ON GR*- CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT
The aim of this paper is to introduce the concept of generalized regular star continuous and generalized regular star irresolute and study some of its properties.

Keywords: rg-continuous, gr- continuous, gr*-continuous, gr*-irresolute.

1. INTRODUCTION
The concept of regular continuous functions was first introduced by Arya.S.P. and Gupta.R[1]. Later Palaniappan.N. and Rao.K.C.[10] studied the concept of regular generalized continuous functions. Also, the concept of generalized regular star continuous functions was introduced by Mahmood.S.I.[7]. Recently, the concept of generalized regular star closed set was introduced by Indirani.K. et al.[6]. The purpose of this paper is to introduce a new class of functions, namely, generalized regular star continuous functions and generalized regular star irresolute functions. Also, we study some of the characterization and basic properties of generalized regular star continuous functions.

2. PRELIMINARIES
Throughout this paper, (X,τ) (or X) represent a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A, respectively. X\A or A^c denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1: A map f: (X,τ)→(Y,σ) is said to be
(i) g-continuous[2] if f^(-1)(V) is g-closed in X for every closed subset V of Y.
(ii) sg-continuous[11] if f^(-1)(V) is sg-closed in X for every closed subset V of Y.
(iii) gs-continuous[3] if f^(-1)(V) is gs-closed in X for every closed subset V of Y.
(iv) gp-continuous[9] if f^(-1)(V) is gp-closed in X for every closed subset V of Y.
(v) gpr-continuous[5] if f^(-1)(V) is gpr-closed in X for every closed subset V of Y.
(vi) gsp-continuous[4] if f^(-1)(V) is gsp-closed in X for every closed subset V of Y.
(vii) rg-continuous[10] if f^(-1)(V) is rg-closed in X for every closed subset V of Y.

3. gr*- CONTINUOUS FUNCTIONS
Definition 3.1: For a subset A of a space X, gr*-cl(A) = ∩ {F: A⊆F, F is gr* closed in X} is called the gr*-closure of A.
Definition 3.2: Let \((X, \tau)\) be a topological space and \(\tau_{gr^*} = \{V \subseteq X: \text{gr}^*\text{-cl}(X \setminus V) = X \setminus V\}\).

Lemma 3.3: For any \(x \in X\), \(x \in \text{gr}^*\text{-cl}(A)\) if and only if \(V \cap A \neq \phi\), for every \(\text{gr}^*\)-open set \(V\) containing \(X\).

Lemma 3.4: Let \(A\) and \(B\) be subsets of \((X, \tau)\). Then
(i) \(\text{gr}^*\text{-cl}(\phi) = \phi\) and \(\text{gr}^*\text{-cl}(X) = X\).
(ii) If \(A \subseteq B\), then \(\text{gr}^*\text{-cl}(A) \subseteq \text{gr}^*\text{-cl}(B)\).
(iii) \(A \subseteq \text{gr}^*\text{-cl}(A)\).
(iv) If \(A\) is \(\text{gr}^*\)-closed, then \(\text{gr}^*\text{-cl}(A) = A\).
(v) \(\text{gr}^*\)-closure of a set \(A\) is not always \(\text{gr}^*\)-closed.

Remark 3.5: Suppose \(\tau_{gr^*}\) is a topology. If \(A\) is \(\text{gr}^*\)-closed in \((X, \tau)\), then \(A\) is closed in \((X, \tau_{gr^*})\).

Definition 3.6: A function \(f\) from a topological space \(X\) into a topological space \(Y\) is called a generalized regular star continuous (\(\text{gr}^*\text{-Continuous}\)) if \(f^1(V)\) is \(\text{gr}^*\)-Closed set in \(X\) for every closed set \(V\) in \(Y\).

Theorem 3.7: A function \(f: X \to Y\) from a topological space \(X\) into a topological space \(Y\) is \(\text{gr}^*\)-Continuous iff \(f^1(V)\) is \(\text{gr}^*\)-open set in \(X\) for every open set \(V\) in \(Y\).

Theorem 3.8: If a map \(f\) is continuous, then it is \(\text{gr}^*\)-continuous but not conversely.
Proof: Let \(f: X \to Y\) be continuous. Let \(F\) be any closed set in \(Y\). Then the inverse image \(f^1(F)\) is closed set in \(X\). Since every closed set is \(\text{gr}^*\)-closed, \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Therefore \(f\) is \(\text{gr}^*\)-continuous.

Theorem 3.9: If a map \(f: X \to Y\) is \(r\)-continuous, then it is \(\text{gr}^*\)-continuous but not conversely.
Proof: Let \(f: X \to Y\) be \(r\)-continuous. Let \(F\) be any closed set in \(Y\). Then the inverse image \(f^1(F)\) is \(r\)-closed set in \(X\). Since every \(r\)-closed set is \(\text{gr}^*\)-closed, \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Therefore \(f\) is \(\text{gr}^*\)-continuous.

The converse need not be true as seen from the following example.

Example 3.10: Let \(X = Y = \{a, b, c, d\}\), \(\tau = \{\phi\}, \{d\}, \{bc\}, \{(bcd), X\}\) and \(\sigma = \{\phi\}, \{ab\}, X\). Let \(f: X \to Y\) is a identity map, then \(f\) is \(\text{gr}^*\)-continuous but not continuous and \(r\)-continuous.

Theorem 3.11: If a map \(f: X \to Y\) is continuous, Then the following holds.
(i) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(g\)-continuous.
(ii) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gr\)-continuous.
(iii) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gs\)-continuous.
(iv) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gsp\)-continuous.
(v) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gsp\)-continuous.
(vi) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gsp\)-continuous.
(vii) If \(f\) is \(\text{gr}^*\)-continuous, then \(f\) is \(gsp\)-continuous.

Proof: (i) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(g\)-closed, then \(f^1(F)\) is \(g\)-closed in \(X\). Hence \(f\) is \(g\)-continuous.

(ii) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(rg\)-closed, then \(f^1(F)\) is \(rg\)-closed in \(X\). Hence \(f\) is \(rg\)-continuous.

(iii) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(gs\)-closed, then \(f^1(F)\) is \(gs\)-closed in \(X\). Hence \(f\) is \(gs\)-continuous.

(iv) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(sg\)-closed, then \(f^1(F)\) is \(sg\)-closed in \(X\). Hence \(f\) is \(sg\)-continuous.

(v) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(gsp\)-closed, then \(f^1(F)\) is \(gsp\)-closed in \(X\). Hence \(f\) is \(gsp\)-continuous.

(vi) Let \(F\) be a closed set in \(Y\). Since \(F\) is \(\text{gr}^*\)-continuous, then \(f^1(F)\) is \(\text{gr}^*\)-closed in \(X\). Since every \(\text{gr}^*\)-closed set is \(gsp\)-closed, then \(f^1(F)\) is \(gsp\)-closed in \(X\). Hence \(f\) is \(gsp\)-continuous.
(vii) Let \( F \) be a closed set in \( Y \). Since \( F \) is \( \text{gr}^*\)-continuous, then \( f^\dagger(F) \) is \( \text{gr}^* \)-closed in \( X \). Since every \( \text{gr}^*\)-closed set is \( \text{gpr}\)-closed, then \( f^\dagger(F) \) is \( \text{gpr}\)-closed in \( X \). Hence \( f \) is \( \text{gpr}\)-continuous.

The converse need not be true as seen from the following example.

**Example 3.12:** Let \( X=Y=\{a,b,c,d\}, \tau=\{\emptyset, \{a\}, \{b\}, \{ab\}, X\} \) and \( \sigma=\{\emptyset, \{a\}, \{a\}, \{b\}, \{ab\}, X\} \). Let \( f: X \rightarrow Y \) be an identity map, then \( f \) is \( g \)-continuous but not \( \text{gr}^*\)-continuous.

**Example 3.13:** Let \( X=Y=\{a,b,c,d\}, \tau=\{\emptyset, \{a\}, \{b\}, \{ab\}, X\} \) and \( \sigma=\{\emptyset, \{a\}, \{b\}, \{ab\}, X\} \). Let \( f: X \rightarrow Y \) be an identity map, then \( f \) is \( \text{gsp}\)-continuous, \( \text{gs}\)-continuous, \( \text{gp}\)-continuous, \( \text{gsp}\)-continuous, \( \text{gpr}\)-continuous but not \( \text{gr}^*\)-continuous.

**Theorem 3.14:** Let \( A \) be a subset of a topological space \( X \). Then \( x \in \text{gr}^*\text{cl}(A) \) if and only if every \( \text{gr}^*\)-open set \( U \) containing \( x \), \( A \cap U \neq \emptyset \).

Proof: Let \( x \in \text{gr}^*\text{cl}(A) \) and suppose that, there is a \( \text{gr}^*\)-open set \( U \) in \( X \) such that \( x \in U \) and \( A \cap U \neq \emptyset \) implies that \( A \subseteq U^c \) which is \( \text{gr}^*\)-closed in \( X \) implies \( \text{gr}^*\text{cl}(A) \subseteq \text{gr}^*\text{cl}(U^c) = U^c \). Since \( x \in U \) implies that \( x \notin U^c \) implies that \( x \notin \text{gr}^*\text{cl}(A) \), this is a contradiction.

Conversely, Suppose that, for any \( \text{gr}^*\)-open set \( U \) containing \( x \), \( A \cap U \neq \emptyset \). To prove that \( x \in \text{gr}^*\text{cl}(A) \). Suppose that \( x \notin \text{gr}^*\text{cl}(A) \), then there is a \( \text{gr}^*\)-closed set \( F \) in \( X \) such that \( x \notin F \) and \( A \subseteq F \). Since \( x \notin F \) implies that \( x \in F^c \) which is \( \text{gr}^*\)-open in \( X \). Since \( A \subseteq F \) implies that \( A \cap F^c = \emptyset \), this is a contradiction. Thus \( x \in \text{gr}^*\text{cl}(A) \).

**Theorem 3.15:** Let \( f: X \rightarrow Y \) be a function from a topological space \( X \) into a topological space \( Y \). If \( f: X \rightarrow Y \) is \( \text{gr}^*\)-continuous, then \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \) for every subset \( A \) of \( X \).

Proof: Since \( f(A) \subseteq \text{cl}(f(A)) \) implies that \( A \subseteq f^\dagger(\text{cl}(f(A))) \). Since \( \text{cl}(f(A)) \) is a closed set in \( Y \) and \( f \) is \( \text{gr}^*\)-continuous, then by definition \( f^\dagger(\text{cl}(f(A))) \) is a \( \text{gr}^*\)-closed set in \( X \) containing \( A \). Hence \( \text{gr}^*\text{cl}(A) \subseteq f^\dagger(\text{cl}(f(A))) \). Therefore \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

**Theorem 3.16:** Let \( f: X \rightarrow Y \) be a function from a topological space \( X \) into a topological space \( Y \). Then the following statements are equivalent:

(i) For each point \( x \) in \( X \) and each open set \( V \) in \( Y \) with \( f(x) \in V \), there is a \( \text{gr}^*\)-open set \( U \) in \( X \) such that \( x \in U \) and \( f(U) \subseteq V \).

(ii) For each subset \( A \) of \( X \), \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

(iii) For each subset \( B \) of \( Y \), \( \text{gr}^*\text{cl}(f^\dagger(B)) \subseteq f^\dagger(\text{cl}(B)) \).

Proof: (i) \( \rightarrow \) (ii) Suppose that (i) holds and let \( y \in f(\text{gr}^*\text{cl}(A)) \) and let \( V \) be any open neighbourhood of \( y \). Since \( y \in f(\text{gr}^*\text{cl}(A)) \) implies that there exists \( x \in \text{gr}^*\text{cl}(A) \) such that \( f(x) = y \). Since \( f(x) \in V \), then by (i) there exists a \( \text{gr}^*\)-open set \( U \) in \( X \) such that \( x \in U \) and \( f(U) \subseteq V \). Since \( x \in f(\text{gr}^*\text{cl}(A)) \), then by theorem 3.14 \( U \cap \{A\} \neq \emptyset \). Therefore we have \( y = f(x) \in \text{cl}(f(A)) \). Hence \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

(ii) \( \rightarrow \) (i) Let if (ii) holds and let \( x \in X \) and \( V \) be any pen set in \( Y \) containing \( f(x) \). Let \( A = f^\dagger(V^c) \) this implies that \( x \notin A \). Since \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \subseteq V^c \) this implies that \( \text{gr}^*\text{cl}(A) \subseteq f^\dagger(V^c) = A \). Since \( x \notin A \) implies that \( x \notin \text{gr}^*\text{cl}(A) \) and by theorem 3.14 there exists a \( \text{gr}^*\)-open set \( U \) containing \( x \) such that \( U \cap A \neq \emptyset \) and hence \( f(U) \subseteq f(A) \subseteq V \).

(iii) \( \rightarrow \) (i) Suppose that (iii) holds and let \( B = f(A) \) where \( A \) is a subset of \( X \). Then we get from (iii) \( \text{gr}^*\text{cl}(A) \subseteq \text{gr}^*\text{cl}(f^\dagger(f(A))) \subseteq f^\dagger(\text{cl}(f(A))) \). Therefore \( f(\text{gr}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

**Theorem 3.17:** Let \( f: X \rightarrow Y \) be a map. Then the following statements are equivalent:

(i) \( f \) is \( \text{gr}^*\)-continuous.

(ii) the inverse image of each open set in \( Y \) is \( \text{gr}^*\)-open in \( X \).
Proof: Assume that $f: X \rightarrow Y$ is gr*-continuous. Let $G$ be open in $Y$. The $G^r$ is closed in $Y$. Since $f$ is gr*-continuous, $f^1(G^r)$ is gr*-closed in $X$. But $f^1(G^r) = X - f^1(G)$. Thus $f^1(G)$ is gr*-open in $X$.

Conversely assume that the inverse image of each open set in $Y$ is gr*-open in $X$. Let $F$ be any closed set in $Y$. By assumption $c$ is gr*-open in $X$. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is gr*-open in $X$ and so $f^{-1}(F)$ is gr*-closed in $X$. Therefore $f$ is gr*-continuous. Hence (i) and (ii) are equivalent.

**Theorem 3.18:** If a function $f: X \rightarrow Y$ is gr*-continuous, then $f(gr*-cl(A)) \subseteq cl(f(A))$ for every subset $A$ of $X$.

Proof: Let $f: X \rightarrow Y$ be gr*-continuous. Let $A \subseteq X$. Then $cl(f(A))$ is closed in $Y$. Since $f$ is gr*-continuous, $f^{-1}(cl(f(A)))$ is gr*-closed in $X$ and $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$, implies gr*-cl$(A) \subseteq f^{-1}(cl(f(A)))$. Hence $f(gr*-cl(A)) \subseteq cl(f(A))$.

**Theorem 3.19:** Let $f: X \rightarrow Y$ be a function. Let $(X, \tau)$ and $(Y, \sigma)$ be any two spaces such that $\tau_{gr^*}$ is a topology on $X$. Then the following statements are equivalent:

(i) For every subset $A$ of $X$, $f(gr*-cl(A)) \subseteq cl(f(A))$ holds, 
(ii) $f: (X, \tau_{gr^*}) \rightarrow (Y, \sigma)$ is continuous.

Proof: Suppose (i) holds. Let $A$ be closed in $Y$. By hypothesis $f(gr*-cl(f^{-1}(A))) \subseteq cl(f^{-1}(A)) \subseteq cl(f(A)) = A$, i.e., gr*-cl$(f^{-1}(A)) \subseteq f^{-1}(A)$. Also $f^{-1}(A) \subseteq gr*-cl(f^{-1}(A))$. Hence, gr*-cl$(f^{-1}(A)) = f^{-1}(A)$. This implies $(f^{-1}(A))^c \subseteq \tau_{gr^*}$. Thus $f^{-1}(A)$ is closed in $(X, \tau_{gr^*})$ and so $f$ is continuous. This proves (ii).

Suppose (ii) holds. For every subset $A$ of $X$, cl$(f(A))$ is closed in $Y$. Since $f: (X, \tau_{gr^*}) \rightarrow (Y, \sigma)$ is continuous, $f^{-1}(cl(f(A)))$ is closed in $(X, \tau_{gr^*})$ that implies by Definition 3.2 gr*-cl$(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Now we have, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$ and by lemma 3.4 (ii), gr-cl$(A) \subseteq gr*-cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Therefore $f(gr*-cl(A)) \subseteq cl(f(A))$.

**Theorem 3.20:** Let $f: X \rightarrow Y$ is gr*-continuous function and $g: Y \rightarrow Z$ is continuous function then $g \circ f: X \rightarrow Z$ is gr*-continuous.

Proof: Let $g$ be a continuous function and $V$ be any open set in $Z$ then $f^{-1}(V)$ is open in $Y$. Since $f$ is gr*-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gr*-open in $X$. Hence $g \circ f$ is gr*-continuous.

**Theorem 3.21:** Let $X = A \cup B$ be a topological space with topology $\tau$ and $Y$ be a topological space with topology $\sigma$. Let $f: (A, \tau/A) \rightarrow (Y, \sigma)$ and $g: (B, \tau/B) \rightarrow (Y, \sigma)$ be gr*-continuous maps such that $f(x) = g(x)$ for every $x \in A \cap B$. Suppose that $A$ and $B$ are gr*-closed sets in $X$. Then the combination $\alpha: (X, \tau) \rightarrow (Y, \sigma)$ is gr*-continuous.

Proof: Let $F$ be any closed set in $Y$. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$, where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. But $C$ is gr*-closed in $A$ and $g$ is gr*-continuous in $X$ and so $C$ is gr*-closed in $X$. Since we have proved that if $B \subseteq A \subseteq X$, $B$ is gr*-closed in $A$ and $A$ is gr*-closed in $X$ then $B$ is gr*-closed in $X$. Also $C \cup D$ is gr*-closed in $X$. Therefore $\alpha^{-1}(F)$ is gr*-closed in $X$. Hence $\alpha$ is gr*-continuous.

**Definition 3.22:** A function $f: X \rightarrow Y$ is said to be gr*-irresolute, if $f^{-1}(V)$ is gr*-open set in $(X, \tau)$ for every gr*-open set $V$ in $(Y, \sigma)$.

**Theorem 3.23:** Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Let $h = g \circ f$. Then:

(i) $h$ is gr*-continuous if $f$ is gr*-irresolute and $g$ is gr*-continuous and 
(ii) $h$ is gr*-continuous if $g$ is continuous and $f$ is gr*-continuous.

Proof: Let $V$ be closed in $Z$. (i) Suppose $f$ is gr*-irresolute and $g$ is gr*-continuous. Since $g$ continuous is gr*-continuous. $g^{-1}(V)$ is gr*-closed in $Y$. Since $f$ is gr*-irresolute, using the Definition 3.22 $f^{-1}(g^{-1}(V))$ is gr*-closed in $X$. This proves (i).

(ii) Let $g$ be continuous and $f$ be gr*-continuous. Then $g^{-1}(V)$ is closed in $Y$. Since $f$ is gr*-
continuous, using the Definition 3.6 \( f^\dagger(g^\dagger(V)) \) is gr*-closed in \( X \). This proves (iii).

**Theorem 3.24:** A function \( f: X \to Y \) is gr*-irresolute if and only if the inverse image of every gr*-open set in \( Y \) is gr*-open in \( X \).

**Definition 3.25:** A function \( f: X \to Y \) is said to be gr*-closed (resp. gr*-open) if for every gr*-closed (resp. gr*-open) set \( U \) of \( X \) the set \( f(U) \) is gr*-closed (resp. gr*-open) in \( Y \).

**Theorem 3.26:** A function \( f: X \to Y \) be a bijection. Then the following are equivalent:
(i) \( f \) is gr*-open,
(ii) \( f \) is gr*-closed,
(iii) \( f^\dagger \) is gr*-irresolute.

Proof: Suppose \( f \) is gr*-open. Let \( F \) be gr*-closed in \( X \). Then \( X \setminus F \) is gr*-open. By Definition 3.25 \( f(X \setminus F) \) is gr*-open. Since \( f \) is bijection, \( Y \setminus f(F) \) is gr*-open in \( Y \). Therefore \( f \) is gr*-closed. This proves (i) \( \Rightarrow \) (ii).

Let \( g = f^\dagger \). Suppose \( f \) is gr*-closed. Let \( V \) be gr*-open in \( X \). Then \( X \setminus V \) is gr*-closed in \( X \). Since \( f \) is gr*-closed, \( f(X \setminus V) \) is gr*-closed. Since \( f \) is a bijection, \( Y \setminus f(V) \) is gr*-closed that implies \( f(V) \) is gr*-open in \( Y \). Since \( g = f^\dagger \) and since \( g \) and \( f \) are bijection \( g^{-1}(V) = f(V) \) so that \( g^{-1}(V) \) is gr*-open in \( Y \). Therefore \( f^\dagger \) is gr*-irresolute. This proves (ii) \( \Rightarrow \) (iii).

Suppose \( f^\dagger \) is gr*-irresolute. Let \( V \) be gr*-open in \( X \). Then \( X \setminus V \) is gr*-closed in \( X \). Since \( f^\dagger \) is gr*-irresolute and \( (f^\dagger)^{-1}(X \setminus V) = f(X \setminus V) = Y \setminus f(V) \) is gr*-closed in \( Y \) that implies \( f(V) \) is gr*-open in \( Y \). Therefore \( f \) is gr*-open. This proves (iii) \( \Rightarrow \) (i).

**Theorem 3.27:** Let \( f: X \to Y \) and \( g: Y \to Z \) are gr*-irresolute, then \( g \circ f: X \to Z \) is gr*-irresolute.

Proof: Let \( g \) be an gr*-irresolute function and \( V \) be any gr*-open in \( Z \), then \( f^\dagger(V) \) is gr*-open set in \( Y \), since \( f \) is gr*-irresolute, \( f^\dagger(g^\dagger(U)) = (g \circ f)^\dagger(U) \) is gr*-open in \( (X, \tau) \). Hence \( g \circ f \) is gr*-irresolute.

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