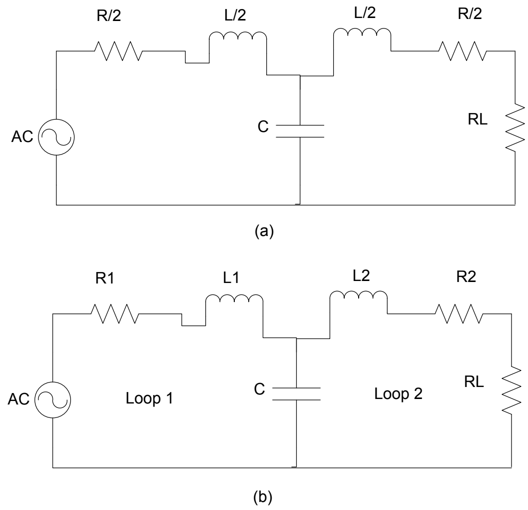

Figure.1 an "L" length cable representation

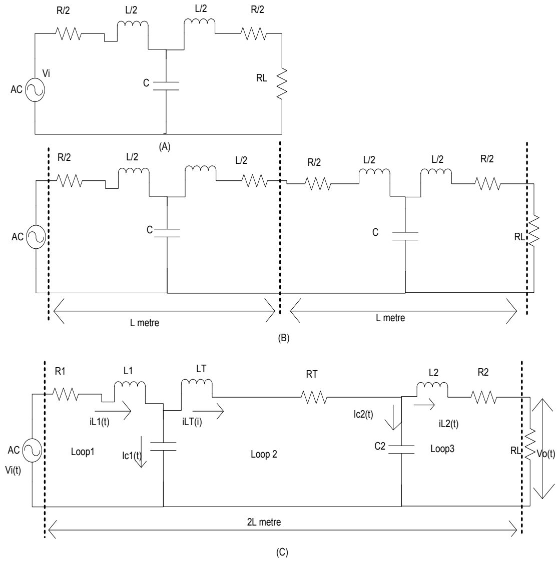

Figure.2 a "2L" conceptualised length cable model

VI. Simulation Results

For simulations, the following assumptions were made to investigate the model response subject to both step and square input signals.

R = 20 Ω/m, L = 0.01 H/m and C= 7.5 μF/m and the load is a100Ω resistor. It is worth mentioning that for a square wave, the signal used in our simulations was 220 V, a 50 Hz.

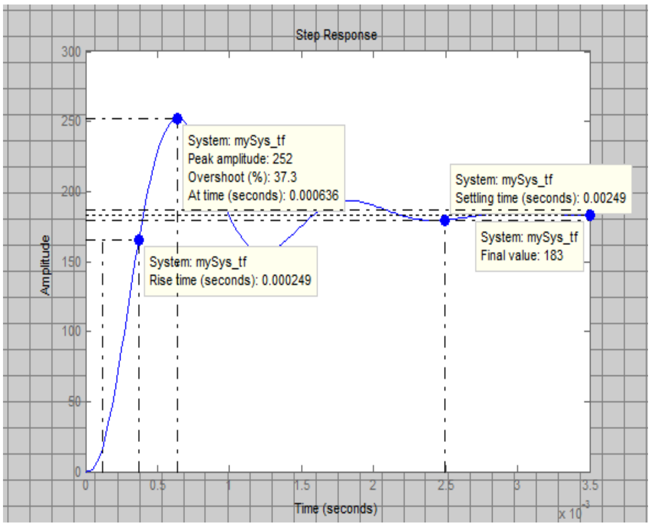

Figure .3 Step signal response for “1L” model

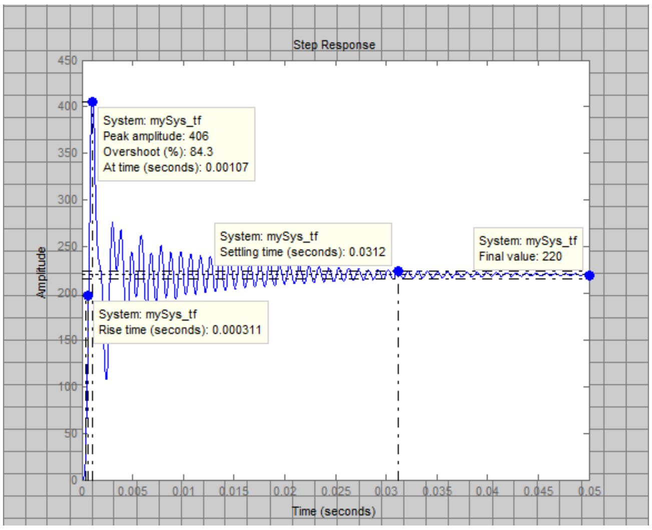

Figure .6 Step signal response for “2L” model

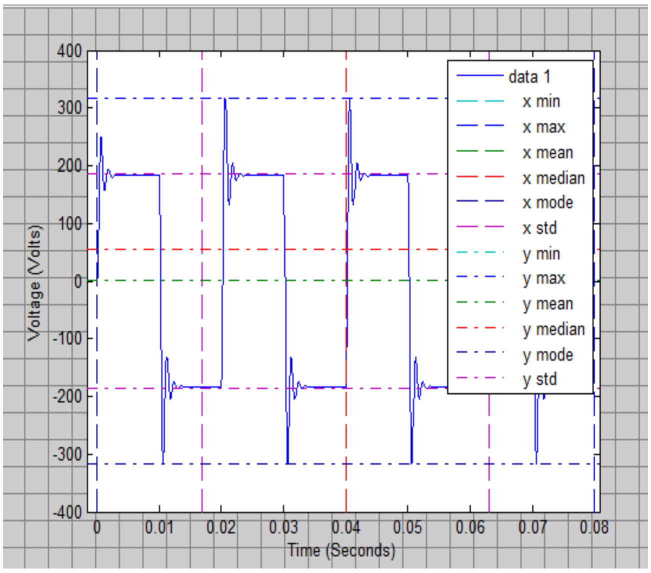

Figure4. Square Signal response for “1L” model

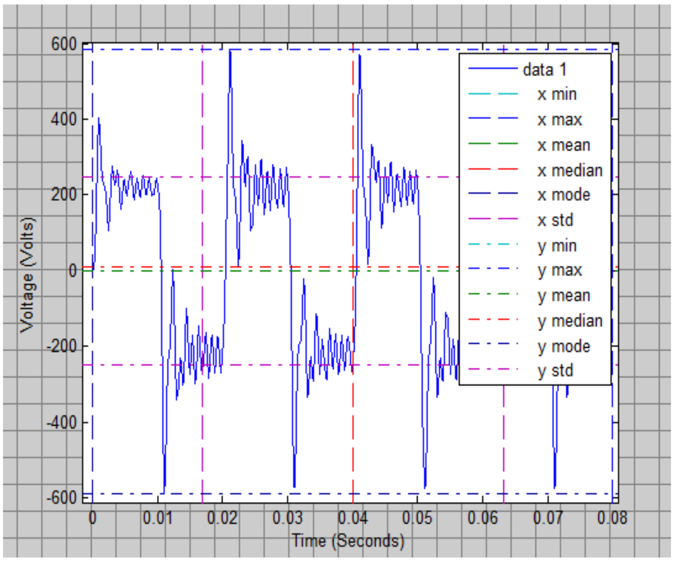

Figure7. Square input signal response for “2L” model

Statistics for data 1

Check to plot statistics on figure:

	X	Y
min	0	-316.4
max	0.08	316.4
mean	0.04	0.2335
median	0.04	55.7
mode	0	-316.4
std	0.02314	186.3
range	0.08	632.8

Figure5. Data statistics for figure 4

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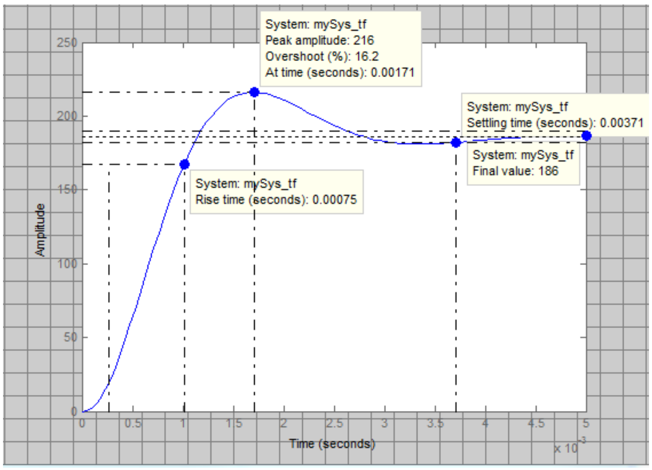

Figure8. Step signal response for "2Va" model

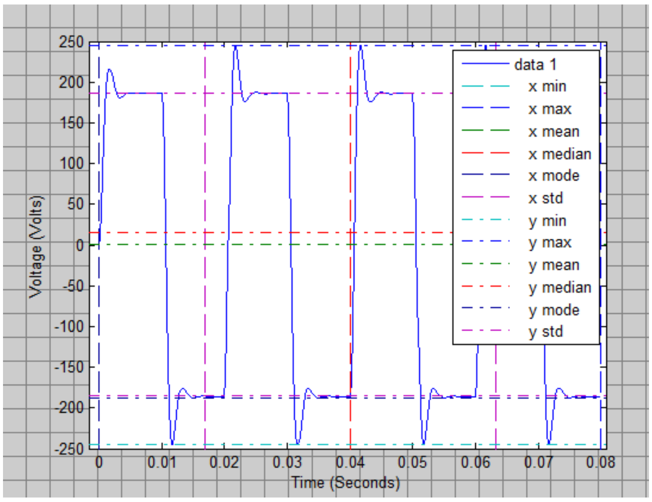

Figure9. Square input signal response for "2Va"model

Table 2 Vertical (Voltage) variable data statistics for square signal response

Model	Min	Max	Mn	Mdn	Mod	Std	Rng
1L	-316.4	316.4	0.2335	55.7	-316.4	186.3	632.8
2L	-588.5	583	0.1451	11.5	-558.5	248	1171
2Va	-244.7	244.7	0.9267	14.87	-187.5	185.3	489.4

The parameter values presented here are Minimum (Min), Maximum (Max), Mean (Mn), Mode (mod), Standard (Std) and Range (Rng)

6. Conclusion

The aim was to model, simulate a conceptualised a low-frequency-signal transmission cable by increasing RLC values versus increasing length by duplicating RLC component along the same conceptualised electrical signal transmission cable and make a comparative analysis.

Both step and square signal input responses showed that increasing length ("2L model") had a poor response if quick settling system is required compared with doubling magnitude of RLC component. This delay in settling was due to high overshoot.

Conceptually, duplicating RLC values along transmission line increases oscillations of an AC signal leading to under damped response thus, not suitable for quick response demanding controls systems.

REFERENCES

[1] Stefan Bilbao and Julius O. , Discrete-Time Lumped Models Smith III , MUS420/EE367A Lecture 7D, Center for Computer Research in Music and Acoustics (CCRMA), Department of Music, Stanford University Stanford, Jan, 2013.

[2] Modeling a RLC Circuit's Current with Differential Equations, Kenny Harwood, May, 2011.

[3] C. Rodriguez Valdez, *Student Member*, R. M. Tallam, *Senior Member*, R.J.Kerman, "A New Frequency-Dependent-Model for Cables" Electro/Information Technology, IEEE International Conference, May, 2008.

[4] Tyco Electronics, Umbilical and Tether cables for Subsea ROV Applications, April, 2008.

[5] L. Horn, G.Taylor "Super Phantom S2 Remotely Operated Vehicle User's Guide", NOAA Cooperative Institute for Ocean Exploration, Research, and Technology.

[6] John O. Attia, "Electronics and Circuit Analysis using MATLAB", CRC Press LLC, 1999.

[7] Analyse RLC second order parallel circuits using duality [online] http://www.dummies.com/how-to/content/analyze-an-rlc-secondorder-parallel-circuit-using-.html

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