A CASE STUDY OF QUALITY CONTROL CHARTS IN A MANUFACTURING INDUSTRY

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***Abstract*— Statistical Process Control (SPC) is a powerful collection of problem solving tools and the most sophisticated useful method in achieving process stability and improving the process capability through the reduction of variability. In the manufacturing process, every product doesn't meet the desired range of quality consistently with the customer specification. This inconsistency occurs due to several sources of variations such as machines, operators, materials etc. The Ultimate target of control chart is to monitor the variations, and subsequently control the process. On account of applying SPC methods, this study deals with the control and improvement of the quality of bolt by inspecting the bolt's height, diameter and weight from a bolt manufacturing company. In this inspection, we have developed X bar chart, S and Range control chart for each three variables. Furthermore, we have also focused on Estimated Weighted Moving Average (EWMA) for detecting small process shifts and multivariate Hotelling's T^2 for simultaneous monitoring of height and diameter of bolt. These inspections show that either the process is in control or out of control. For the out of control situation, the assignable reasons behind it should be identified and prevented by taking necessary steps.**

***Keywords*— EWMA, Hotelling's T^2, Mean, Range, Statistical process control.**

I. INTRODUCTION

A control chart is a graphical display of quality characteristics that has been measured or computed from a sample versus the sample number or time. The chart contains a centre line which represents the average value of the quality characteristic corresponding to the in-control state. Two control limits (UCL, LCL) are chosen so that if the process is in control, nearly all of the sample points will fall between them. It is also an estimating device which exhibits statistical control of such process parameters like the mean, standard deviation, fraction, non- conforming or fall- out and so forth. These estimates may then be used to determine the capability of the process to produce acceptable products. [1]

II. METHODOLOGY

According to the approach of control chart, three variables under study were X1 = Height (cm), X2 =diameter (mm) and X3 = weight (mg). In dataset, there were 20 sample numbers with 5 sample size. The following approaches based upon the principles of Statistical Quality Control(SQC) were applied here.

A. The Range and the X bar control charts

The upper and lower control limits for Range chart are

$$LCL = D_3 \overline{R},$$

Where,

$$D_3 = 1\text{-}3(d_3/d_2)$$

$$UCL = D_4 \overline{R}$$

Where,

$$D_4 = 1+3(d_3/d_2)$$

The upper and lower control limits for mean are [4]

$$UCL = \overline{\overline{X}} + A_2\overline{R}$$
$$LCL = \overline{\overline{X}} - A_2\overline{R}$$

B. The Exponentially Weighted Moving Average (EWMA) Control Chart

The upper and lower control limits are

$$\frac{3\sigma}{(n)^{\wedge}0.5}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)]^{\wedge}2t} \pm \overline{\overline{X}}$$

Where ,

$$\lambda = 0.3$$

C. The Multivariate Quality Control Chart

The Hotelling's T^2 statistic is [5]

$$T^2 = m\,(\overline{X}j - \overline{\overline{X}})^T S^{-1} (\overline{X}j - \overline{\overline{X}})$$

Plotting the T^2 values on the time axis. The lower control limit is zero, and the upper control limit is [1]

$$UCL = \frac{p(n-1)(m-1)}{nm-n-p+1} F_{2,nm-n-p+1}(0.01)$$

III. RESULT AND DISCUSSION

A. X bar, Range, S, EWMA control charts for height of the bolt

Table -1 Data set for height of the specific type of Bolt

Sample No	X1(height)	X2	X3	X4	X5	$\bar{X}$	S	R
01	4.297	4.286	4.298	4.262	4.286	4.2858	0.0145	0.036
02	4.285	4.293	4.267	4.283	4.288	4.2832	0.0098	0.026
03	4.261	4.273	4.271	4.303	4.301	4.2818	0.019	0.042
04	4.273	4.287	4.303	4.304	4.296	4.2926	0.013	0.031
05	4.264	4.278	4.298	4.301	4.291	4.2864	0.0153	0.037
06	4.269	4.296	4.293	4.277	4.289	4.2848	0.0114	0.027
07	4.286	4.303	4.281	4.273	4.271	4.2828	0.0128	0.032
08	4.31	4.298	4.301	4.291	4.269	4.2938	0.0154	0.041
09	4.277	4.269	4.287	4.263	4.263	4.2718	0.0103	0.024
10	4.269	4.301	4.269	4.302	4.303	4.2888	0.0181	0.034
11	4.293	4.295	4.273	4.294	4.301	4.2912	0.0106	0.028
12	4.281	4.291	4.291	4.261	4.278	4.2804	0.0123	0.03
13	4.301	4.272	4.304	4.274	4.296	4.2894	0.0153	0.032
14	4.292	4.308	4.296	4.282	4.263	4.2882	0.0169	0.045
15	4.272	4.291	4.281	4.287	4.286	4.2834	0.0073	0.019
16	4.281	4.273	4.293	4.302	4.29	4.2878	0.0112	0.029
17	4.294	4.286	4.302	4.291	4.281	4.2908	0.0079	0.021
18	4.262	4.267	4.301	4.297	4.303	4.286	0.0198	0.041
19	4.291	4.303	4.296	4.279	4.29	4.2918	0.0088	0.024
20	4.302	4.279	4.279	4.281	4.286	4.2854	0.0097	0.023
						4.28631	0.01297	0.0311

For the data set, the X bar control chart is shown in fig.1

For X bar chart, the LCL= 4.2684, UCL= 4.3043 and CL=4.2863;

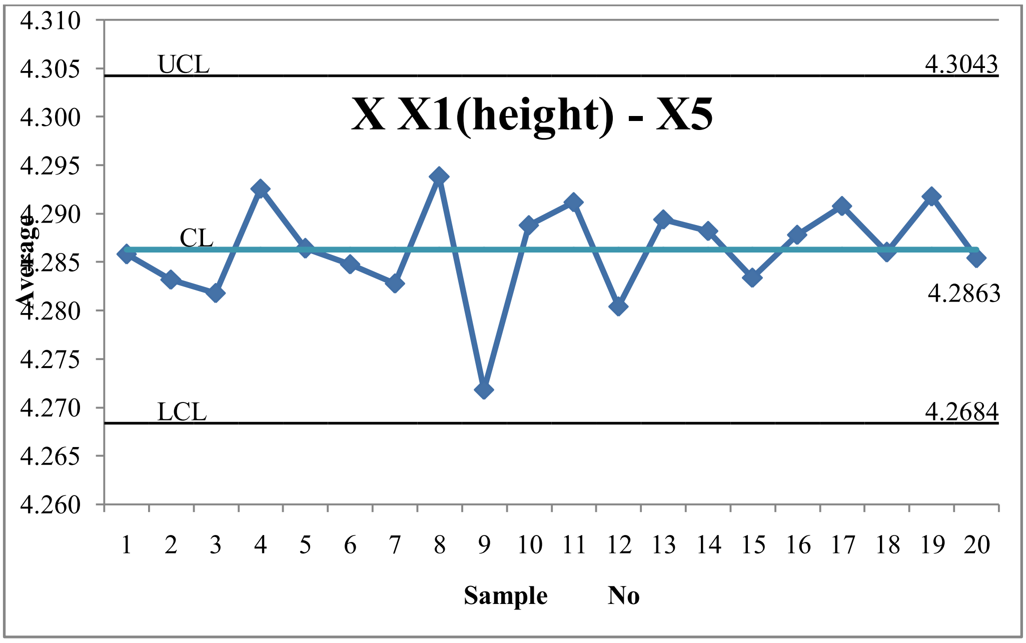

Fig.1 Sample mean Control Chart for height (cm)

Any point falling outside the control limits indicates that assignable causes had affected the process and the process is out of control. Looking into the X bar chart it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For Range chart, the LCL= 0, UCL= 0.0657 and CL=0.0311;

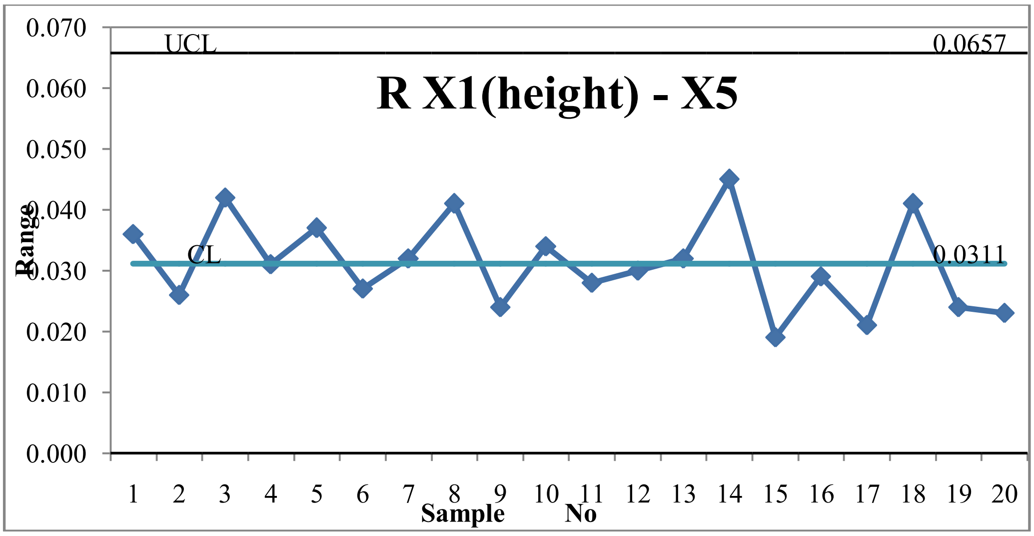

Fig. 2 Range chart for height of the bolt

Looking into the R chart Fig.2 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For Standard Deviation chart the LCL= 0 , UCL= 0.0271 and CL=0.0130;

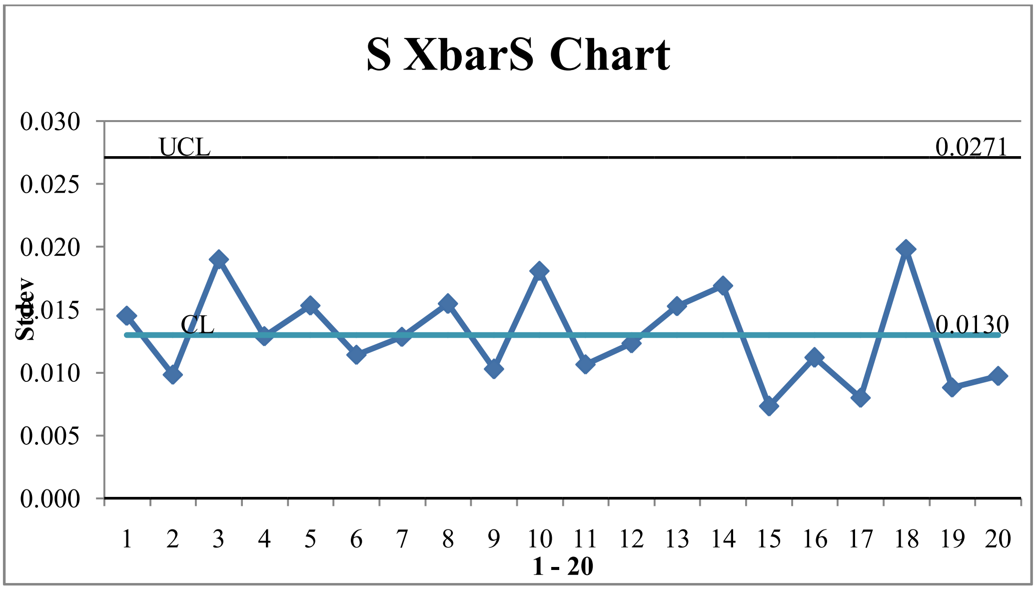

Fig.3 S chart for height of the bolt

Looking into the S chart Fig.3 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For the same dataset, Exponentially Weighted Moving Average control chart is shown in Fig.4.

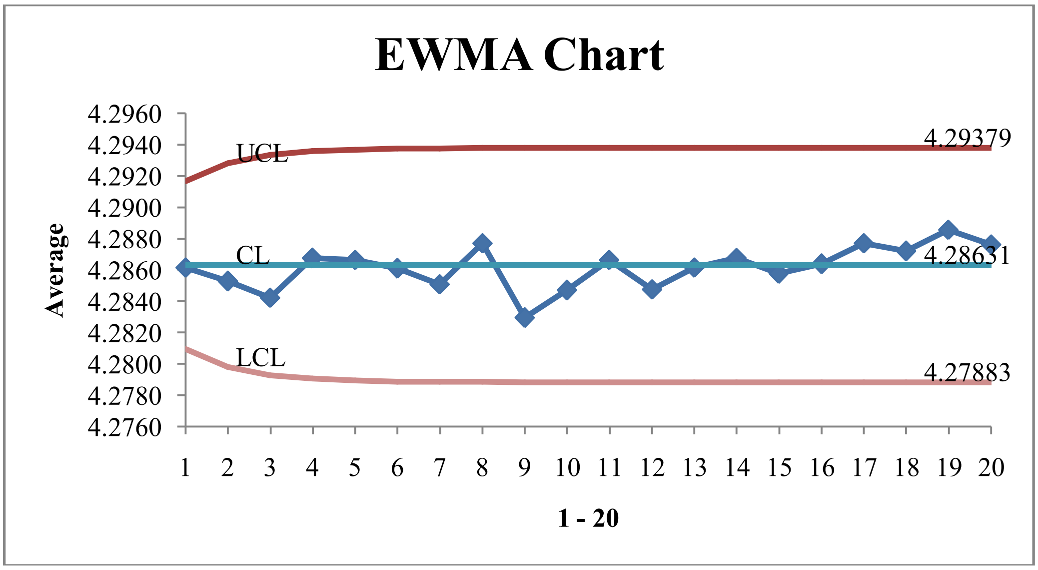

Fig.4 EWMA chart for height of the bolt

Here,

UCL= 4.29379,

LCL = 4.27883,

Center line = 4.28631;

In the EWMA control chart of X, it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

For the same data set, Exponentially Weighted Moving Average control chart for S is shown in Fig.5

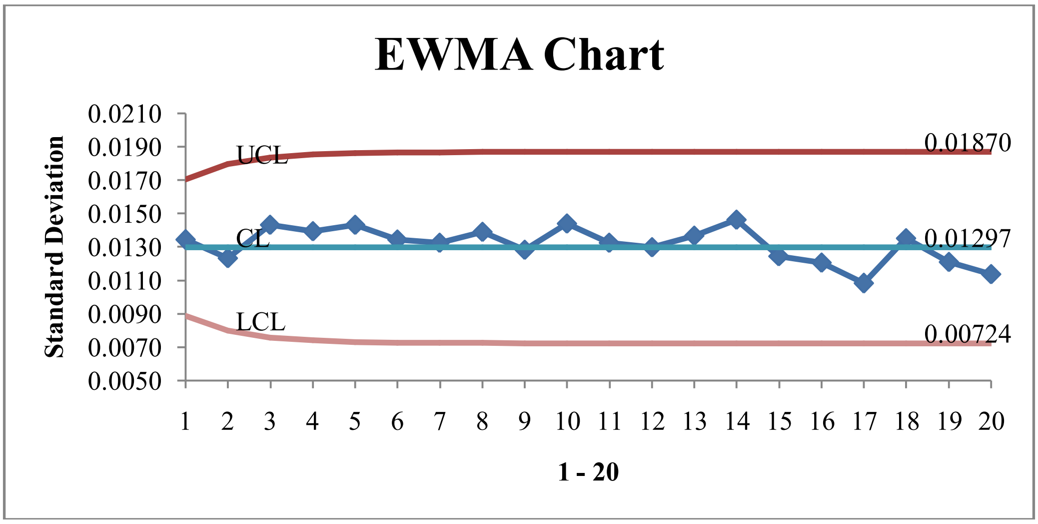

Fig.5 EWMA chart for Standard Deviation

Here,

UCL= 0.01870,

LCL = 0.00724,

Center line = 0.01297;

In this EWMA control chart for S it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

For the same data set, Exponentially Weighted Moving Average control chart for Range is shown in Fig.6

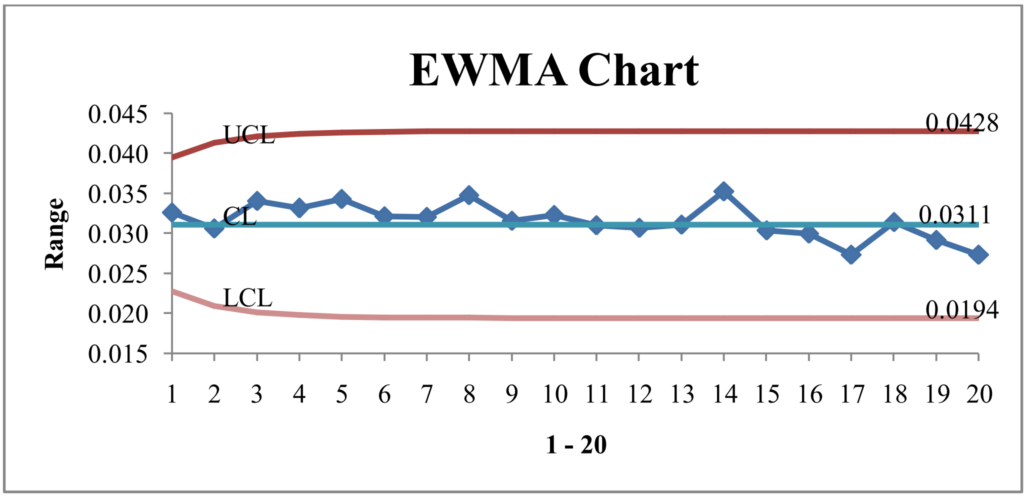

Fig.6 EWMA chart for Range

Here,

UCL= 0.0428,

LCL = 0.0194,

Center line = 0.0311;

In the EWMA control chart for Range it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

B. X bar, Range, S, EWMA control charts for diameter of the bolt

Table -2 Data set for diameter of the specific type of Bolt

Sample No	X_1(Diameter)	X_2	X_3	X_4	X_5	$\overline{X}$	S	R
01	7.521	7.531	7.508	7.546	7.513	7.524	0.015	0.038
02	7.592	7.503	7.521	7.581	7.529	7.545	0.039	0.089
03	7.501	7.561	7.520	7.561	7.545	7.538	0.026	0.060
04	7.543	7.556	7.569	7.508	7.541	7.543	0.023	0.061
05	7.575	7.541	7.577	7.551	7.530	7.555	0.021	0.047
06	7.583	7.543	7.526	7.546	7.536	7.547	0.022	0.057
07	7.521	7.554	7.513	7.533	7.567	7.538	0.023	0.054
08	7.503	7.509	7.505	7.526	7.581	7.525	0.033	0.078
09	7.512	7.561	7.506	7.514	7.521	7.523	0.022	0.055
10	7.569	7.567	7.519	7.559	7.525	7.548	0.024	0.050
11	7.550	7.541	7.526	7.561	7.519	7.539	0.017	0.042
12	7.521	7.509	7.567	7.545	7.504	7.529	0.026	0.063
13	7.549	7.513	7.589	7.516	7.567	7.547	0.033	0.076
14	7.589	7.589	7.582	7.521	7.581	7.572	0.029	0.068
15	7.521	7.523	7.503	7.580	7.524	7.530	0.029	0.077
16	7.531	7.540	7.509	7.560	7.532	7.534	0.018	0.051
17	7.502	7.554	7.511	7.513	7.545	7.525	0.023	0.052
18	7.530	7.534	7.536	7.519	7.551	7.534	0.012	0.032
19	7.578	7.516	7.546	7.526	7.549	7.543	0.024	0.062
20	7.545	7.505	7.560	7.543	7.561	7.524	0.015	0.042
						7.5391	0.0240	0.0584

For the data set the X bar control chart is shown in Fig.7

For X bar chart, the LCL= 7.5048 , UCL= 7.5734 and CL=7.5391;

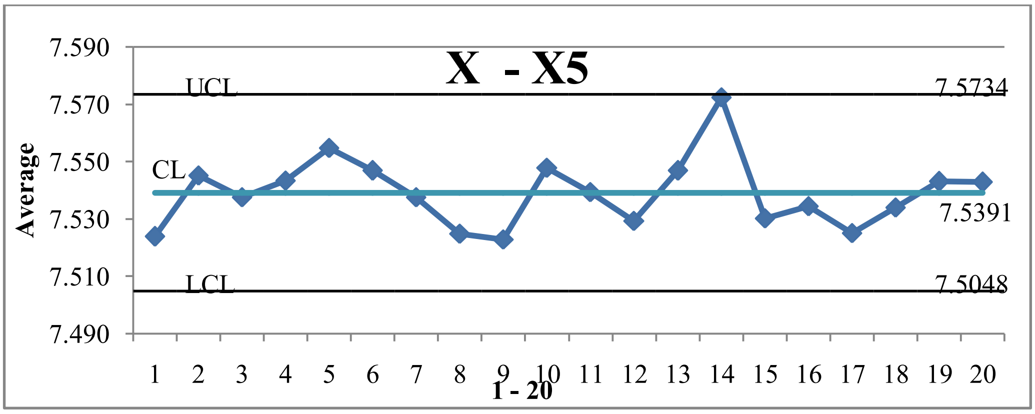

Fig.7 Sample mean for diameter of the bolt

Any point falling outside the control limits indicates that assignable causes had affected the process and the process is out of control. Looking into the X bar chart it is observed that all the points are falling with-in the control limits. It means that the assignable cause does not affect the process.

For Standard Deviation chart, the LCL= 0, UCL= 0.0502 and CL=0.0240;

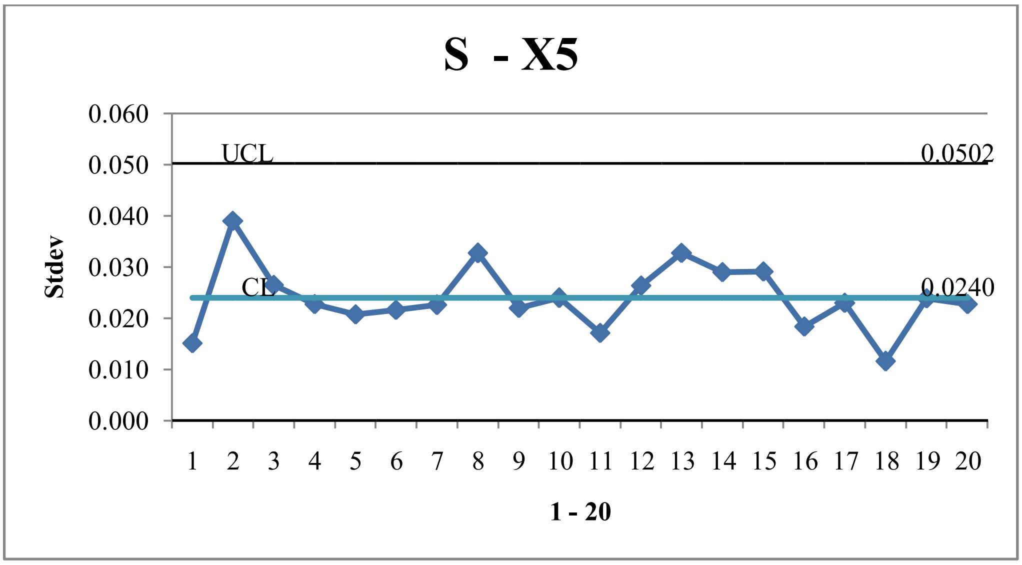

Fig.8 Standard Deviation chart for diameter of the bolt

Looking into the S chart Fig.8 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For Range chart, the LCL= 0, UCL= 0.1235 and CL=0.0584;

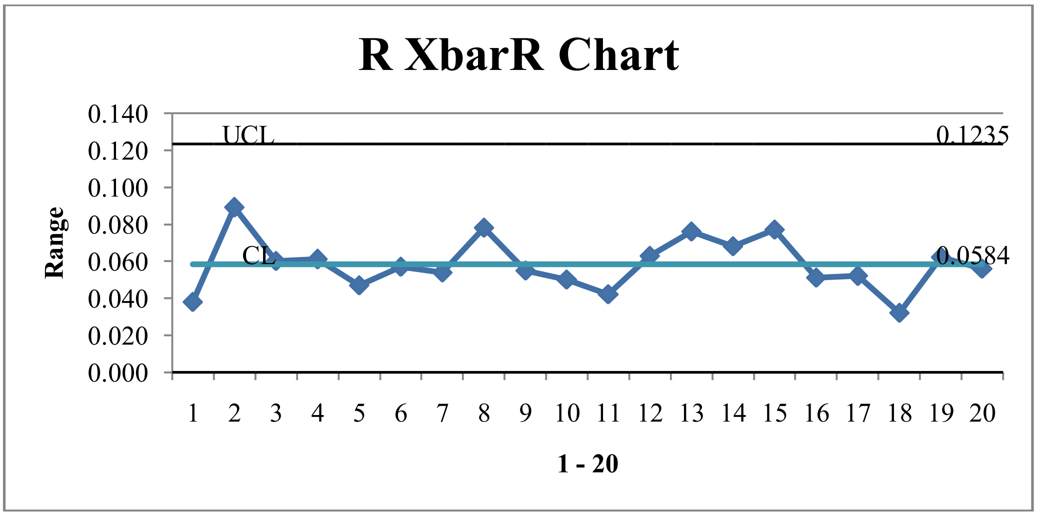

Fig.9 Range chart for diameter of the bolt

Looking into the R chart Fig.9 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For the same data set, Exponentially Weighted Moving Average control chart is shown in Fig.10

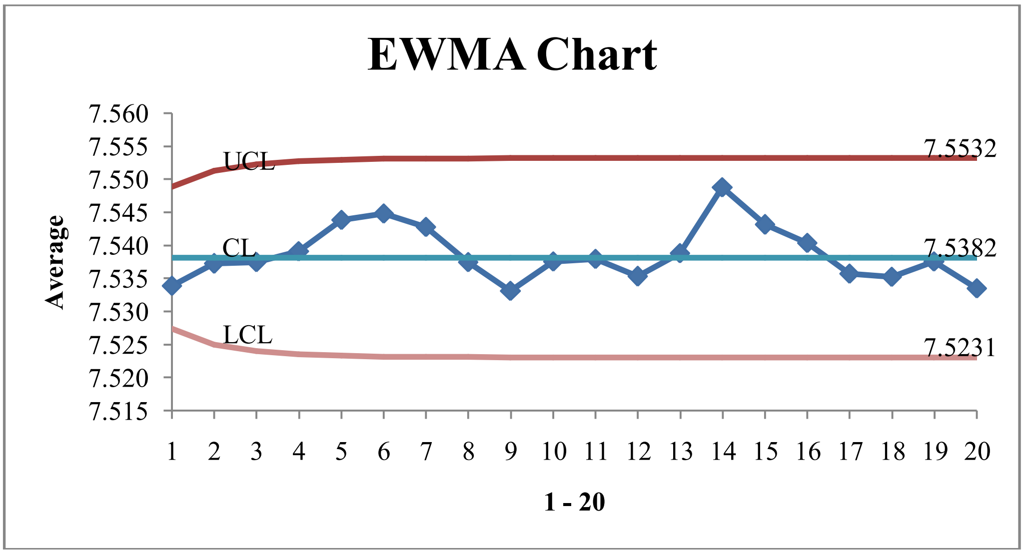

Fig.10 EWMA chart for Average

Here,

UCL= 7.5532,

LCL = 7.5231,

Center line = 7.5382;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

For the same data set, Exponentially Weighted Moving Average control chart is shown in Fig.11

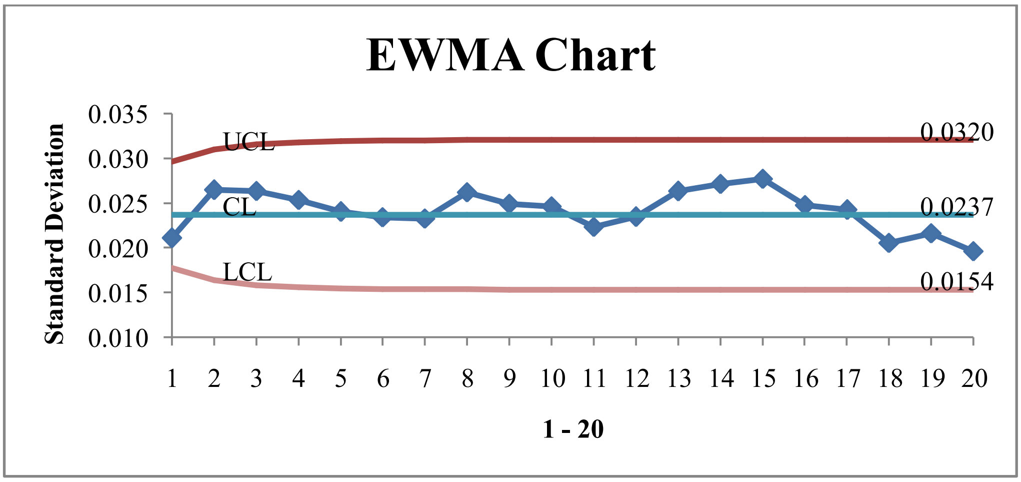

Fig.11 EWMA chart for Standard Deviation

Here,

UCL= 0.0320,

LCL = 0.0154,

Center line = 0.0237;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

For the same data set Exponentially Weighted Moving Average control chart is shown in Fig.12

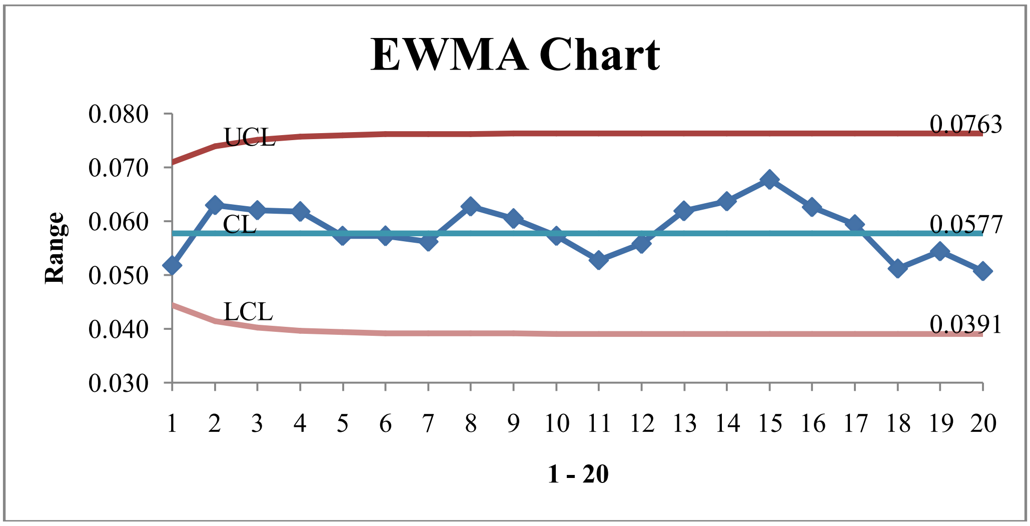

Fig.12 EWMA chart for Range

Here,

UCL= 0.0763,

LCL = 0.0391,

Center line = 0.0577;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

C. X bar, Range, S, EWMA control charts for weight of the bolt

Table -3 Data set for weight of the specific type of Bolt

Sample No.	X1(weight)mg	X2	X3	X4	X5	X	S	R
1	15.52	15.55	15.54	15.56	15.52	15.54	0.02	0.04
2	15.51	15.54	15.51	15.55	15.55	15.53	0.02	0.04
3	15.53	15.56	15.56	15.52	15.56	15.55	0.02	0.04
4	15.52	15.54	15.53	15.53	15.53	15.53	0.01	0.02
5	15.51	15.55	15.55	15.53	15.53	15.53	0.02	0.04
6	15.53	15.53	15.51	15.54	15.52	15.53	0.01	0.03
7	15.53	15.56	15.53	15.56	15.56	15.55	0.02	0.03
8	15.53	15.57	15.54	15.55	15.54	15.55	0.02	0.04
9	15.52	15.55	15.54	15.55	15.53	15.54	0.01	0.03
10	15.51	15.54	15.52	15.52	15.54	15.53	0.01	0.03
11	15.54	15.54	15.56	15.57	15.56	15.55	0.01	0.03
12	15.51	15.55	15.54	15.56	15.51	15.53	0.02	0.05
13	15.55	15.54	15.54	15.54	15.55	15.54	0.01	0.01
14	15.54	15.56	15.54	15.55	15.56	15.55	0.01	0.02
15	15.54	15.53	15.51	15.53	15.54	15.53	0.01	0.03
16	15.53	15.56	15.52	15.56	15.52	15.54	0.02	0.04
17	15.52	15.55	15.56	15.54	15.54	15.54	0.01	0.04
18	15.53	15.55	15.53	15.52	15.56	15.54	0.02	0.04
19	15.53	15.57	15.53	15.51	15.56	15.54	0.02	0.06
20	15.54	15.53	15.54	15.56	15.57	15.55	0.02	0.04
					15.539		0.015	0.035

For the data set the X bar control chart is shown in Fig.13

For X bar chart, the LCL=15.519 , UCL= 15.559, and CL=15.539;

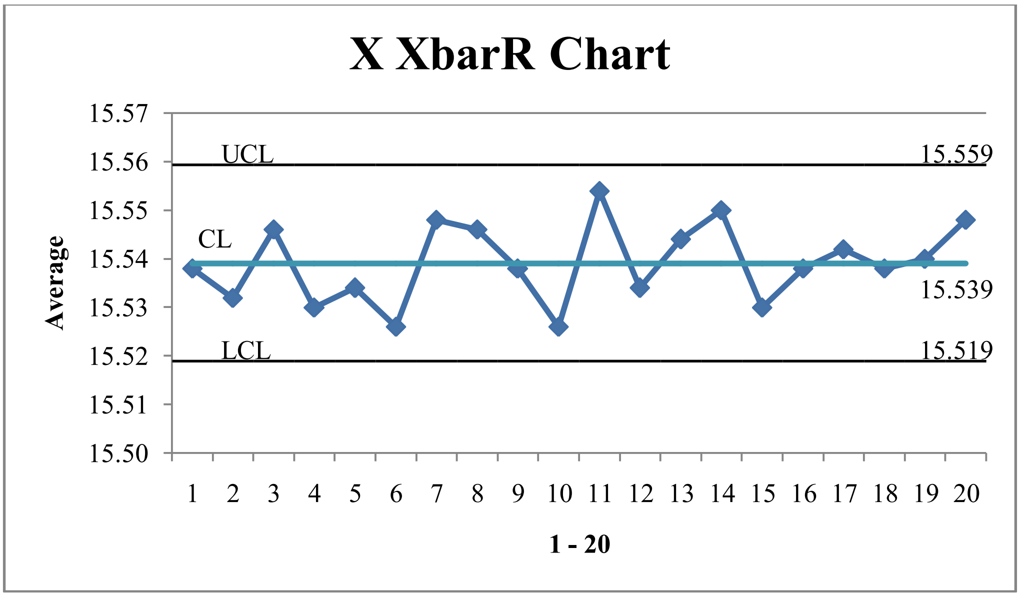

Fig.13 Sample mean chart for weight of the bolt

Looking into the X bar chart it is observed that all the points are falling with-in the control limits. It means that the assignable cause does not affect the process.

For Range chart the LCL= 0, UCL= 0.1235 and CL=0.0584;

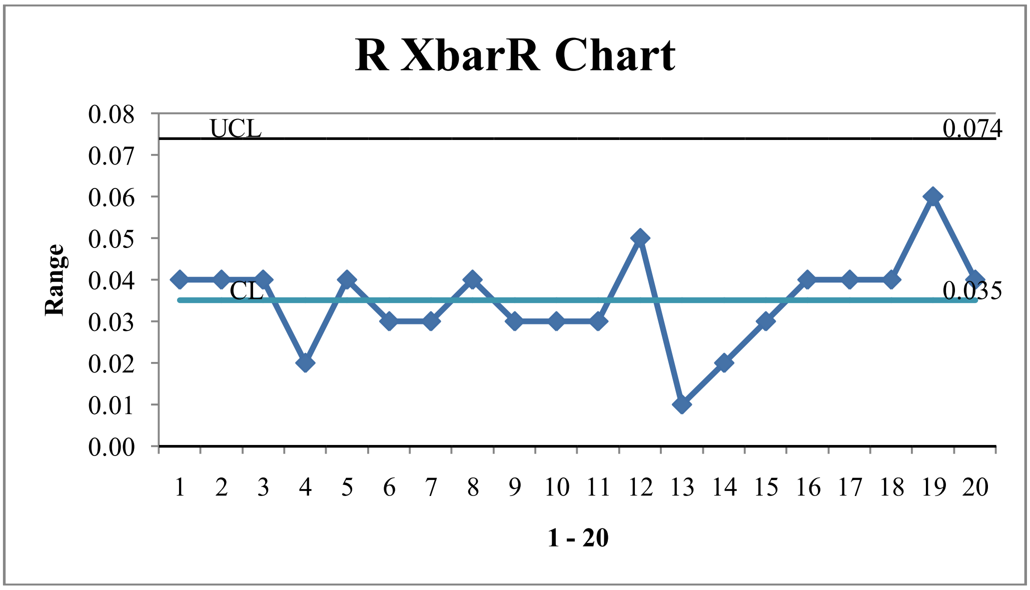

Fig.14 Range chart for weight of the bolt

Looking into the R chart Fig.14 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For Range chart the LCL= 0, UCL= 0.1235 and CL=0.0584;

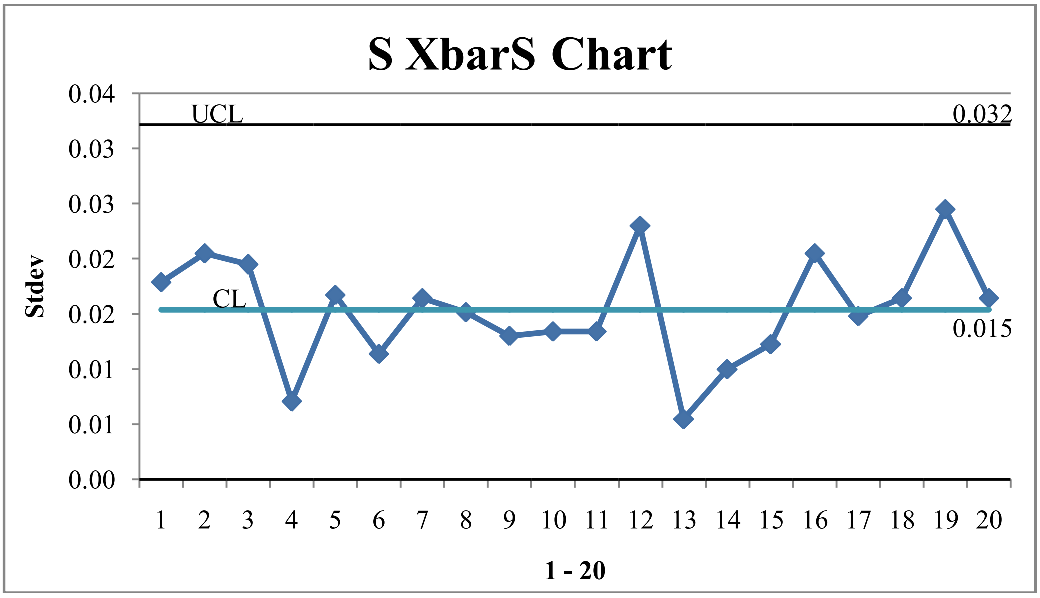

Fig.15 Standard Deviation chart for weight of the bolt

Looking into the S chart Fig.15 it is observed that all the points are falling within the control limits. It means that the assignable cause does not affect the process.

For the same data set, Exponentially Weighted Moving Average control chart is shown in Fig.16

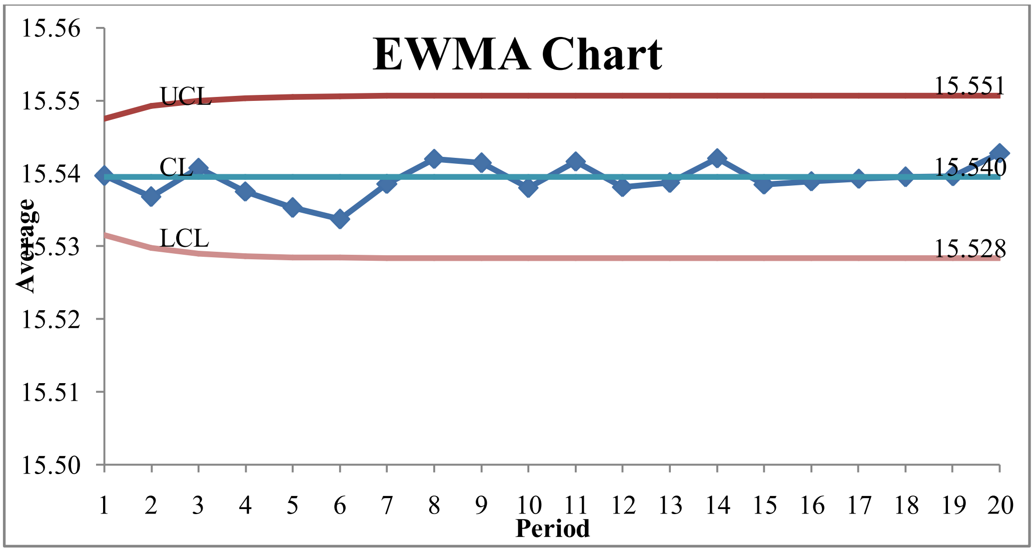

Fig.16 EWMA chart for Average

Here,

UCL= 15.551,

LCL = 15.528,

Center line = 15.540;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

For the same data set Exponentially Weighted Moving Average control chart is shown in Fig.17.

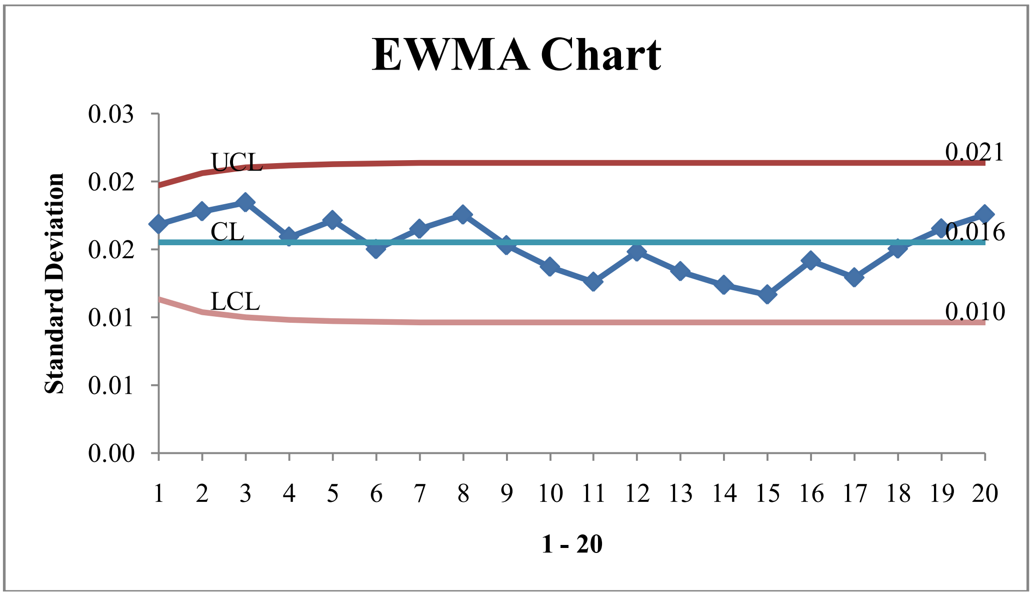

Fig.17 EWMA chart for Standard Deviation

Here,

UCL= 0.021,

LCL = 0.010,

Center line = 0.016;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.
For the same data set Exponentially Weighted Moving Average control chart is shown in Fig.18

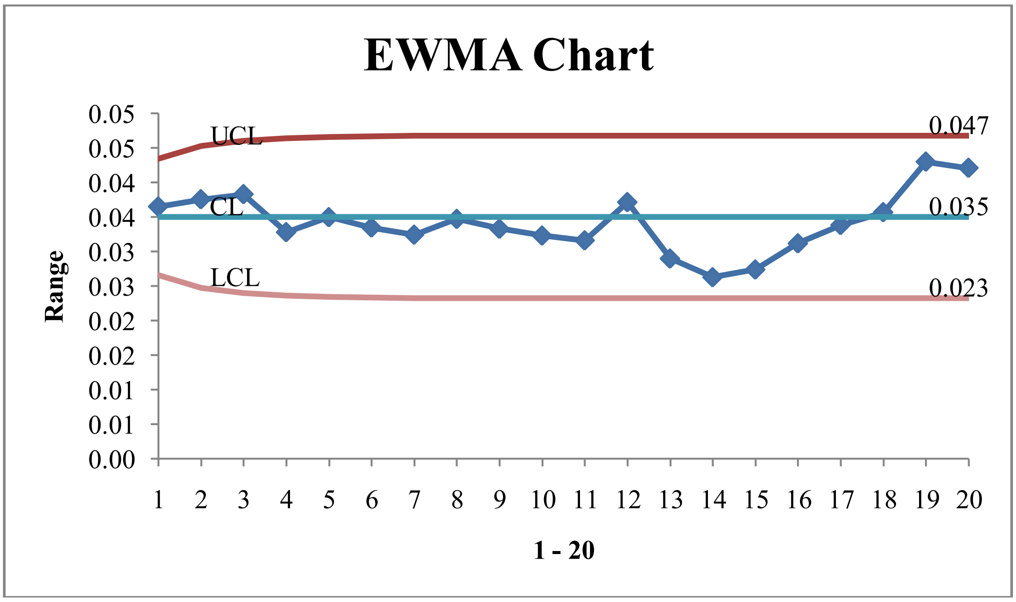

Fig.18 EWMA chart for Range

Here,

UCL= 0.047,

LCL = 0.023,

Center line = 0.035;

In the EWMA control chart of X it is observed that all the points falling inside the control limits and there is no sharp shift is observed so the process is in control.

D. Multivariate control chart (Hotelling's T^2)

Multivariate control charts monitor multiple process characteristics. Independent variables can be put individually. But, when we observe a relationship between two or more variables, we can use this chart to determine whether the process is in control. Multivariate control charts can detect shifts in the mean or the relationship between several related variables. But, one difficulty encountered with any multivariate control chart is practical interpretation of out-of-control signal that means if the process is out of control, it is difficult to find the assignable cause.

In this case study, we have used Hotelling's T^2 chart to represent the dual inspection result between two variables (height, diameter) of our samples.

From the following chart, we came to the conclusion that all the points are in controlled region except a point which almost touched the UCL. This irregular behaviour has occurred for some assignable causes indicating that the process is not fully in control.

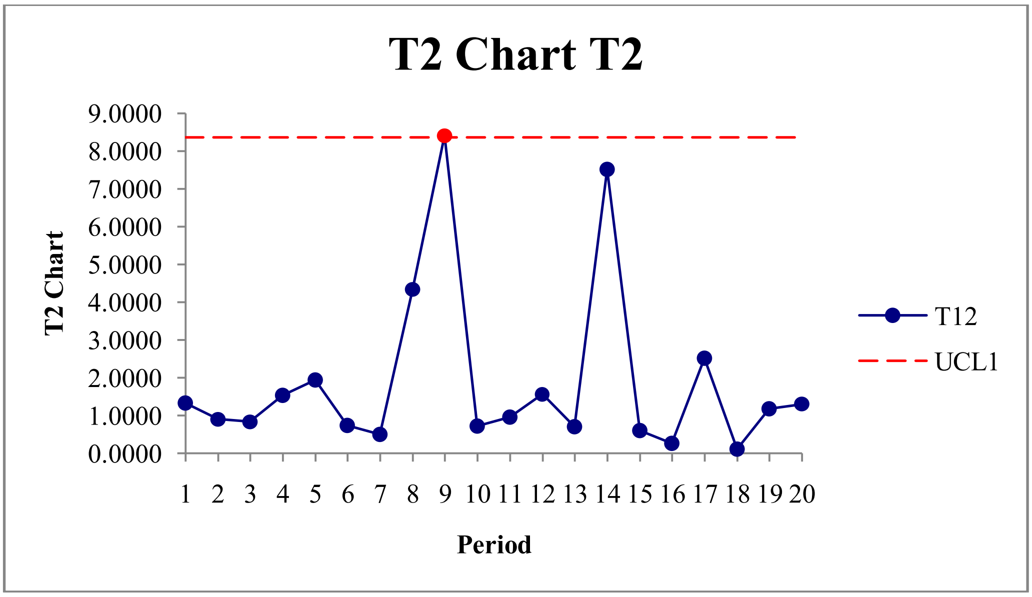

Fig.19 Hotelling's T^2 chart for height and diameter

IV. CONCLUSION

Control charts are among the most effective management control tools, and they are as important as cost control and material control. This verifies, graphically, variation in output quality characteristics against pre-fixed upper and lower limits. In our case study, we exactly did the same thing by experimenting with three variables; height, diameter and weight of our samples from bolt manufacturing industry and put them in different control charts to inspect the quality of the samples. We have come in the settlement that in all the figures, points are within the control limit representing the process is in control. All the figures have been plotted by popular PC based statistics package.

V. APPENDIX

Factors for Computing Central Lines and 3s Control Limits for X, s, and R Charts

	Chart for Averages			Chart for Ranges						Chart for Standard Deviations				
	Factors for Control Limits			Factor for Central Line	Factors for Control Limits					Factor for Central Line	Factors for Control Limits			
Observations in Sample, n	A	A_2	A_3	d_2	d_1	D_1	D_2	D_3	D_4	c_4	B_3	B_4	B_5	B_6
2	2.121	1.880	2.659	1.128	0.853	0	3.686	0	3.267	0.7979	0	3.267	0	2.606
3	1.732	1.023	1.954	1.693	0.888	0	4.358	0	2.574	0.8862	0	2.568	0	2.276
4	1.500	0.729	1.628	2.059	0.880	0	4.698	0	2.282	0.9213	0	2.266	0	2.088
5	1.342	0.577	1.427	2.326	0.864	0	4.918	0	2.114	0.9400	0	2.089	0	1.964
6	1.225	0.483	1.287	2.534	0.848	0	5.078	0	2.004	0.9515	0.030	1.970	0.029	1.874
7	1.134	0.419	1.182	2.704	0.833	0.204	5.204	0.076	1.924	0.9594	0.118	1.882	0.113	1.806
8	1.061	0.373	1.099	2.847	0.820	0.388	5.306	0.136	1.864	0.9650	0.185	1.815	0.179	1.751
9	1.000	0.337	1.032	2.970	0.808	0.547	5.393	0.184	1.816	0.9693	0.239	1.761	0.232	1.707
10	0.949	0.308	0.975	3.078	0.797	0.687	5.469	0.223	1.777	0.9727	0.284	1.716	0.276	1.669
11	0.905	0.285	0.927	3.173	0.787	0.811	5.535	0.256	1.744	0.9754	0.321	1.679	0.313	1.637
12	0.866	0.266	0.886	3.258	0.778	0.922	5.594	0.283	1.717	0.9776	0.354	1.646	0.346	1.610
13	0.832	0.249	0.850	3.336	0.770	1.025	5.647	0.307	1.693	0.9794	0.382	1.618	0.374	1.585
14	0.802	0.235	0.817	3.407	0.763	1.118	5.696	0.328	1.672	0.9810	0.406	1.594	0.399	1.563
15	0.775	0.223	0.789	3.472	0.756	1.203	5.741	0.347	1.653	0.9823	0.428	1.572	0.421	1.544
16	0.750	0.212	0.763	3.532	0.750	1.282	5.782	0.363	1.637	0.9835	0.448	1.552	0.440	1.526
17	0.728	0.203	0.739	3.588	0.744	1.356	5.820	0.378	1.622	0.9845	0.466	1.534	0.458	1.511
18	0.707	0.194	0.718	3.640	0.739	1.424	5.856	0.391	1.608	0.9854	0.482	1.518	0.475	1.496
19	0.688	0.187	0.698	3.689	0.734	1.487	5.891	0.403	1.597	0.9862	0.497	1.503	0.490	1.483
20	0.671	0.180	0.680	3.735	0.729	1.549	5.921	0.415	1.585	0.9869	0.510	1.490	0.504	1.470

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