A Resource Allocation Scheme for Decode and Forward Relay Cognitive Networks

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Abstract—*Relayed transmission in a Cognitive Radio (CR) environment increases the coverage area and capacity of the communication system. There are many types of cooperative communications, amplify and forward (AAF) and decode and forward (DAF). Adaptive relaying protocol (ARP) is used here to overcome the disadvantages of AAF and DAF. In this paper we are mainly focusing on the joint power allocation in a cognitive radio system in a cooperative mode that operates ARP in a multi-carrier mode, the multi-carrier scenario is used in an orthogonal frequency division multiplexing (OFDM) mode, and problem is formulated to maximize the end-to-end rate by providing the best power allocation at the transmitters. The simulation results shows that DAF will achieve more capacity than AAF, but if we use ARP in comparison to other relaying techniques we can achieve even more capacity*.

Keywords-*cognitive radio ; cooperative communication ; Amplify_and_forward; Decode_and_forward; Power allocation ; Pairing ;Adaptive relaying protocol* .

1. Introduction

The growth of technology has affected directly modern communication systems. This expansion can be observed when a comparison is made between the earlier systems with some bits per second as a communication rate and the 300 Mbps already considered in the long-term evolution (LTE) wireless communication systems. The growth of the data rate in wireless standards and services was accompanied by a rise in applications and costumers which implies a strong increase in the demand for the limited frequency spectrum. This means that the actual available spectrum resource may not be able to respond to the emerging and future technology demands.

In current systems, the frequency allocation, the type of service, the maximum transmission powers, and the time duration of the licenses are managed by governmental agencies, which apply the 'command-and-control' allocation model by assigning a fixed frequency block for each communication service. This statistic scheme is inflexible in spectrum management and results in an inefficient use of the available spectrum because licensed users are not using the allocated portion of spectrum at all times or over all the spatial locations, and at the same time

Cognitive radio (CR) can manage the spectrum utilization by detecting spectrum holes and avoiding the occupied spectrum using the available part of the spectrum. Spectrum utilization can be improved by allowing the secondary users (SUs) to use the vacant channels left by the licensed users (PUs) [1]. Such systems have to distribute their limited resources among the SUs in order to maximize the capacity without causing harmful interference to the PUs (see, e.g., [2,3]). Since OFDM is widely used in various wireless systems and shows a high spectral efficiency and flexibility, it is often recommended for cognitive radio systems [4].

To increase coverage and to obtain achievable capacity of the communication system, relays (R) are used. It transfers the information from the cognitive source (CS) to the destination (D) when the direct link is not available [5] (in some cases even if a direct link exists, the relays are used to improve the performance of the communication systems). The resource allocation problem for the non-cognitive OFDM based relay system has been widely studied [6,7]. In [8], a cooperative scheme of decode-and-forward technique is combined with the cognitive radio to produce high performance and higher coverage area. Note that in cognitive cooperative communication systems, both transmitters, namely source and relay, have to be aware about the interference threshold tolerated by the PU.

The relaying techniques are amplify-and-forward (AAF) [9] and decode-and-forward (DAF) [10]. We know In the AAF case, the relay amplifies the received signal from the source (S) by some fixed factor, and then forwards it to destination. However, the relay using the DAF strategy decodes 'perfectly' the received signal from S and then encodes it again (with the same code known by S and D) and forwards it to D. Note that these procedures are done at each sub-carrier. The disadvantages of these two techniques of relaying come with the fact that: (1) AAF relaying can amplify the noise coming from the (S-R) link, which degrades the signal quality, and (2) DAF relaying causes a propagation of error in case of un correct decoding of the information symbols.

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Adaptive Relaying Protocol (ARP), as named in [6], is one of the proposed solutions that benefits from the advantages of DAF and AAF, and aims to minimize the disadvantages of these relaying techniques. In [7,11] the relay can execute AAF and DAF, and there is a technique based on the signal-to-noise-ratio (SNR) which triggers the switching between the AAF and DAF strategies. It assumes that at high SNR (for an SNR above some SNR thresholds), the relay can decode perfectly, so it is better to operate with DAF and for low SNRs (below certain threshold), when it is harder to decode correctly, it is preferable to use the AAF to avoid propagation errors.

The main motto of this paper is to provide an efficient procedure to integrate the adaptive relaying technique in a CR based environment for a power allocation at the transmitters (S and R) to reach high capacity, without causing harmful interference to the primary user. The proposed solution goes through an algorithm based on the dual problem and sub-gradient method [12-14]. we begin by selecting the sub-carrier and assume that the relay uses the same sub-carrier for receiving (from S) and for transmission (to D).

Related work:

Many researchers have been proposed different concepts to solve the power allocation problem for the cooperative cognitive radio communication systems. In [15], study of the spectrum sensing of the cognitive system using a cooperative relay cognitive radio system is mentioned. Their work focuses on the tradeoff between the spectrum sensing and SU transmission. The use of relays is presented also in [16], where we study the outage probability of the SU when DAF is used. The AAF and DAF are both used in [18], where the system decides to use one of these schemes according to the known CSI. If the relay operates in AAF mode, it will amplify the received signal. For the DAF mode, if decoding is unsuccessful, the relay will remain silent. Otherwise, the relay re-encodes the decoded data and transmits it to the destination.

We use a different scheme in our work focus on the cognitive context, and this yields another optimization problem. In this paper we use an adaptive scheme of relaying based on switching between both techniques, i.e., AAF and DAF.

1.2 Summary of contribution:

A new adaptive relaying protocol based on the AAF and DAF modes is proposed here. The relay is able to perform the AAF and the DAF according to the capability of the relay to decode successfully the signal. Note that the decision of switching, between both modes, is based not only on the channel information but also on the received SNR which will add more complexity on the optimization problem to maximize the total rate subject to power and interference constraints; the ARP is described in [6], but it is not used for cognitive radio as it is in this paper. Also, the optimization problem to maximize the total rate is missing in [6]. This paper uses the dual problem and sub-gradient algorithm to solve numerically the optimization problem.

1.1 Outline of the paper:

The remainder of the paper is organized as follows. In Section 2, we present the system model. The proposed solution is illustrated by some selected numerical results to compare the performance over the different types of relaying (AAF, DAF, and ARP). In Section 3, we investigate the pairing problem by including the pairing parameters within the optimization problem studied in Section 2. Finally, we conclude this work in Section 4, with a summary of the main results.

2. Near-optimal algorithm for sub-carrier matching scheme:

We focus here on the simple case in which the power allocation of a cognitive system with one relay system using a matching pairing strategy is used. In particular, we assume that the relay forwards the signal over the same received sub-carrier and study the difference in performance between the three types of relaying schemes introduced in the previous section.

2.1 System model:

An OFDM-based relay CR system is considered here. The CR relay system coexists with the primary system in the same geographical location. We assume that there is no direct link between the CS and the D, so S tries to communicate with D through the relay (see Figure 1).

The frequency spectrum of the CR system is divided into N sub-carriers, each having a Δf bandwidth. We assume that the CR system can transmit through the unused PU Band without exceeding the maximum interference power I_{th} that can be tolerated by PU. The relay is assumed to be half duplex, so receiving and forwarding at two different time slots. In the first time slot, S transmits to R, while in the second time slot, R forwards the signal to D with the ARP technique. It has been assumed that we have one relay which can work with different channels. Note that the relay can forward the data using two techniques (DAF and AAF) by switching between them, as presented in Figure 2. The calculation of the mutual interference between PU, SU, and the relay is presented in the next part.

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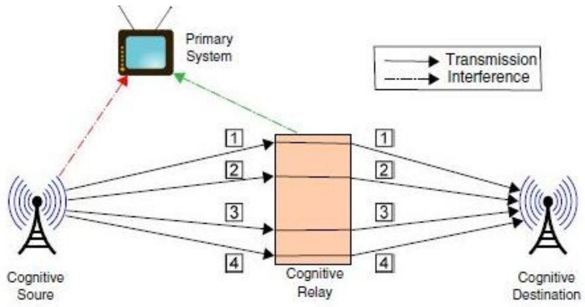

Figure 1. Illustration of a cooperative relay cognitive radio network with $N = 4$.

2.2 Interference analysis:

The mutual interference introduced to PU by the i_{th} sub-carrier in OFDM systems is presented in [23]. Assume that $\O_i$ is the power spectrum density (PSD) of the i_{th} sub-carrier. The form of the PSD depends directly on the multi-carrier wave form technique. In our case, when an OFDM-based system is used, the PSD at the i_{th} sub-carrier band can be written as

$$\Phi_i(f)=P_iT_s\left(\frac{\sin \pi f\,Ts}{\pi f\,Ts}\right)^2 \quad (1)$$

Where P_i is the total transmit power emitted by the i_{th} sub-carrier, and T_s is the symbol duration. Hence, the mutual interference introduced by the i_{th} sub-carrier to PU, $I_i(d_i, P_i)$, can be found by integrating the PSD of the i_{th} sub-carrier over the PU band, B, and can be obtained using the following expression [8]

$$I_i\,(d_i, p_i)= \int_{di-b/2}^{di+B/2} Gi\ \Phi i(f)df \triangleq P_i\Omega^i \quad (2)$$

Where d_i and G_i denote the spectral distance and the channel gain, respectively, between the i_{th} sub-carrier and the PU band, while Ω^i is the interference factor of the i_{th} subcarrier to the PU band [24]a. Note that (2) expresses the interference in terms of the total transmit power P_i of the i_{th} sub-carrier linearly, which will be used to solve the optimization problem in the next subsections.

By the same analysis, the interference power introduced by PU signal into the band of the i_{th} sub-carrier is expressed as [8]

$$j_i = \int_{di-\Delta f/2}^{di+\Delta f/2} Yi\ \Psi(f)df \quad (3)$$

Where $\Psi(f)$ is the PSD of PU signal, and Y_i is the channel gain between the i_{th} sub-carrier and the PU signal. By completing the interference analysis of the different agents of the cognitive system, we can formulate the optimization problem before proceeding to the solution

2.3 Capacity analysis and problem formulation

Let us first define the variables of the problem. Let $(P^i_{SR};\ P^i_{RD})$ be the power transmitted over the i_{th} subcarrier in the (S-R;R-D) link. The i_{th} sub-carrier channel gain over the (S-R;R-D) link is given by $(H^i_{SR};H^i_{RD})$.The noise variance is assigned by $\sigma^2_i= \sigma^2_{AWGN}+J_i$, where σ^2_{AWGN} is the variance of the additive white Gaussian noise (AWGN), and J_i is the interference introduced by the PU signal into the i_{th} sub-carrier which is evaluated using (3). This interference can be modeled as an AWGN as described in [2]. To make the analysis more clear, the noise variance σ^2 is assumed to be the same for all sub-carriers and both time slots.

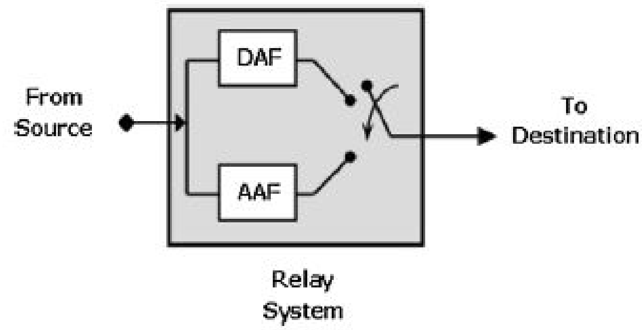

Figure 2 Block diagram for the structure of an adaptive relay.

2.3.1 Processing during the first time slot

Let $x_{s,i}$ be the transmitted signal from S over the i_{th} channel. The received signal at the relay R over the i_{th} sub-carrier in the first time slot is given by

$$\gamma^i_{SR} = \sqrt{H^i_{SR}P^i_{SR}}x^i_{s,i} + n^i_{SR} \quad (4)$$

Where n^i_{SR} is the noise between S and R with a variance$\sigma^2_{SR,i} = \sigma^2$, and $i = \{1, 2, . . .N\}$ denote the i_{th} sub-carrier. According to the Shannon capacity formula, the transmission rate of the i_{th} sub-carrier between the source and the relay $R_{1,i}$ can be calculated as

$$R_{1,i} = \frac{1}{2}\log_2\left(1 + \frac{P^i_{SR}H^i_{SR}}{\sigma^2}\right) \quad (5)$$

As it has been mentioned above, we should limit the Interference caused by the CS to the PU, which gives us the following interference constraint [8]:

$$\sum_{i=1}^{N} P^i_{SR}\Omega^i_{SP} \le I_{th} \quad (6)$$

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Where Ω_{SP}^{i} denotes the interference factor of the i_{th} subcarrier to the PU band.

2.3.2 Capacity in the second time slot:

In the second time slot, the relay decodes and re-encodes or amplifies the signal over the i_{th} channel depending on the received SNR, then forwards it to the destination. This means that the transmit signal from the relay over the i_{th} channel is

$$x_{RD}^{i} = \begin{cases} \sqrt{P_{RD}^{i}}\, x_{s,i} & \text{for } P_{SR}^{i}\gamma_{SR}^{i} \geq \gamma_{th} \\ \beta_i \sqrt{P_{RD}^{i}}\, \gamma_{sr}^{i} & \text{for } P_{SR}^{i}\gamma_{SR}^{i} < \gamma_{th} \end{cases} \quad (7)$$

Where $P^{i}_{SR}\gamma^{i}_{SR} = P^{i}_{SR}H^{i}_{SR}/\sigma^2$ is the received SNR via the source-relay link, and γ_{th} is the threshold SNR to ensure successful decoding. We assume that we have successful decoding when $P_{SR}^{i}\gamma_{SR}^{i}$ is above γ_{th}. In (7), βi is an amplification factor used by the relay to amplify the signal using the AAF mode. The choice of βi should assure the normalization of the total transmit power to the same value with all the AAF channels. It is defined in ([25], Eq. (9)) as

$$\beta[i] = \frac{1}{\sigma\sqrt{P_{SR}^{i}\gamma_{SR}^{i} + 1}} \quad (8)$$

At the destination, the received signal over the i_{th} channel can be written as

$$y_{RD}^{i} = \sqrt{H_{RD}^{i}}\, x_{RD}^{i} + n_{RD}^{i} \quad (9)$$

Let us define two variables: $\gamma^{i}_{SR} = H^{i}_{SR}/\sigma^2$ and $\gamma^{i}_{RD} = H^{i}_{RD}/\sigma^2$ .Using (7) and (9), we derive the expression of the total SNR delivered via the i_{th} channel as

$$\gamma_{ARP}^{i} = \begin{cases} P_{RD}^{i}\gamma_{RD}^{i} & \text{for } P_{SR}^{i}\gamma_{SR}^{i} \geq \gamma_{th} \\ \gamma_{AF}^{i} & \text{for } P_{SR}^{i}\gamma_{SR}^{i} < \gamma_{th} \end{cases} \quad (10)$$

Where γ^{i}_{AF} is the SNR for the set of channels that work on amplify-and-forward, and it is given by

$$\gamma_{AF}^{i} = \frac{\mathbb{E}\left[y_{RD,AAF}^{i}\right]^2}{\mathbb{E}[(y_{RD,AAF}^{i})2] - \mathbb{E}\left[y_{RD,AAF}^{i}\right]^2} = \frac{P_{SR}^{i}\gamma_{SR}^{i}P_{RD}^{i}\gamma_{RD}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i} + 1} \quad (11)$$

Where E[Y] denotes the expected value of the random variable Y.

By Shannon capacity formula, we calculate the rate of the channel in the second time slot for the two cases as

$$\begin{cases} R_{2,DAF,i} = \frac{1}{2}\log_2\left(1 + \frac{P_{RD}^{i}H_{RD}^{i}}{\sigma^2}\right) & \textit{for the DAF case} \\ R_{2,AAF,i} = \frac{1}{2}\log_2\left(1 + \frac{P_{RD}^{i}\gamma_{RD}^{i}P_{SR}^{i}\gamma_{SR}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i} + 1}\right) & \textit{for the AAF case} \end{cases} \quad (12)$$

Note that $R_{2,AAF,i}$ is not jointly concave in P^{i}_{RD} and P^{i}_{SR}. To make the analysis simpler, we adopt the following approximation:

$$R_{2,AAF,i} \approx \frac{1}{2}\log_2\left(1 + \frac{P_{RD}^{i}\gamma_{RD}^{i}P_{SR}^{i}\gamma_{SR}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i}}\right) \quad (13)$$

This approximation is used in [26], and it is based on the assumption that the system has a high SNR for the amplified signal between the relay and the destination. It is proved in [27] that this approximation is also accurate even in the moderate-low SNR regime.

To make things more clear, a new binary variable α_i is defined in a way that it takes the values '0' or '1' to indicate if the relay uses the DAF case ($\alpha_i = 0$) or the AAF case ($\alpha_i = 1$).We denote also by A the set of index of channels that work on AAF, and D as the set of index of channels that work on DAF

$$A = \{i, \propto_i = 1\}\,; D = \{i, \propto_i = 0\} \quad (14)$$

As in the DAF case, we calculate the interference caused by the relay to the PU for the AAF case. Using the interference analysis done above and the expression given in ([28], Eq. (17-18)), we get the following interference constraint
in the second time slot $(R \longrightarrow D)$:

$$\sum_{i\in A} P_{RD}^{i}\Omega_{RP}^{i} + \sum_{i\in D} P_{RD}^{i}\Omega_{RP}^{i} \leq I_{th} \quad (15)$$

2.3.3 Total capacity:

It is clear that the capacity has different expression in each time slot and forwarding technique. Therefore, we need to find a unified expression that will be used as the objective function of the upcoming optimization problem. In fact, the achievable rate in each sub-carrier is the minimum rate between both time slots. Thus, the transmission rate is given by

$$R_i = \propto_i \min\{R_{1,i}, R_{2,DAF,i}\} + (1 - \propto_i)\min\{R_{1,i}, R_{2,AAF,i}\} \quad (16)$$

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The maximum capacity is achievable when the rate in the first time slot is equal to the rate in the second time slot for every sub-carrier if it is possible. Thus, from (16), we should have the following capacity relation for the *i*th sub-carrier to achieve maximum rate:

$$R_{1,i} = \begin{cases} R_{2,DAF,i} & for\ i\ \in D \\ R_{2,AAF,i} & for\ i\ \in A \end{cases} \tag{17}$$

Hence, we get two cases. For the DAF case, the equality is achievable by assembling (17), (5), and (12) to derive the following relation between the transmission powers:

$$P_{RD}^{i} = \frac{P_{SR}^{i} H_{SR}^{i}}{H_{RD}^{i}}\ for\ i\ \in D \tag{18}$$

However, if we look to the formula of the rates in the AAF case (12), we can see that the rate in the second time slot is always less than the rate in the first time slot and cannot reach it. In fact, we have

$$\frac{P_{RD}^{i}\gamma_{RD}^{i}P_{SR}^{i}\gamma_{SR}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i}} < \frac{P_{SR}^{i}\gamma_{SR}^{i}}{\sigma^2} \quad \text{for } i \in A \tag{19}$$

This means that the achievable rate of these channels is equal to the rate in the second time slot. According to these derivations, we find the total expression of the rate in our model as

$$R = \sum_{i\in A}\frac{1}{2}\log_2\left(1 + \frac{P_{RD}^{i}\gamma_{RD}^{i}P_{SR}^{i}\gamma_{SR}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i}}\right) + \sum_{i\in D}\frac{1}{2}\log_2(1 + P_{SR}^{i}\gamma_{SR}^{i}) \tag{20}$$

2.3.4 Optimization problem of the sub-carrier matching Technique

Our objective is to maximize the total capacity of the CR system while the interference introduced to the primary user is below the tolerated threshold. Therefore, the optimization problem can be formulated as follows:

$$\max_{P_{SR}^{i}\,P_{RD}^{i}} \sum_{i=1}^{n} R_i$$

-(Interference at first time slot))

$$\sum P_{SR}^{i}\Omega_{SR}^{i} \leq I_{th} \tag{21}$$

-(Interference at second time slot)

$$\sum_{i\in A} P_{RD}^{i}\Omega_{RP}^{i} + \sum_{i\in D} P_{RD}^{i}\Omega_{RP}^{i} \leq I_{th}$$

$$P_{SR}^{i}\geq 0; \quad P_{RD}^{i}\geq 0$$

In this problem, the power constraints at each transmitter (S and R) are missed. However, when we take a look at the interference constraints, we note that the power constraint is defined indirectly. Moreover, we use the identity $\Omega_{SP}^{i} \geq \min \Omega_{SP}^{i}$ to ensure the following inequality Ni=1 $\sum P^{i}_{SR} \leq I_{th}/\min\ j\ (\Omega^{i}_{SP})$. Thus, the interference constraint implies, indirectly, a power constraint in the two time slots; even if the resulting interference is too small, the problem remains approximately the same without significant change. This analysis can help us in this chapter because the problem is relatively simple and is not a mixed integer programming problem, so the power constraint can be omitted. However, in the next section, the problem is more complex, and we should define the power constraints at the transmitters to avoid non-convergence of the algorithm.

We assume that all fading gains are perfectly known. The channel gains between the CR system parts (S, R, and D) can be obtained by channel estimation techniques; the channel gains between the CR system and the PU can be obtained by estimating the received signal power from the primary terminal when it transmits [29]. At the end of this part and by assembling the previous equations and relations, we can re-write the optimization problem given in (21) as

$$\max_{P_{SR}^{i},P_{RD}^{i}} \frac{1}{2}\sum_{i\in A}\log_2\left(1 + \frac{P_{RD}^{i}\gamma_{RD}^{i}P_{SR}^{i}\gamma_{SR}^{i}}{P_{RD}^{i}\gamma_{RD}^{i} + P_{SR}^{i}\gamma_{SR}^{i}}\right) + \sum_{i\in D}\frac{1}{2}\log_2(1 + P_{SR}^{i}\gamma_{SR}^{i})$$

Show that

$$\sum P_{SR}^{i}\Omega_{SR}^{*i} \leq I_{th} \tag{22}$$

$$\sum_{i\in A} P_{RD}^{i}\Omega_{RP}^{i} + \sum \frac{P_{SR}^{i}H_{SR}^{i}}{H_{RD}^{i}}\Omega_{RP}^{i} \leq I_{th}\ ;$$

$$P_{SR}^{i}\geq 0; \quad P_{RD}^{i}\geq 0$$

.

Under the previous assumption of perfect knowledge of the channel coefficient and the noise variance, the problem is a convex optimization problem with the parameter P^{i}_{RD} and P^{i}_{SR}. In the next part, we solve this problem using the Lagrangian method and the Karush-Kuhn-Tucker (KKT) conditions.Moreover, using the fact that the problem is convex, the dual solution and the primal solution are the same, so the problem can be solved using the dual formulation.

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2.4 Solution

For simplicity reasons and for making the mathematical notation easy to follow, we denote the following: P^i_{SR} by P^i_1, P^i_{RD} by P^i_2, γ^i_{SR} by γ^i_1, γ^i_{RD} by γ^i_2, Ω^i_{SP} by Ω^i_1, and Ω^i_{RP} by Ω^i_2.

2.4.1 Dual problem

The Lagrangian function with Lagrangian multipliers λ, μ can be written as presented in (23).

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N}\left[\propto_i \log_2\left(1+\frac{P_1^i\gamma_1^i P_2^i\gamma_2^i}{P_1^i\gamma_1^i+P_2^i\gamma_2^i}\right)+(1-\propto_i)\left[\frac{1}{2}\log_2(1+P_1^i\gamma_1^i)\right]+\mu(I_{th}-\sum P_1^i\Omega_1^i)+\lambda(I_{th}-\sum\propto_i P_2^i\Omega_2^i)\right]+(1-\propto_i)\frac{P_1^i H_{SR}^i}{H_{RD}^i}\Omega_2^i) \quad (23)$$

Note that we substitute A and D by their definition, and we include α_i in the Lagrangian to simplify the computation. We develop the Lagrangian to get the following expression:

$$\mathcal{L} = \sum_{i=1}^{N}\propto_i\left[\frac{1}{2}\log_2\left(1+\frac{P_1^i\gamma_1^i P_2^i\gamma_2^i}{P_1^i\gamma_1^i+P_2^i\gamma_2^i}\right)-\lambda P_2^i\Omega_2-\mu P_1^i\Omega_i^*\right]+(1-\propto_i)[\frac{1}{2}\log_2(1+P_1^i\gamma_1^i)-\lambda\frac{P_1^i H_{SR}^i}{H_{RD}^i}\Omega_i-\mu P_1^i\Omega_1^*]+(\lambda+\mu)I_{th} \quad (24)$$

Now, we solve this problem using the dual approach. But first, let us define the dual problem and the dual function as

$$\min(\mu\geq 0,\lambda\geq 0)\quad g(\mu,\lambda) \quad (25)$$

where

$$g(\mu,\lambda)\triangleq \max_{P_1^i P_2^i \alpha_i}\mathcal{L} \quad (26)$$

From (24) and for a given set of α_i, the problem can be divided into N independent problems. Thus, we divide the dual function (the Lagrangian) into N dual functions (Lagrangian), such that g_i (L_i). For each sub-carrier i, for given λ and μ, and according to the value of α_i (which can take two values, 0 or 1), it appears that there are two cases:

• Case $\alpha_i = 1$. In this case, the relay is working on AAF for the i th sub-carrier. As we have in (23), only the terms related to the AAF approach. Hence, the dual function can be simplified as follows:

$$g(\mu,\lambda) = \max_{P_1^i P_2^i\geq 0}\mathcal{L}_i = \max_{P_1^i P_2^i\geq 0}\frac{1}{2}\log_2\left(1+\frac{P_1^i\gamma_1^i P_2^i\gamma_2^i}{P_1^i\gamma_1^i+P_2^i\gamma_2^i}\right)-\lambda P_2^i\Omega_i^*-\mu P_1^i\Omega_i^*$$

The maximum of $\mathcal{L}_i$ can be found by searching the partial derivative of Li subject to P^i_1 and P^i_2 which leads to

$$\frac{\partial\mathcal{L}_1}{\partial P_1^i}=\frac{(P_1^i)^2(\gamma_2^i)^2\gamma_1^i}{(P_2^i\gamma_2^i+P_1^i\gamma_1^i)(P_2^i\gamma_2^i+P_1^i\gamma_1^i+P_2^i\gamma_2^i P_1^i\gamma_1^i)}-\mu\Omega_1^* \quad (27)$$

$$\frac{\partial\mathcal{L}_1}{\partial P_2^i}=\frac{(P_2^i)^2(\gamma_2^i)^2\gamma_1^i}{(P_2^i\gamma_2^i+P_1^i\gamma_1^i)(P_2^i\gamma_2^i+P_1^i\gamma_1^i+P_2^i\gamma_2^i P_1^i\gamma_1^i)}-\lambda\Omega_i \quad (28)$$

We then equal both (27) and (28) to zero. The solution of these equations leads to $Pi*$ 1= $ciPi*$

$$P_2^{i*}=[\frac{\gamma_2^i}{\mu c_i\Omega_i^*(\gamma_2^i+c_i\gamma_1^i)}-\frac{1}{c_i\gamma_1^i}-\frac{1}{\gamma_2^i}]^+ \quad (29)$$

Case $\alpha_i = 0$. For this case, the relay switches to the DAF technique at the i th sub-carrier having $i \in D$. The problem of the DAF relaying has been solved in [8]. We just have to know the value of $Pi*$ 1 which can be obtained from the following relation $P^{i*}_2 = P^{i*}_1 H^i_{SR}/H^i_{RD}$. The solution is found to be given, in this case, by the following expression:

$$P_1^{i*}=[\frac{1}{\mu\Omega_1^i+\lambda\frac{H_{SR}^i}{H_{RD}^i}\Omega_2^i}-\frac{1}{\gamma_1^i}]^+ \quad (30)$$

By obtaining the optimal values of the transmitted powers P^{i*}_1 and P^{i*}_2 , the dual function is now a function of μ and λ. In the next subsection, we use an algorithm named sub-gradient algorithm [12] that proceeds to the search of the optimum values of μ and λ iteratively.

2.4.2 Sub-gradient method to solve the dual problem

With the obtained optimal values of primal variables (P^{i*}_1 and P^{i*}_2), the dual problem can be solved using the subgradient method [12-14]. In fact, our algorithm is based on the calculation of the Lagrangian multipliers λ and μ in each iteration. The decision about the type of relaying mode over each sub-carrier is made using (10). The implementation procedures is described in the power allocation algorithm depicted in Algorithm 1.

Algorithm 1 Power Allocation Algorithm

```
1:  Initialize λ = λ0 and μ = μ0
2:  for k = 1 to Iter_max do
3:     Compute c_i, P2^i and P1^i using (29), ∀ i
4:     Set α_i by the decision rule presented in (10)
5:     Compute P1^j and P2^j using (30), ∀ j
6:     Set α_j by the decision rule presented in (10)
7:     if α_i = α_j = 1 then
8:        Choose P1 and P2 according to Step-3
9:     else if α_i = α_j = 0 then
10:       Choose P1 and P2 according to Step-5
11:    else
12:       Choose α that maximize the capacity
13:    end if
```


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14: $\mu^{(k+1)} \leftarrow \mu^{(k)} - \delta^{(k)}(I_{th} - \sum_{i=1}^{N} P_1^i \Omega_i^*)$
15: $\lambda^{(k+1)} \leftarrow \lambda^{(k)} - \delta^{(k)}(I_{th} - \sum_{i=1}^{N} P_2^i \Omega_2^*)$
16: end for

The parameter $\delta^{(k)}$ appears in lines 14 and 15 of Algorithm 1, denoting the step size of the *k*th iteration. This algorithm is well described in [12-14], where many types of step size can be used in the sub-gradient algorithm. In our model, we tried different step sizes and then used the best one in terms of best performance and less complexity. In the proposed scheme, the optimal power requires (N^2) function evaluations for every sub-carrier to be matched in the second time slot. Therefore, the complexity of the proposed algorithm is $O(TN^2)$, where T is the number of iterations required for convergence. A comparison between the different schemes used in this paper is derived in Section 3.3.

2.5 Simulation results

The simulations are performed under the scenario given in Section 2.1. An OFDM system of N sub-carriers ($N \in \{16, 32, 64\}$) at the source and destination and one relay system is assumed. The values of T_s, Δf, and I_{th} are assumed to be 4 μs, 0.3125 MHz, and −20 dBm, respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1.

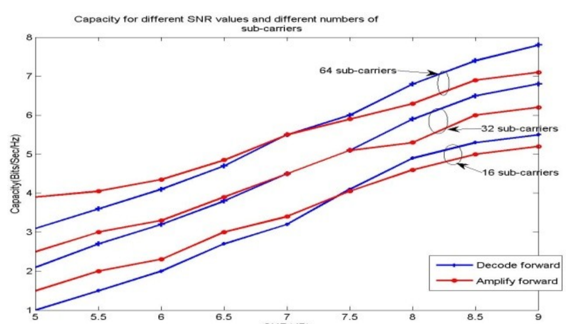

Fig. 3. Achieved capacity for different SNR, different number of sub-carriers,and $I_{th} = 10^{-5} W$

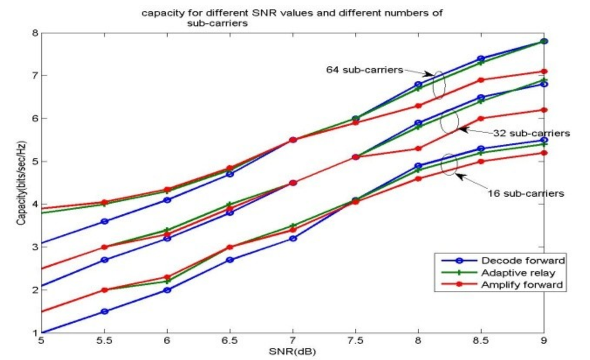

Fig. 4. Achieved capacity for different SNR, different number of sub-carriers,and $I_{th} = 10^{-5} W$

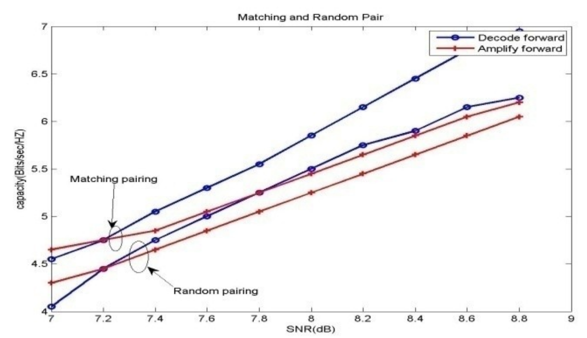

Fig.5. Achieved capacity for different SNR, $I_{th} = 10^{-5}$ W, 32 sub-carriers, and two types of sub-carrier pairing: sub-carrier matching and random

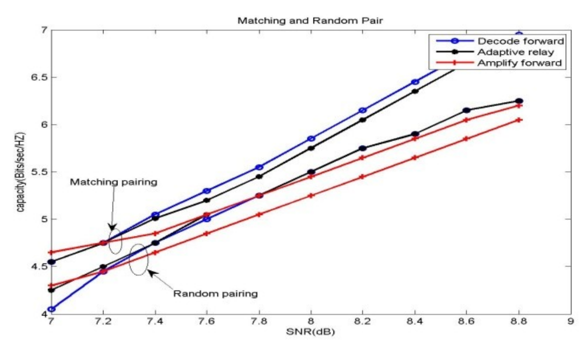

Fig.6. Achieved capacity for different SNR, $I_{th} = 10^{-5}$ W, 32 sub-carriers, and two types of sub-carrier pairing: sub-carrier matching and random

Figure 3 plots the average capacity using the different schemes (AAF, DAF) vs. the SNR and using different values of total number of sub-carriers with $N = \{16, 32, 64\}$. It is shown that for low values of SNR $(1/2\sigma^2 < 7)$ and for each value of N, the DAF relay decoding procedure is not perfect. Therefore, the AAF performs better than the DAF and provides higher capacity. However, at high SNR (> 7) values, the behavior of the system becomes inverse to the previous situation. Here, decoding can be done 'perfectly', and the propagation of errors due to the amplification in AAF process has more chances to occur. Thus, in this SNR region, the performance achieved by the DAF mode is higher than that achieved by the AAF.

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Figure 4 plots the average capacity using the different schemes (AAF, DAF, and ARP) vs. the SNR and using different values of total number of sub-carriers with $N = \{16, 32, 64\}$. It can be also shown that the ARP relaying protocol achieves, for the different depicted values of SNR, the best results. This can be explained by the fact that the ARP Protocol is able to switch (in an adaptive way) from one relaying mode to another (AAF or DAF) using in each moment the relaying mode that achieves the best performance. In other words, the ARP tends to use the AAF Relaying protocol for low values of SNR, and use the DAF for higher SNRs. Thus, ARP is able to take advantage of each relaying mode, depending on the SNR range. Figure 4 shows, finally, how the system capacity scales as function of the increase in the total number of carriers of the system.

Figure 5 and 6 shows capacity performance comparison using the matching and random pairing techniques for different values of SNR. In matching pairing technique, the same carrier k is used in both time slots (in S-R and R-D links). However, with random pairing technique, the assigned carrier in the second time slot will be randomly. It can be shown in this figure that higher capacity is achieved by matching carrier pairings than using the random assignment process of the carriers from S-R to R-D. It can be also observed that the ARP relaying technique achieves the best performances in both cases, matching and random pairing for different values of SNRs. We can conclude that using the matching pairing technique with ARP relaying strategy, higher capacity performance could be achieved for a wide range of SNR values.

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