MHD Flow of an Elastico-viscous fluid in porous medium in a slip flow regime

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Abstract— In this paper an electrically conducting fluid with elastico-viscous character over a porous plate is considered. The surrounding medium is packed with a homogenous porous matrix. The effects of various physical parameters such as magnetic parameter M, elastic parameter Rc, Porosity parameter Kp and rotational parameter E and Rarefaction parameter or slip flow parameter R have studied with help of graphs.

Keywords: MHD flow, elastico-viscous, porous plate, slip fow

1. Introduction

In the slip flow regime the study of flow problems has attracted the researchers due to its application in the high speed flight problems. The problem of high speed flight in the upper atmosphere besides the kinetic approach have been studied through an continuous approach along with consideration of the velocity slip at a rigid boundary . A rarefied gas can neither be considered as an absolutely continuous medium with Knudsen number Kn (which is the ratio of the molecular mean free path to some characteristic length) or satisfying $Kn < 10^{-3}$ nor a free molecular medium with $Kn > 10$.

Chaudhary and Chand [1] have studied hydromagnetic flow past a long vertical channel embedded in porous medium with transpiration cooling. Kurtcebe and Erim [2] have analyzed heat transfer of a visco-elastic fluid in a porous channel. Panda *et al.* [3] have studied free convection of conducting viscous fluid between two vertical walls filled with a porous material. Sharma and Yadav [4] have reported steady MHD boundary layer flow and heat transfer between two long vertical wavy walls. Dash [5] has studied the effects of radiation and chemical reaction in MHD flow past a stretched vertical permeable surface through a porous medium with constant suction. Sharma *et al.* [6] have analyaed the steady MHD flow and heat transfer between two rotating porous disk. Sharma and Sharma [7] have studied the effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium. Das and Panda [8] have reported the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow regime. Panda and Das [9] have analyzed the MHD free convection flow of a particulate suspension past an infinite porous inclined flat plate with heat absorption. Das *et al.* [10] have analyzed mass transfer effect of MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillating suction and heat source.

In the present paper we have considered an electrically conducting fluid exhibiting the character of elastico-viscous over a porous plate. The surrounding medium is packed with a homogenous porous matrix. Moreover presence of magnetic field has created enormous interest to study the effects of various physical parameters such as magnetic parameter M, elastic parameter Rc, Porosity parameter Kp and rotational parameter E and Rarefaction parameter or slip flow parameter R .

2. Mathematical Formulation

The steady flow of an incompressible elastico-viscous (Walters B' model] electrically conducting fluid with uniform velocity U_∞ is considered in this problem. The flow is along x-axis past on infinite porous flat plate. The plate lies in the plane z=0. The fluid and the plate are rotating in uniform with constant angular velocity Ω about z-axis, and the flow is permeated by a uniform transverse magnetic field B_0 along z-axis. The pressure gradient $\frac{\partial p}{\partial x}$, far away from the plate is balanced by the coriolis force $2\,\Omega\,U_\infty$. All the physical variables are function of z only since the present set up is homogeneous to the horizontal plane. If (u, v, w) be the components of the velocity field then the equations governing flow of elastico-viscous fluid of Walters B′ model

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a rotating frame of reference through porous medium are given by

$$\frac{dw}{dz} = 0 \qquad (1)$$

$$w\frac{du}{dz} = \nu\frac{d^2u}{dz^2} + 2\Omega v - \frac{\sigma B_0^2}{\rho}\left(u - U_\infty\right) + \frac{K_0}{\rho}w\frac{d^3u}{dz^3} - \frac{\nu}{K_P^8}\left(u - U_\infty\right) \qquad (2)$$

$$w\frac{dv}{dz} = \nu\frac{d^2v}{dz^2} - 2\Omega\left(u - U_\infty\right) - \frac{\sigma B_0^2 v}{\rho} + \frac{K_0}{\rho}w\frac{d^3v}{dz^3} - \frac{\nu v}{K_P^*} \qquad (3)$$

Where σ is the electrical conductivity of the fluid, B_0 is the uniform magnetic field along z-axis, K_0 is the elastic parameter of the fluid, K^*_p is the porosity parameter . it is assumed that the magnetic Reynolds number is very small so that induced magnetic field can be neglected.

Now integrating equation (1) we get w = constant = - w_0 every where in the flow, where $w_0 > 0$ stands for suction and $w_0 < 0$ for blowing at the plate.

Multiplying 'i' = $\left(\sqrt{-1}\right)$ in equation (3) and adding with equation (2) we obtain

$$-w_0\left(\frac{du}{dz} + i\frac{dv}{dz}\right) = \nu\left(\frac{d^2u}{dz^2} + i\frac{d^2v}{dz^2}\right) + 2\Omega\left(v - iu + iU_\infty\right) - \frac{\sigma B_0^2}{\rho}\left(u - U_\infty + iv\right) - \frac{K_0 w}{\rho}\left(\frac{d^3u}{dz^3} + i\frac{d^3v}{dz^3}\right) - \frac{\nu}{K_P^*}\left(u - U_\infty + iv\right)$$

The boundary conditions at the plate correspond to slip flow are given by

$$u = L_1\frac{d\mu}{dz},\ v = L_1\frac{dv}{dz} \quad at\ z = 0 \qquad (4)$$

$$u \to U_\infty,\ v \to 0 \qquad as\ z \to \infty \qquad (5)$$

Hence $L_1 = \left(\frac{2}{f} - 1\right)l$

where f and l are the fraction of molecules reflected diffusely from the plate and the molecular mean free path of the fluid respectively.

Introducing the following dimensionless quantities

$$\eta = \frac{U_\infty}{\nu}z,\ S = \frac{w_0}{U_\infty},\ E = \frac{2\Omega\nu}{U_\infty^2},\ M^2 = \frac{\sigma B_0^2 \nu}{\rho U_\infty^2},$$

$$F(\eta) = \frac{u}{U_\infty} - 1 + i\frac{v}{U_\infty}$$

$$Rc = \frac{K_0\ w_0\ U_\infty}{\rho\nu^2},\ Kp = \frac{K_p^\infty U_\infty^2}{\nu},$$

$$Q^2 = M^2 + \frac{1}{K_p} \qquad (6)$$

and combining (2) and (3) we get

$$\frac{d^2F}{d\eta^2} + S\frac{dF}{d\eta} - \left(Q^2 + iE\right)F(\eta) - Rc\frac{d^3F}{d\eta^3} = 0 \qquad (7)$$

In view of (6) the boundary conditions take the form

$$F = R\frac{dF}{d\eta} - 1 \quad at\ \eta = 0$$

$$F \to 0 \qquad as\ \eta \to \infty \qquad (8)$$

where $R = \frac{L_1}{\nu}U_\infty$ is Rarefaction parameter

3. Solution of Problem

A small perturbation technique is used, with Rc as the small perturbation parameter, where

$$F(\eta) = F_0(\eta) + RcF_1(\eta) + 0(Rc^2) \qquad (9)$$

Substituting (9) in (7) we get zeroth order equation

$$\frac{d^2F_0}{d\eta^2} + S\frac{dF_0}{d\eta} - \left(Q^2 + iE\right)F_0(\eta) = 0 \qquad (10)$$

and the first order equation

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$$\frac{d^2F_1}{d\eta^2}+S\frac{dF_1}{d\eta}-\left(Q^2+iE\right)F_1(\eta)-\frac{d^3F_0}{d\eta^3}=0 \quad (11)$$

The boundary condition (8) now reduce to

$$F_0(\eta) = R\frac{dF_0}{d\eta} - 1,\; F_1(\eta) = R\frac{dF_1}{d\eta}, \text{ at } \eta = 0$$

$$F_0 \to 0,\; F_1 \to 0 \text{ as } \eta \to \infty$$

(12)

The solution of (10) and (11) satisfying the boundary condition (12) are given by

$$F_0(\eta) = \frac{e^{-\alpha\eta}}{A_1}\left[(1+R\alpha)\cos\beta\eta - R\beta\sin\beta\eta\right]$$

$$+ i\frac{e^{-\alpha\eta}}{A_1}\left[R\beta\cos\beta\eta + (1+R\alpha)\sin\beta\eta\right] \quad (13)$$

$$F_1(\eta) = \frac{e^{-\alpha\eta}}{A_1}\left[(A_5-\eta A_6)\cos\beta\eta + (A_7+\eta A_8)\sin\beta\eta\right]$$

$$+ i\frac{e^{-\alpha\eta}}{A_1}\left[(A_7+\eta A_8)\cos\beta\eta - (A_5-\eta A_6)\sin\beta\eta\right] \quad (14)$$

Substituting $F_0(\eta)$ and $F_1(\eta)$ in (9), $F(\eta)$ is found out. Again utilizing (6) we get

$$u^* = \frac{u}{U_\infty} = 1 - \frac{e^{-\alpha\eta}}{A_1}\left[(1++R\alpha)\cos\beta\eta - R\beta\sin\beta\eta\right]$$

$$+ Rc\frac{e^{-\alpha\eta}}{A_1}\left[(A_5-\eta A_6)\cos\beta\eta + (A_7+\eta A_8)\sin\beta\eta\right] \quad (15)$$

$$v^* = \frac{v}{U_\infty} = \frac{e^{-\alpha\eta}}{A_1}\left[R\beta\cos\beta\eta + (1+R\alpha)\sin\beta\eta\right]$$

$$+ Rc\frac{e^{-\alpha\eta}}{A_1}\left[(A_7+\eta A_8)\cos\beta\eta - (A_5-\eta A_6)\sin\beta\eta\right] \quad (16)$$

where

$$\alpha = \frac{S}{2} + \frac{1}{2\sqrt{2}}\left[\left(S^2+4Q^2\right) + \left[\left(S^2+4Q^2\right)^2 + 16E^2\right]^{1/2}\right]^{1/2} \quad (17)$$

$$\beta = \frac{1}{2\sqrt{2}}\left[\left(S^2+4Q^2\right)^2 + 16E^2\right]^{1/2} - \left(S^2+4Q^2\right)^{1/2} \quad (18)$$

$$A_1 = (1+R\alpha)^2 + R^2\beta^2$$

$$A_2 = \alpha(1+R\alpha) + R\beta^2$$

$$A_4 = \left[2\alpha\beta A_2 - \beta\left(\beta^2-\alpha^2\right)\right] / A_1$$

$$A_5 = \frac{\{R\beta A_4 - (1+R\alpha)A_3\}\,(S-2\alpha) - 2\beta\{(1+R\alpha)A_4 + 2\beta A_3\}}{(S-2\alpha)^2 + 4\beta^2}$$

$$A_6 = \frac{\{2\alpha\beta^2 + (\beta^2-\alpha\;)A_2\}(S-2\alpha) + 2\beta\{2\alpha\beta A_2 - \beta(\beta^2-\alpha^2)\}}{(S-2\alpha)^2 + 4\beta^2}$$

$$A_7 = \frac{\{(1+R\alpha)A_4 + R\beta A_3\}(S-2\alpha) + 2\beta\{R\beta A_4 - (1+R\alpha)A_3\}}{(S-2\alpha)^2 + 4\beta^2}$$

$$A_8 = \frac{\{2\alpha\beta A_2 - \beta(\beta^2-\alpha^2)\}(S-2\alpha) - 2\beta\{2\alpha\beta^2 + (\beta^2-\alpha^2)A_2\}}{(S-2\alpha)^2 + 4\beta^2}$$

In case of blowing $S < 0$, taking $S = -S_1$, where, $S_1 > 0$, we can find

$$\alpha_1 = -\frac{S_1}{2} + \frac{1}{2\sqrt{2}}\left\{\left(S_1^2+4Q^2\right) + \left[\left(S_1^2+4Q^2\right)^2 + 16E^2\right]^{1/2}\right\}^{1/2} \quad (19$$

$$\beta_1 = \frac{1}{2\sqrt{2}}\left\{\left[\left(S_1^2+4Q^2\right)^2 + 16E^2\right]^{1/2} - \left(S_1^2+4M^2\right)\right\}^{1/2} \quad (20)$$

and the velocity distribution is from (15) and (16) the dimensionless shear stress components at this plate.

$$\tau_1 = \left.\frac{du^*}{d\eta}\right|_{\eta=0} = \frac{\alpha + R(\alpha^2+\beta^2) + Rc(\beta A_7 - A_5 - A_6)}{(1+R\alpha)^2 + R^2\beta^2}$$

$$\tau_2 = \left.\frac{dv^*}{d\eta}\right|_{\eta=0} = \frac{\beta + Rc\left[A_8 - \beta A_5 - \alpha A_7\right]}{(1+R\alpha)^2 + R^2\beta^2}$$

5. Results and Discussion

In this paper hydromagnetic flow and heat transfer of elastico-viscous fluid of Walters B′ model over a porous plate bounded by a porous medium in the slip flow regime has been studied. The effects of various parameters like rarefaction parameter, elastic parameter, temperature jump parameter, rotation parameter, magnetic parameter and porosity parameter have been discussed. The equation (15) and (16) represent the velocity components u* and v* on the flow field respectively.

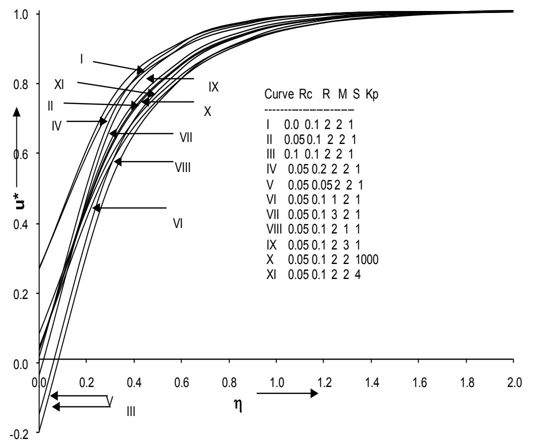

Fig. 1 Primary velocity profiles for E=2.0

We observe from Fig. 1 that an increase in permeability parameter Kp, magnetic parameter M and elastic parameter Rc lead to decrease the primary velocity at all points. On the other hand the opposite effect is observed in case of rarefaction parameter R and suction parameter S. On careful observation to the curves (I, II, III), it is concluded that the elasticity of the fluid decelerates the flow at all points and further increase in Rc is indicated by curve (III) where Rc=0.1 for which the primary velocity assumes the minimum value.

Presence of elastic element causes thinning of the boundary layer thickness. Moreover curve X represent the case of absence of porous medium (i.e. Kp=1000). Due to coincidence of curve X (Kp=1000) and curve XI (i.e. Kp=4.0), it is concluded that the porosity of the medium also decelerates the flow but not significantly as the case of elasticity .For strong suction and highly elastic fluid under lower rarefaction the velocity assumes negative value at the plate. The positive velocity may be attributed to centrifugal force and it is duly compensated giving rise to the negative values on the plate.

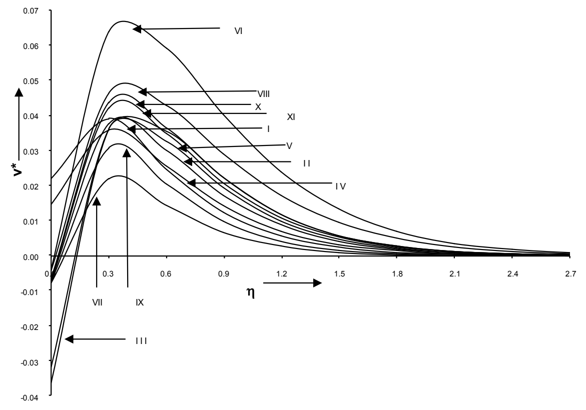

Fig-2 Secondary Velocity Profile For E-2.0

Curve	Rc	R	M	S	Kp
I	0.0	0.1	2	2	1
II	0.05	0.1	2	2	1
III	0.1	0.1	2	2	1
IV	0.05	0.2	2	2	1
V	0.05	0.05	2	2	1
VI	0.05	0.1	1	2	1
VII	0.05	0.1	3	2	1
VIII	0.05	0.1	2	1	1
IX	0.05	0.1	2	3	1

Fig.2 exhibits the secondary velocity distribution. It is seen that the reverse effect due to all parameters except magnetic parameter are observed on the secondary component velocity. The effect of magnetic parameter remains same as that of primary one. Further it is revealed that the profiles attain maximum value at $\eta = 0.3$ (approx.). It is also remarked from table (1)-(5) that the shear stress components at the plate τ_1 for primary and τ_2 for secondary increase as the elastic parameter increases, but reverse effect is observed in case of rarefaction parameter. In the absence of porous medium τ_1 and τ_2 decrease as elastic parameter rarefaction parameter increase. Thus it may be concluded that due to presence of porous medium the effect of elasticity is reversed but the effect of rarefaction parameter remains unchanged.

6. CONCLUSION

1. The magnetic parameter affects primary and secondary velocity alike but other parameters affect adversely.
2. Presence of elastic element causes thinning of the boundary layer
3. The presence of elasticity slows down the process to attain the ambient stream temperature.

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