Modelling and Simulation Study on Fractional and Integer Order PI Controller for a SISO Process

S.Divya and P.Anbumalar

***Abstract*— In recent years it is remarkable to see increasing number of studies related to the theory and application of fractional order controller, specially PI^{λ} and $PI^{\lambda}D^{\mu}$controller in many areas of science and engineering. Research activities are focused on developing new methods and analysis for fractional order controller as an extension to classical control theory. This paper deals with the designof fractional orderPI^{λ}controller for a SISO (two interacting tank level) process. Integer order Proportional integral (IO-PI) controller and fractional order proportional integral (FO-PI) controller has been designed for integer order system(IOS) by approximated M_s constrained integral gain optimization (A-MIGO) and fractional M_s constrained integral gain optimization (F-MIGO) techniquerule respectively. Further the performance of closed loop response of IOS with fractional and integer order PI controller has been analyzed based on integral squared error (ISE)criterion. From the simulation study it has been inferred that the designed FO-PI controller show better performance than the IO-PI controller in terms of set point tracking, robustness, disturbance rejection and stability analysis.**

***Keywords*— Fractional order controller, integer order controller and two interacting tank level system**

I. INTRODUCTION

The PID controller is by far the most dominating form of feedback in use today. Due to its functional simplicity and performance robustness, the proportional-integral-derivative controller has been widely used in the process industries. Design and tuning of PID controllers have been a large research area ever since Ziegler and Nichols presented their methods in 1942. Specifications, stability, design, applications and performance of the PID controller have been widely treated since then. On the other handin recent years, fractional order dynamic systems and controllers based on fractional order calculus have gained an increasing attention in control community [1]. This is mainly due to the fact that many real physical systems are well characterized by fractional order differential equations involving non integer order derivatives.

S.Divya, *Department of Electronics and Instrumentation Engineering, SRM University, Kattankulathur, Tamil Nadu, India,Mobile No.9884818529*

P.Anbumalar, *Department of Electronics and Instrumentation Engineering B.S.Abdur Rahman University, Vandalur, Chennai-48, Tamil Nadu, India, Mobile No.9994727033*

The concept of fractional order $PI^{\lambda}D^{\mu}$controller which has an integrator of real order λ and differentiator of real order μ is proposed by Podlubny [2]. The fractional order controller structure is given by

$$C(s) = K_P + \frac{K_i}{s^{\alpha}} + K_d s^{\mu}$$

where$K_P$is the proportional constant, $K_i$is the integration constant and $K_d$is the differentiation constant. The $PI^{\lambda}D^{\mu}$ algorithm is represented by a fractional integro-differential equation of type as follows

$$c(t) = K_P e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t)$$

where D is the integro-differential operator[3], e(t) is the controller input and c(t) is the controller output. Clearly, depending on the values of the orders λ and μ, the numerous choices for the controller's type can be made. For instance, taking λ=1 and μ=1 yields the classical PID controller. Moreover, the selection of λ=1 and μ=0 leads to the PI controller, λ=0 and μ=1 gives the PD controller, and also λ=0 and μ=0 results in P controller.

The paper is organized as follows: Section II gives a brief description of the process modelling. In section III, integer and fractional order PI controller is designed for IOS using F-MIGO and A-MIGO tuning technique. The comparative simulation results for the control performance for fractional and integer order PI controller based on ISE criterionare presented in section IV. Finally, results and conclusion were discussed in section V.

II. PROCESS MODELING

A. Process description

The two interacting tank process as shown in Fig 1, consists of two identical cylindrical tanks with equal cross-sectional area (A). These two tanks are connected by cylindrical pipes of uniform cross sectional area (α). From the reservoir the process liquid is pumped into the first tank through a control valve. The input flow to tank 1 is q_i. The two tanks are interconnected through manual valves. The objective is tocontrol the level of liquid in tank 2 by varying the inflow rate (q_i) in the tank 1.

ISSN: 2278 – 7798

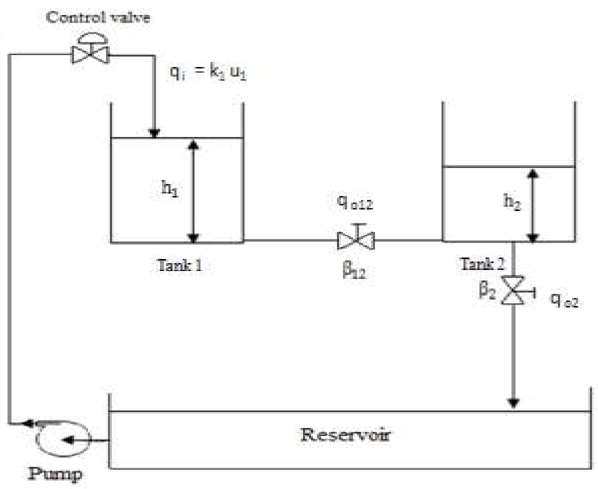

Fig.1 Two interacting tank level system

Where,
h_1, h_2= height of fluid in tank 1 and tank 2 respectively (cm).
A = cross sectional area of tank 1 and tank 2 (cm).
u_1= input to tank 1 (v).
q_i= flow rate of fluid into tank 1(cm^2/s).
q_{o12}= flow rate of fluid between tank 1 and tank 2 (cm^2/s).
k_1= pump flow rate into tank 1(cm^3/vs).
β_{12}= valve ratio between tank 1 and tank 2.
β_2 = valve ratio at the outlet of tank 2.
a_{12}= cross sectional area of jointed pipe between tank 1and tank 2 (cm^2).
a_2= cross sectional area of pipe at the outlet of tank 2 (cm^2).
g = gravity (cm/s^2).

B. Mathematical modelling of two interacting tank level process

The non-linear equations describing the open loop dynamics of the TITL process is derived using the mass balance equation and Bernoulli's principle as follows, According to mass balance equation, rate of accumulation= inflow rate- outflow rate.

For tank 1

$$\frac{dh_1}{dt}A = q_i - q_{o12} \quad (1)$$

Where $q_i$and q_{o12} are the inflow and outflow of tank 1, which is given by

$$q_i = k_1 u_1 \quad (2)$$

$$q_{012} = \beta_{12}\, a_{12}\sqrt{2g(h_1(t) - h_2(t))} \quad (3)$$

Substituting (2) and (3) in equation (1) gives

$$\frac{dh_1(t)}{dt} = \frac{k_1 u_1}{A} - \frac{\beta_{12} a_{12}\sqrt{2g(h_1(t) - h_2(t))}}{A} \quad (4)$$

Similarly for tank 2

$$\frac{dh_2}{dt}A = q_{o12} - q_{o2} \quad (5)$$

Whereq_{o12} and q_{o2} are the inflow and outflow of tank 2, which is given by

$$q_{012} = \beta_{12}\, a_{12}\sqrt{2g(h_1(t) - h_2(t))} \quad (6)$$

$$q_{02} = \beta_2\, a_2\sqrt{2gh_2(t)} \quad (7)$$

Substituting (6) and (7) in equation (5) gives

$$\frac{dh_2(t)}{dt} = \frac{\beta_{12}\, a_{12}\sqrt{2g(h_1(t) - h_2(t))}}{A} - \frac{\beta_2 a_2\sqrt{2gh_2(t)}}{A} \quad (8)$$

Linearized integer order model at different operating points are listed in table 1.

TABLE: 1 INTEGER ORDER MODEL

Regions (u_1) v	**Operating point (u_1) v**	**Height (h_1) cm**	**Height (h_2) cm**	**Integer order system $P_i(s)$**
(0.3-0.8)	0.54	0.29	0.23	$\frac{0.8613}{52.56s+1}e^{-3.96s}$
(0.8-1.5)	1.18	1.41	1.09	$\frac{1.915}{116.5s+1}e^{-8.66s}$
(1.5-2.1)	1.84	3.40	2.64	$\frac{2.997}{180.1s+1}e^{-13.4s}$
(2.1-3)	2.58	6.66	5.17	$\frac{3.332}{212.5s+1}e^{-18.5s}$
(3-4.5)	3.87	14.06	10.92	$\frac{7.156}{442.3s+1}e^{-28.9s}$

III. CONTROLLER DESIGN

A. Fractional order PI^λ controller (FOC) design

Let P(s) and C(s) be the plant and controller transfer function respectively. The transfer function of the fractional order PI^λcontroller is

$$C_f(s) = K_P + \frac{K_i}{s^\lambda} \quad (9)$$

The fractional order PIcontroller in time domain is represented by as:

$$c(t) = K_P e(t) + K_i D_t^{-\alpha} \quad (10)$$

where$D_t^{-\alpha}$is the fractional integro-differential operator. The general calculus operator (including fractional order and integer order) is defined as:

$$ {}_aD_t^\alpha = \begin{cases} d^\alpha / dt^\alpha, R(\alpha) > 0 \\ 1, R(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha}, R(\alpha) < 0 \end{cases} $$

where$a$ and t are respectively the lower and upper bounds, and α is the order of derivative or integrals, which can be non-integers, or even complex numbers

ISSN: 2278 – 7798

The tuning rules for FO-PI controller based on Fractional Ms constrained integral gain optimization method (F-MIGO) for generic FOPDT model [4], [5] is given by

$$K_P = \frac{1}{k_p}\left(\frac{0.2978}{\tau + 0.000307}\right) \quad (11)$$

$$T_i = T\left(\frac{0.8578}{\tau^2 - 3.402 + 2.405}\right) \quad (12)$$

$$\alpha = \begin{cases} 1.1 & if \tau \geq 0.6 \\ 1 & if\ 0.4 \leq \tau < 0.6 \\ 0.9 & if\ 0.1 \leq \tau < 0.4 \\ 0.7 & if \tau < 0.1 \end{cases} \quad (13)$$

$$\tau = \frac{L}{L+T} \quad (14)$$

where$k_p$and T are the steady state gain and time constant and L is the delay.

B. Integer order controller(IOC) design

The transfer function of integer order PI controller is represented by

$$C_f(s) = K_P + \frac{K_i}{s} \quad (15)$$

The tuning rules for integer order PI controller based on Approximated Ms constrained integral gain optimization method (A-MIGO) for generic FOPDT model [6] is given by

$$K = \frac{0.14}{k_P} + \frac{0.28T}{k_p L} \quad (16)$$

$$K_i = K\left[0.33L + \frac{6.8LT}{10L+T}\right]^{-1} \quad (17)$$

The controller parameters of fractional and integer order PI controller for integer order system (IOS) of are shown in Table 2.

TABLE: 2 CONTROLLER PARAMETERS

Regions (u_1) v	**Integer order PI controller $C_i(s)$**	**Fractional order PI controller $C_f(s)$**
(0.3-0.8)	$4.477 + \frac{0.268}{s}$	$4.917 + \frac{0.236}{s^{0.7}}$
(0.8-1.5)	$2.040 + \frac{0.0556}{s}$	$2.578 + \frac{0.0568}{s^{0.7}}$
(1.5-2.1)	$1.302 + \frac{0.0297}{s}$	$1.433 + \frac{0.020}{s^{0.7}}$
(2.1-3)	$1.007 + \frac{0.013}{s}$	$1.112 + \frac{0.013}{s^{0.7}}$
(3-4.5)	$0.6183 + \frac{0.00481}{s}$	$0.690 + \frac{0.00401}{s^{0.7}}$

IV. SIMULATION RESULT AND DISCUSSION

A. Set point tracking:

Fig 2 shows the servo performance of IOS with integer and fractional order PI controller. From the figure it is observed that the overshoot of IOS with FO-PI controller is significantly reduced when compared with IO-PI controller for IOS. Table 3 shows the performance analysis of closed loop response of integer and fractional order PI controller based on ISE criterion.

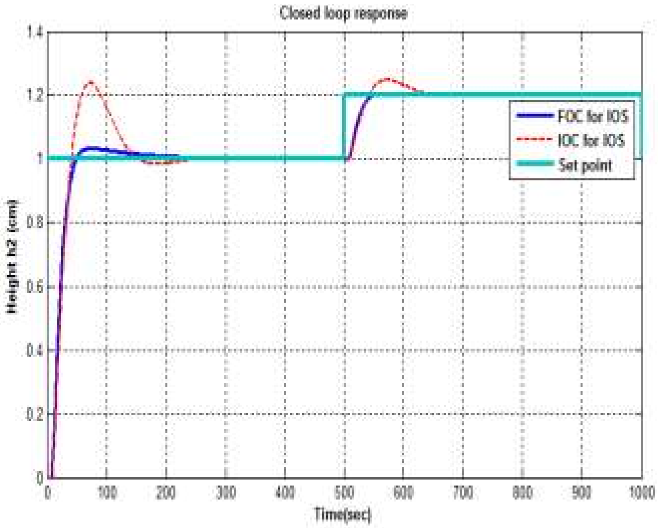

Fig 2: Set point tracking performance of fractional/integer order controller for IOS

TABLE 3: ISE PERFORMANCE ANALYSIS FOR SET POINT TRACKING

Regions (u_1) v	**ISE**	
	FOC with IOS	**IOC with IOS**
(0.3-0.8)	2.826	3.213
(0.8-1.5)	17.8	21.51
(1.5-2.1)	85.11	101.4
(2.1-3)	288.2	318.1
(3-4.5)	632.5	722.3

B. Robustness Test

The uncertainties in the plant parameters like changes in the gain or the time constant are considered for checking the robustness of both control systems. By apply ±5% change in gain the unit step responses of each control systems for the parametric uncertainties are shown in Fig. 3 and 4. From these figures, it is shown that the robustness of the FOC for IOS is better than one of the IOC for IOS. The performance indices of robustness test for two control systems are shown in Table 4 and 5, respectively.

ISSN: 2278 – 7798

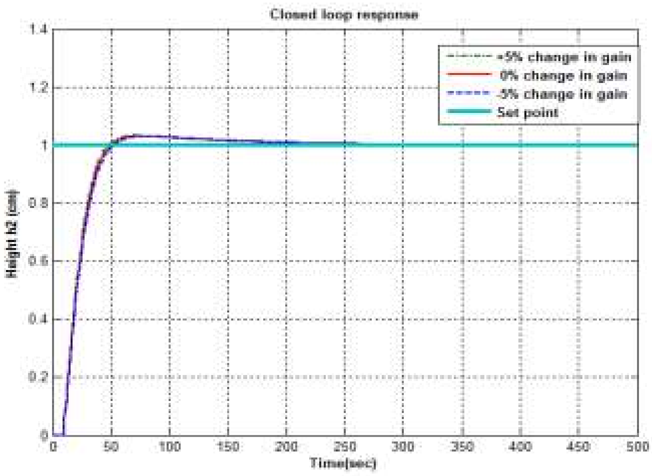

Fig. 3 Robustness test for fractional PI controller with IOS

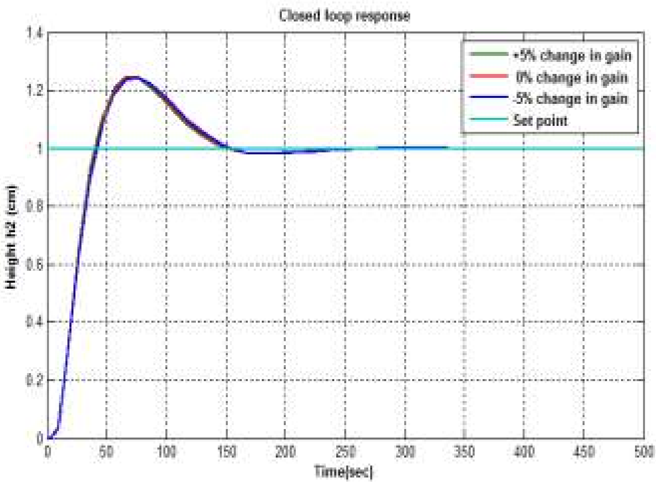

Fig. 4 Robustness test for fractional PI controller with IOS

TABLE 4: ROBUSTNESS TEST FOR FO- PI CONTROLLER WITH IOS

Regions (u_1) v	ISE		
	+5% change in gain	0% change in gain	-5% change in gain
(0.3-0.8)	2.008	2.078	2.156
(0.8-1.5)	16.65	16.89	17.15
(1.5-2.1)	109.8	92.24	112.6
(2.1-3)	281.8	306.2	303.4
(3-4.5)	975.3	979.5	983.7

TABLE 5: ROBUSTNESS TEST FOR IO- PI CONTROLLER WITH IOS

Regions (u_1) v	ISE		
	+5% change in gain	0% change in gain	-5% change in gain
(0.3-0.8)	2.299	2.367	2.443
(0.8-1.5)	20.49	20.76	21.04
(1.5-2.1)	109.6	110.4	117.3
(2.1-3)	307.1	338.2	335.7
(3-4.5)	1121	1099	1123

C. Disturbance rejection

As shown Fig 5, FO-PI controller for IOS has a good disturbance rejection and attains the set point at the faster rate when compared to IO-PI controller for IOS. The performance analysis of disturbances rejection for integer and fractional order PI controller is shown in Table6.

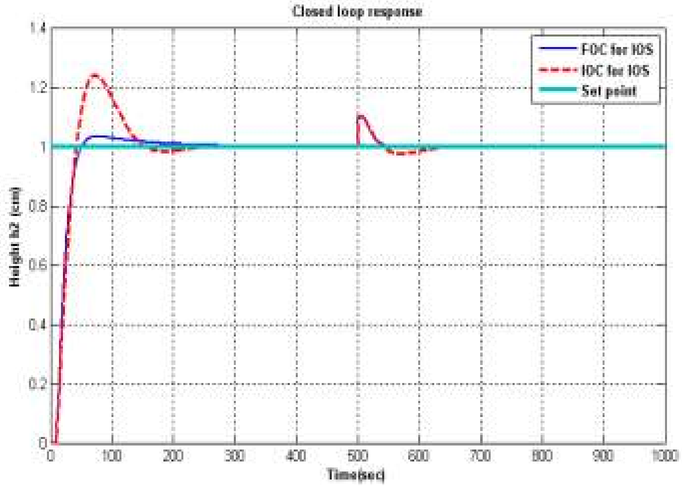

Fig. 5 Disturbance rejection performance of fractional/integer order controller for IOS

TABLE 6: ISE PERFORMANCE ANALYSIS FOR DISTURBANCE REJECTION

Regions (u_1) v	ISE	
	FOC with IOS	IOC with IOS
(0.3-0.8)	2.16	2.457
(0.8-1.5)	17.06	21.52
(1.5-2.1)	92.59	110.6
(2.1-3)	306.6	338.3
(3-4.5)	982.4	1120

D. Stability analysis

It is well known from the general stability theory that a linear time invariant (LTI) system is stable if all roots of characteristic equation are negative or have negative real parts. It meansthat they are located on the left half of the complex plane.

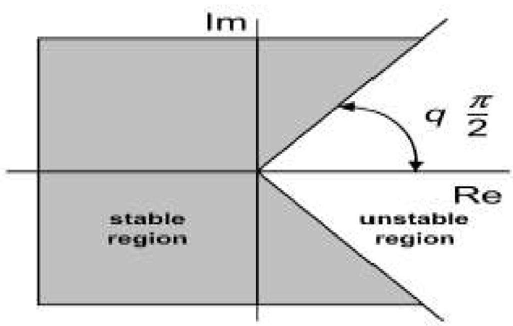

Fig.6 Stability region of LTI FOS with order <q≤1

In the fractional-order LTI case, where the stability of a fractional order control system is analyzed

ISSN: 2278 – 7798

withMatignon'sstability theorem [7],[8], the left half of the plane is different from the integer one. As shown in Fig 6, the vertical axis of the complex plane is changed with an angle depending on the fractional order. Interesting notion is that a stable fractional system may have roots in the right half of complex plane.The pole position plot of the FOC for IOS obtained using the MATLAB is shown in Fig 7. This figure shows that all poles of the FOS are in the fractional left half plane and thus the FOS is stable. Similarly, the pole position plot of IOC for IOS is shown in Fig 10.

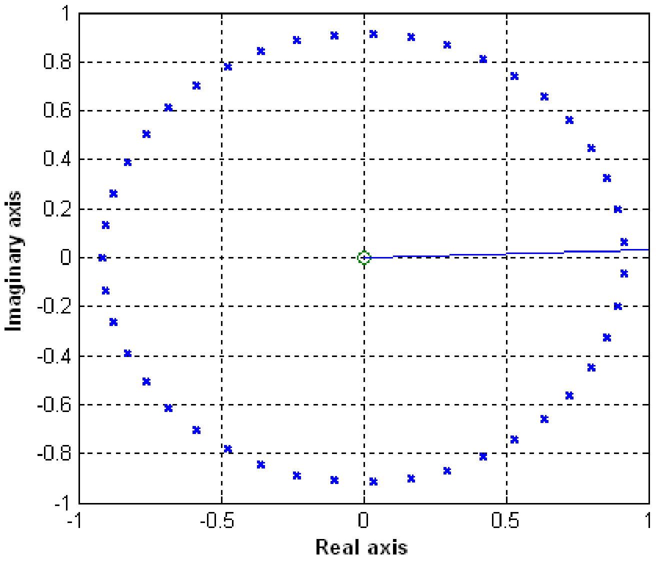

Fig.9.Poles of FOC for IOS

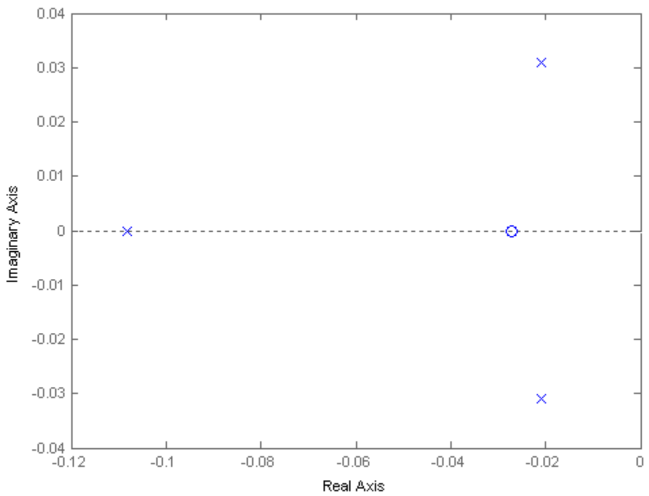

Fig. 10Poles of IOC for IOS

V. CONCLUSION AND FUTURE WORK

In this paper, the fractional and integer PI controller is designed for a class of linear FOPDT model.From the simulation and performance analysis it is inferred that the designed fractional order PI controller for integer order system shows better result, when compared to the integer order PI controller for integer order system in terms of set point tracking, robustness, disturbance rejection and stability. Further,the design and implementation of fractional and integer order controllerfor MIMO process with will be carried out in future.

REFERENCES

[1] Y. Q. Chen. Ubiquitous fractional order controls?.*in Proc. FDA 2006 Fractional Derivatives and Appl.,* Porto, Jul. 19-21, 2006

[2] I. Podlubny, "Fractional-order systems and PID"-controllers," *IEEE Trans. Autom. Control,* vol. 44, no.I, pp. 208-214, 1999.

[3] Yang Quan Chen, Ivo Petras and DingyuXue, "Fractional Order Control - A Tutorial", *American Control Conference Hyatt Regency Riverfront*, St. Louis, MO, USA June 10-12, 2009.

[4] VarshaBhambhani and Yang Quan Chen, "Experimental Study of Fractional Order Proportional Integral (FOPI) Controller for Water Level Control", *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, Dec. 9-11, 2008.

[5] TriptiBhaskaran, YangQuan Chen and Dingy u Xue "Practical tuning of fractional order proportional and integral controller", *Proceedings of the ASME 2007 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference IDETC/CIE* 2007, September 4-7, 2007, Las Vegas, Nevada, USA.

[6] TriptiBhaskaran,"Practical tuning of fractional order proportional and integral controller",M.Sc thesis 2007.

[7] D. Matignon, "Generalized Fractional Differential and Difference Equations: Stability Properties and Modelling Issues" *Proeerdings of Math. Theory of Networks and Systems Symposium, Padova*, Italy, 1998.

[8] PrabooNagarajendraNarayanaswamy, PalaniappanKanthabhabha, Fractional order PI control strategy for liquid level system, "*Second World Congress on Nature and Biologically Inspired Computing",* Dec. 15-17, 2010 in Kitakyushu, Fukuoka, Japan.

ISSN: 2278 – 7798