Finding Optimal Path in a Network Problem Using Intuitionistic Fuzzy Arc Length

P.Jayagowri and G.Geetharamani

***Abstract*— A.Nagoor Gani and M.M.Mohammed Jabarulla Jabarulla(On searching Intuitionistic fuzzy shortest path in a network, Applied mathematical sciences, vol. 4, 2010, no.69, 3447-3454) proposed a new algorithm for solving intuitionistic fuzzy shortest path problem on a network an edge with shortest length is computed and using Euclidian distance to find shortest path. In this paper proposed new algorithm to find shortest path in a network problem which an intuitionistic fuzzy number , is assigned to each arc length. To show the advantages of the proposed algorithm over existing algorithm the numerical examples presented in (A.Nagoor Gani and M.Mohammed Jabarulla, 2010) are solved using the proposed algorithm and obtained results are discussed.**

***Index Terms*— Intuitionistic fuzzy shortest path problem, Intuitionistic Triangular fuzzy number, Ranking function for intuitionistic fuzzy number.**

I. INTRODUCTION

The main objective of the shortest path problem is to find a path with minimum distance. The shortest path problem concentrates on finding the path with minimum distance. The shortest path problem is the basic network problem. The fuzzy shortest path with problem was first analyzed by Dubois and Prade [1] using fuzzy number instead of a real number is assigned to each edges. Okada[8] concentrated on a shortest path problem and introduced the concept of degree of possibility in which an arc is on the shortest path. Takahasi and Yamakami[6] discussed the shortest path problem from a specified node to every other node on a network .Chuang and kung [5] pointed out that there are several methods to solve this kind of problem in the open literature. Amit kumar and Manjot kaur [10] introduced a existing algorithm about shortest path problem with fuzzy arc length.

This paper is organized as follows. In section 2 provides preliminary concept required for analysis. In section 3, required Notations are explained. In section 4 an algorithm is proposed for solving network flow problems. In section 5 illustrative Examples presented in A.Nagoor gani and .M.Mohammed Jabarulla are solved by using the proposed algorithm. In section 6 ,shortcoming of existing algorithm are discussed. In section **7,** shows the advantages of the proposed algorithm over existing algorithm the numerical examples presented in A.Nagoor Gani and M.Mohammed Jabarulla, 2010. In section 8 the obtained results are discussed. Finally, in section 9 some conclusions are drawn.

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II. PRELIMINARIES

In this section , some basic definitions relating to Fuzzy sets and Intuitionistic Fuzzy sets are given

A. Intuitionistic Fuzzy Set

Let X be an Universe of discourse, then an Intuitionistic fuzzy set(IFS) A in X is given by $A = \{(x, \mu_A), \gamma_A(x)/x \in X\}$.Where the function $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ determine the degree of membership and non membership of the element $x \in X$, respectively and for every $x \in X, 0 \le \mu_A(x) + \gamma_A(x) \le 1$.

B. Intuitionistic Fuzzy Number

Let $A = \{(x, \mu_A), \gamma_A(x)/x \in X\}$ be an IFS, then we call the paper ($\mu_A(x), \gamma_A(x)$) an intuitionistic fuzzy number. For convenience, we denote an intuitionistic fuzzy number by A=(<a,b,c><l,m,n>)where $\langle a, b, c\rangle \in F(I) < l, m, n >\in F(I)$. I=[0,1] $0 \le c + n \le 1$.

C. Triangular Intuitionistic FuzzyNumber

A Triangular Intuitionistic fuzzy number A is denoted by $\{(\mu_A, \gamma_A)/x \in R\}$ where μ_A and γ_A are triangular fuzzy number with $\gamma_A \le \mu_A^C$ .So a triangular intuitionistic fuzzy number A is given by $\widetilde{A} = \left(\langle a_1, a_2, a_3\rangle \langle a_1', a_2, a_3'\rangle\right)$

that is either $a_1' \ge a_2$ and $a_3' \ge a_3$ (or) $a_2 \le a_1$ and $a_3' \le a_2$ are membership and non membership fuzzy numbers of A.

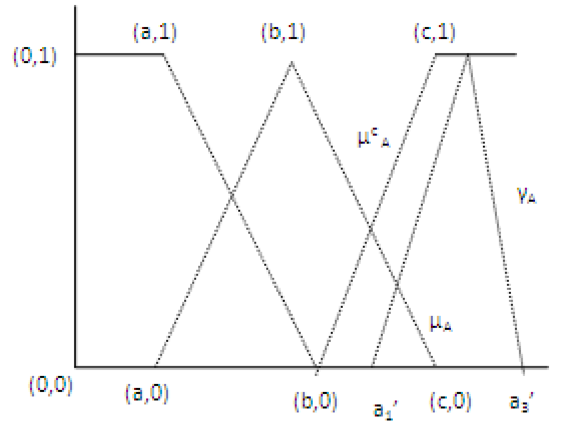

Figure 1 Triangular Intuitionstic fuzzy number

D. Operation on Triangular intuitionistc fuzzy number

$\tilde{A} = \left(\langle a_1, a_2, a_3\rangle \langle a_1', a_2, a_3'\rangle\right)$ and $\tilde{B} = \left(\langle b_1, b_2, b_3\rangle \langle b_1', b_2, b_3'\rangle\right)$ be two intuitionistic triangular fuzzy number then the arithmetic operation are as follows:

$$Addition: \tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3,)\ (a_1' + b_1', a_2 + b_2, a_3' + b_3')$$

$$Subtraction: \tilde{A} \Theta \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1,)\ (a_1' - b_3', a_2 - b_2, a_3' - b_1')$$

-

E. Ranking of Triangular intuitionistic fuzzy number

$\tilde{A} = \left(\langle a_1, a_2, a_3\rangle \langle a_1', a_2, a_3'\rangle\right)$ be a triangular intuitionstic fuzzy number, then we define a score function for membership and non-membership values respectively as

$$S(\tilde{A}_\mu) = \frac{a_1 + 2a_2 + a_3}{4} \quad \& \ S(\tilde{A}_\gamma) = \frac{a_1' + 2a_2 + a_3'}{4}$$

Let $\tilde{A} = \left(\langle a_1, a_2, a_3\rangle \langle a_1', a_2, a_3'\rangle\right)$ and $\tilde{B} = \left(\langle b_1, b_2, b_3\rangle \langle b_1', b_2, b_3'\rangle\right)$ be two intuitionistic triangular fuzzy number and $S(\tilde{A}_\mu)$, $S(\tilde{A}_\gamma)$ &$S(\tilde{B}_\mu)$, $S(\tilde{B}_\gamma)$ be the score of $\tilde{A}$ & $\tilde{B}$ respectively then

i) if $S(\tilde{A}_\mu) \le S(\tilde{B}_\mu)$ & $S(\tilde{A}_\gamma) \le S(\tilde{B}_\gamma)$ then $\tilde{A} < \tilde{B}$

ii)if $S(\tilde{A}_\mu) \ge S(\tilde{B}_\mu)$ & $S(\tilde{A}_\gamma) \ge S(\tilde{B}_\gamma)$ then $\tilde{A} > \tilde{B}$

iii_if $S(\tilde{A}_\mu) = S(\tilde{B}_\mu)$ & $S(\tilde{A}_\gamma) = S(\tilde{B}_\gamma)$ then $\tilde{A} = \tilde{B}$

III. NOTATIONS

The notations that will be used through out the paper are as follows.

$N = \{1, 2, \ldots. n\}$ The set of all nodes in a network.

Nm(j): The set of all predecessor nodes of node j.

$\overline{m_i}$: The intuitionistic fuzzy membership function distance between node i and source node

$\overline{m_{ij}}$: The intuitionistic fuzzy distance between node i and j.

$\overline{n_i}$: The intuitionistic fuzzy non membership function distance between node i and source node.

$\overline{n_{ij}}$: The intuitionistic fuzzy non membership function distance between node i and j.

p_i : The path in the i th node.

Remark:

A node i is said to be predecessor node of node j if (i) Node i is directly connected to node j

(ii) The direction of path connecting node i and j is from i to j.

IV. PROPOSED ALGORITHM

Step 1: Assume $\overline{m_1} = (0,0,0)$ and $\overline{n_1} = (0,0,0)$ are the label source node.

Step 2: Find $\overline{m_j} = \min(\overline{m_i} \oplus \overline{m_{ij}})$ / $i \in N(m(j)), j \ne 1$, j =2,3,4….n

$\overline{n_j} = \min(\overline{n_i} \oplus \overline{n_{ij}}) / i \in N(n(j))\ j \ne 1$, j = 2,3,4….n

Then find $p_i = i \to j$ where i and j are the values of the minimum value which is in m_j.

Step 3: Repeat step 2 until p_i is calculated for all the nodes starting from source node to destination node, but fuzzy distance along all paths is $\tilde{d}_j$.

Step 4: Discard the path which don't have either direct or indirect link with the source node and destination node.

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Step 5: Now the fuzzy optimized path can be obtained by combining the remaining paths starting from source node to destination node

Step 6 : The Intuitionistic fuzzy shortest distance between source node to destination node is calculated by sum of the value of the Intuitionistic fuzzy number which are involved in our Intuitionistic fuzzy optimized path in step 5.

V. IILUSTRATIVE EXAMPLE

Example1: Consider a small network shown in figure, where each arc length is represented as a triangular Intuitionistic fuzzy number.

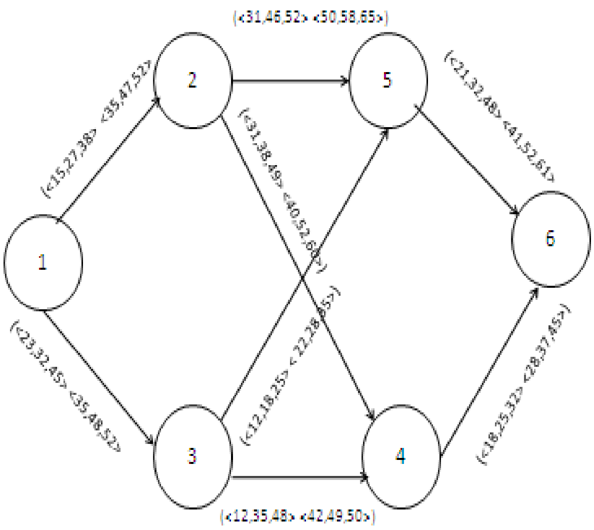

Fig 2

Solution:

Since node 6 is the destination node, so n=6.

Step 1:

Assume $\overline{m_1} = (0,0,0)$ and $\overline{n_1} = (0,0,0)$

Step 2&3 :

(i) put i =1 , j=2

$$\overline{m_j} = \min\{m_i \oplus m_{ij}\}$$

$$\overline{m_2} = \min\{m_1 \oplus m_{12}\} = \min\{(0,0,0) \oplus (15,27,38)\}$$

$\overline{m_2} = (15,27,38)$ and $\overline{n_j} = \min\{n_i \oplus n_{ij}\}$

$$\overline{n_2} = \min\{n_1 \oplus n_{12}\} = \min\{(0,0,0) \oplus (35,47,52)\}$$

$$\overline{n_2} = (35,47,52)$$

$$\therefore \quad p_1 = 1 \to 2$$

(ii) Put i = 1,2 ,j= 3

$$\overline{m_3} = \min\{m_1 \oplus m_{13}, m_2 \oplus m_{23}\}$$

$\overline{m_3} = \min\{(0,0,0) \oplus (23,32,45)\}$ (since there is no connection between node 2 and node 3. Therefore we neglect Second term)

$\overline{m_3} = (23,32,45)$ and

$$\overline{n_3} = \min\{n_1 \oplus n_{13}, n_2 \oplus n_{23}\}$$

$$\overline{n_3} = \min\{(0,0,0) \oplus (35,48,52)\}$$

$$\overline{n_3} = (35,48,52) \quad \therefore \quad p_1 = 1 \to 3$$

(iii)Put i=1,2,3 , j =4

$$\overline{m_4} = \min\{m_1 \oplus m_{14}, m_2 \oplus m_{24}, m_3 \oplus m_{34}\}$$

$$\overline{m_4} = \min\{(15,27,38) \oplus (31,38,49), (23,32,45) \oplus (12,35,48)\}$$

$\overline{m_4} = \min\{(46,65,87), (35,67,93)\}$ $\overline{m_4} = (35,67,93)$ and

$$\overline{n_4} = \min\{n_1 \oplus n_{14}, n_2 \oplus n_{24}, n_3 \oplus n_{34}\}$$

$$\overline{n_4} = \min\{(35,47,52) \oplus (40,52,60), (35,48,52) \oplus (42,49,50)\}$$

$$\overline{n_4} = \min\{(75,99,112), (77,97,102,)\}$$

$$\overline{n_4} = (77,97,102) \quad \therefore p_3 = 3 \to 4$$

(iv) i =1,2,3,4 , j=5

$$\overline{m_5} = \min\{m_1 \oplus m_{15}, m_2 \oplus m_{25}, m_3 \oplus m_{35}, m_4 \oplus m_{45}\}$$

$$\overline{m_5} = \min\{(15,27,38) \oplus (31,46,52), (23,32,45) \oplus (12,18,25)\}$$

$$\overline{m_5} = \min\{(46,73,90), (35,50,70)\}$$

$\overline{m_5} = (35,50,70)$ and

$$\overline{n_5} = \min\{n_1 \oplus n_{15}, n_2 \oplus n_{25}, n_3 \oplus n_{35}, n_4 \oplus n_{45}\}$$

$$\overline{n_5} = \min\{(35,47,52) \oplus (50,58,65), (35,48,52) \oplus (22,28,35)\}$$

$$\overline{n_5} = \min\{(85,105,117), (57,76,87)\}$$

$$\overline{n_5} = (57,76,87) \therefore p_3 = 3 \to 5$$

(v) i=1,2,3,4,5 ,j =6

$$\overline{m_6} = \min\left\{\begin{matrix} m_1 \oplus m_{16}, m_2 \oplus m_{26}, m_3 \oplus m_{36}, \\ m_4 \oplus m_{46}, m_5 \oplus m_{56} \end{matrix}\right\}$$

$$\overline{m_6} = \min\{(35,67,93) \oplus (18,25,32), (35,50,70) \oplus (21,32,48)\}$$

$$\overline{m_6} = \min\{(53,92,125), (56,82,118)\}$$

$\overline{m_6} = (56,82,118)$ and

$\overline{n_6} = \min\{n_1 \oplus n_{16}, n_2 \oplus n_{26}, n_3 \oplus n_{36}, n_4 \oplus n_{46}, n_5 \oplus n_{56}\}$

$\overline{n_6} = \min\{(77,97,102) \oplus (28,37,45), (57,76,87) \oplus (41,52,61)\}$

$\overline{n_6} = \min\{(105,134,14\ 7), (98,128,148)\}$

$\overline{n_6} = (98,128,148) \quad \therefore p_5 = 5 \rightarrow 6$

Step 4&5

TABLE 1: Direct or Indirect link between source node and destination node

j	mp_I	np_I	D/ID Link with source node	D/ID Link with Distination node	Direct/Indirect Link withSource &Distination node	Distance
2	$mp_1 = 1 \rightarrow 2$	$np_1 = 1 \rightarrow 2$	Yes DIR Link	No	Discard	Discard
3	$mp_1 = 1 \rightarrow 3$	$np_1 = 1 \rightarrow 3$	Yes DIR Link	Yes ID Link	YES	**<23,32,45> <35,48,52>**
4	$mp_3 = 3 \rightarrow 4$	$np_3 = 3 \rightarrow 4$	Yes ID Link	Yes ID Link	NO	Discard
5	$mp_2 = 3 \rightarrow 5$	$mp_2 = 3 \rightarrow 5$	Yes ID Link	Yes ID Link	YES	<35,50,70> <57,76,87>
6	$mp_4 = 5 \rightarrow 6$	$np_4 = 5 \rightarrow 6$	Yes ID Link	Yes DIR Link	YES	**<56,82,118> <98,128, 148>**

VI. Short coming of existing method

In this section some shortcomings of the existing algorithm A.Nagoor Gani and M.Mohammed Jabbarulla are pointed out.

1. By using existing algorithm, to find the all possible intuitionistic fuzzy shortest path and intuitionistic fuzzy distance of each node from source node to destination node. It is very difficult and take more time for large network. For example: let applying the existing algorithm on a network having more than 8 or 9 nodes it takes more time to calculate all possible path , and then calculate L_{min} using similarity measure and solving problem on a network an edge with shortest length is computed and using Euclidean distance to find shortest path.
2. For using the algorithm, proposed by A.Nagoorgani and M.Mohammed jabbarulla a decision maker should have a good knowledge in programming language.

VII. Advantage of proposed algorithm

In this section it is shown that if we apply the proposed algorithm with existing comparison method (A.Nagoor Gani and M.Mohammed Jabburulla, 2010) to solve the intuitionistic fuzzy shortest path problems then it overcomes all the shortcomings describe in section 6.

1. Using proposed algorithm with existing comparison method (A.Nagoorgani and Mohammed jaburulla,2010) the intuitionistic fuzzy shortest path between node 1 and 6 of the network shown in fig.2 are same.(1→3→5→6)
2. The proposed algorithm is very easy to understand. The decision maker can see each and every step derived from the direct or indirect link , the decision maker can decided whether to select the shortest path or to discard the path. So no need to calculate all the possible path , it saves time, and very simple. In the meanwhile decision maker can find intuitionistic fuzzy distance also.
3. The procedure of finding the shortest path is simple without any complexity. For using the proposed algorithm a decision maker should have the knowledge of ranking function and additive arithemetic operation of Intuitionistic fuzzy numbers. There is no need of much knowledge of Intuitionistic Fuzzy linear programming, crisp linear programming and use of any Operation research technique for finding the shortest path.
4. It is very easy to learn and implemented in to a programming language for any huge problem.

VIII. Results and discussion

1. To compare the proposed algorithm with existing algorithm(A.Nagoor Gani and Mohammed Jaburalla, 2010) the numerical examples presented in A.Nagoor Gani and M. Mohammed Jaburalla(2010) then the obtained intuitionstic shortest path is same as obtained by the existing algorithm, but the existing algorithm is very confusing to understand and take more time to calculate to apply for finding

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the optimal solution of intuitionistic shortest path problems for a new decision maker while the proposed algorithm is very easy to understand and to apply for the same.

2. If the proposed algorithm is applied with the existing comparison method (A.Nagoor Gani and M. Mohammed Jaburalla, 2010) then it overcomes all the shortcoming described in section 6 and the intuitionstic fuzzy shortest path is same as obtained by the existing algorithm.
3. In addition that in the proposed method we find intuitionistic fuzzy shortest distance also.

IX. CONCLUSION

The shortcoming of the existing algorithm for finding intuitionstic fuzzy shortest path of any node from source node are pointed out and to overcome these shortcomings a new algorithm is proposed for the same . In most of the real life networks the number of nodes are large. Fuzzy shortest path length and shortest path are the useful information for the decision makers in a Intuitionistic fuzzy shortest path problem. The paper justifies that, each and every step derived from the Direct or Indirect link , the reader can decided whether to select the shortest path or to discard the path. The procedure of finding the shortest path is simple without any complexity. In this paper we have developed an algorithm to find the intuitionistic fuzzy shortest paths of a network with its arc length as triangular Intuitionistic fuzzy numbers. The results obtained by using proposed algorithm and the existing algorithm (A.Nagoorgani and M.Mohammed Jabarulla) are compared and it is show that it is better to use the proposed algorithm for solving intuitionstic fuzzy shortest path problems as compare to the existing algorithm.

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