Mathematical Modeling and Computer Simulation For Mechanical Timer Runway Escapement Mechanism

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***Abstract*— Aim of this paper is to develop realistic and workable computer program to simulate dynamic behavior of the runaway escapement. A theoretical analysis was conducted and the mathematical model of the motion of mechanical timer device consisting of the rotor (drive gear), three pairs of involutes gears and pallet was developed. Mathematical model, developed on the basis of the equations of the dynamics of rigid material system and impact mechanics, includes differential equations of motion for coupled motion, free motion, and phase of impact.**

***Index Terms*—** Clock mechanism,Escapement, Mathematical modeling ,Equation of motion, Differential equation.

Abbreviations and Acronyms

a - Distance between Pivot Points
b - Radius of Escape Wheel
c - Distance of Pin from centre of Pallet
r - Radius of Pin
α - Escape Wheel Tooth Half Angle
δ -Tooth Angle
ϕ - Angular Position of Escape wheel
φ -Angular Position of Pallet
$\ddot{\phi}$ - Angular acceleration of escape wheel
$\ddot{\varphi}$ - Angular velocity of pallet.
U - Pallet/escape wheel geometry relationship coefficient.
V - Pallet/escape wheel geometry relationship coefficient
I_p - Moment of inertia of Pallet
I_s - Moment t of inertia of Escape Wheel
G - Distance between the pin contact point to the tip of the ratchet tooth.

I. INTRODUCTION

The timer is designed in such a way that it gives a Variable time. The complete working mechanism of the timer ,the distinct feature of timer includes the pallet and ratchet arrangement. The required Variable time is obtained by this arrangement only. Mechanical timer consists of many components. Among these components the escapement is the most important. It determines the timekeeping accuracy ,and is often regarded to as the brain of the movement. It was the aim of this present investigation is to develop a realistic and workable computer simulation of the dynamic behavior of the pin pallet.

The energy required to drive the complete assembly is stored in a coil spring which in turn is rotated manually by externally provided profile on main shaft. The energy stored in the spring tries to move the whole assembly in reverse direction. The reverse motion of spring is restricted by the activating key which ,under normal circumstances remains aligned with the ratchet. For starting timer ,this key is moved out which makes the ratchet free to rotate.

For creating the time delay in the mechanism ,the first and the foremost requirement is producing a forced oscillating (damping) effect for dissipation of the energy in the system. System gets energy from clock spring which should be wounded after some interval to get constant energy out of the spring. This damping effect actually dissipates the spring energy. Thus the time required to relive the spring becomes far more as compared to that for loading, which can be advantageously used for a time delay after making certain required calculations.

As far as the timer is considered ,the oscillating motion of pallet produces this damping effect. The two pins provided on pallet engages with first and third teeth of ratchet alternately which causes an increment in energy loss rate, which gives us a specific time delay.

The last given aspect served to define the aim of this theoretical paper Their objective is to provide an educational tool for mechanical engineers or anyone who are interested to understand the working principle of the mechanical watch movement. In this paper, it consists of four different escapements: the Graham escapement, the English lever escapement, the Swiss lever escapement, and the Daniels co-axial double- wheel escapement. [1].MarkHeadrick's Horology Page. This is the homepage for the book "Clock and Watch Escapement Mechanics" authored by Headrick in 1997. The book showed how to draw an escapement step by step. Both

ISSN: 2278 – 7798

escapements of mechanical clocks and watches are covered. Its descriptions are so detailed that one can learn the geometry and the drawing techniques. Although it gives detailed explanation of the escapement, the animations on the website are rather crude. It shows only a simple wireframe model missing all the details. Hence, it is impractical to learn the details through these animations[3]. The design process for a spring driven runaway escapement having a flat sided pallet has been developed and a computer model to simulate its operation established .Simulation runs were in excellent agreement with experimental results[5].Technical Report ARAED-TR-94011**.**This investigation developed a computer simulation, which serves as a design tool for a complete safety and arming mechanism, which operates in a projectile that experiences spin, precession, and notation. This mechanism contains a straight-sided verge runaway escapement, a two-pass clock gear step-up train, and is powered by a spin-driven rotor. The mathematical model of the escapement recognizes three motion regimes, i.e., coupled motion, free motion, and impact. The model of the clock gear meshes differentiates between round-on-round and round-on-flat contact of the mating teeth. The use as a design tool is furnished by the fact that the computer program allows an infinite variation in verge and rotor mass properties as well as in component centre distances and clock tooth geometries with the circular arc and radial flank geometries. Similarly, the limits of operation of the mechanism when used in conjunction with a projectile having path logistic motion may be observed.

Runway escapement.

The simplest escapement use a non - resonat oscillating mass rather than an oscillating spring-mass as the basic timekeeper .A modern version of this type of escapement is shown in fig-1.The verge is the oscillating mass. It interferes with the rotation of the escape wheel as the latter attempts to rotate under the influenece of a mainspring-driven gear train .One tooth or pallet of the verge is always positioned to interfere with the motion of the escape wheel or "scape" wheel, as it is sometimes known, so the latter is never allowed to rotate freely.

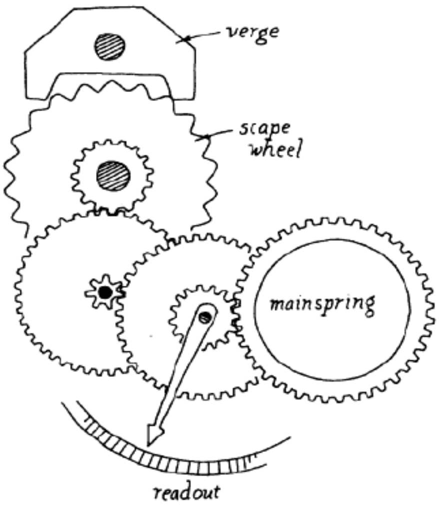


Fig.1-Complete timer mechanism showing drive spring, gear train and runway escapement

If the torque applied to the escape wheel is increases the mass will oscillate more rapidly and the wheel will rotate more rapidly ,since the verge is not a true oscillator. Hence the name "runway" escapement for this type a mechanism.Fig-2 shows atypical torque-speed curve for average escapement. This type of escapement incidentally, has a very high "beat rate". If sounds more like a bumblebee than a watch. The runway finds a great deal of application in such things as portable range timers, parking meters use in military products such as bomb fuses ,rockets. Because it is very rugged and economical. Furthermore, it is a self-starting escapement, which is necessary in many military situations. Also its characteristics are similar to those of the fluid damper and runway escapement is often used as a mechanical damper for speed control.

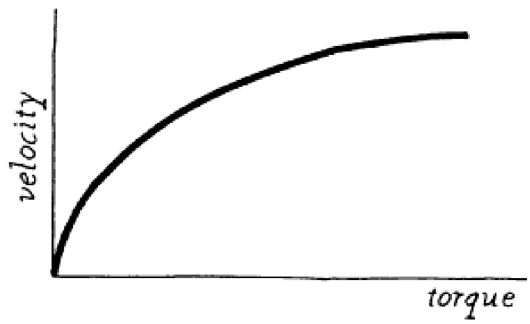
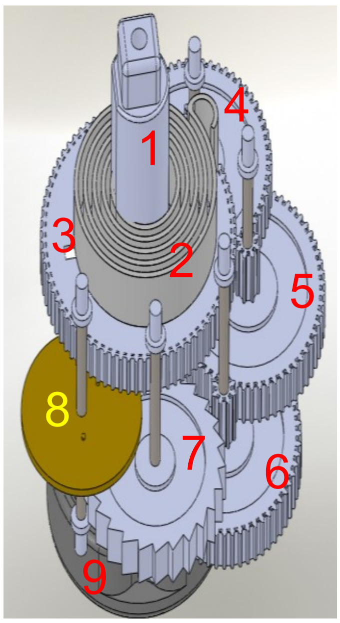


Fig 2- Torque-speed curve for a runway escapement.

II. PROPOSED DIGRAM

After the specification from respective agency 1st step is to draw the basic diagram of mechanical timer. First of all drawing regarding mechanical timer. Figure shows the components of the mechanical timer assembly in fig.3

The complete body of the timer can be basically divided in 3 parts.viz Top plate contains main central shaft, middle portion containing all moving parts (gears, ratchet, spring and pallet) and lastly the base plate contains electronic switches.



1.Main Shaft
2.Torsional Spring
3.Gear – 1
4.Gear – 2
5.Gear – 3
6.Gear – 4
7.Escape Wheel
8.Pallet
9.Lower Cam

Fig .3 Gear Assembly

ISSN: 2278 – 7798

III Working Principal

The timer is designed for giving time and motion to the respective cam for variable time. The basic requirement for timer is that the loading and unloading of timer should be from the central shaft. Hence, the kinematic chain has been designed with a torsion spring on the central shaft. As all arrangements are mounted on the central shaft.

The main design challenge is that, the torsion spring is mounted on the central shaft and when it is fully wound it has a tendency to rotate (unwound) very fast. To restrict its speed gears and an energy dispenser (i.e. pallet arrangement) have been added for constant torque reduction. It is possible to compensate for energy loss in a damped system by applying an external force that does positive work on the system. At any instant, energy can be put into the system by an applied force that acts in the direction of motion of the oscillator. The amplitude of motion remains constant if the energy input per cycle exactly equals the energy lost as a result of damping. Any motion of this type is called forced oscillation.

These force oscillations are responsible for regulating time in the mechanical timer. In this case the external energy like push force in above example is provided by the torsion spring wounded on central shaft. This torsion spring is a clock spring which provides constant torque for first 360° of winding. This spring gives energy to the central gear from where it is transferred to escapement wheel(ratchet) which rotates and forces oscillation of the pallet.

This arrangement is responsible for getting accurate time as an output by restricting the free motion of escapement wheel.

***Working of Central Gear and Gear Train*:**

The central shaft is fitted with a gear which transfers rotational motion initiated by the unwinding of the torsion spring. This torque is transferred from gear to gear through specified gear ratio. This gear ratio is selected for reduction of torque from the first to the last gear. This torque reduction is helpful for regulated energy dispense by the pallet arrangement.

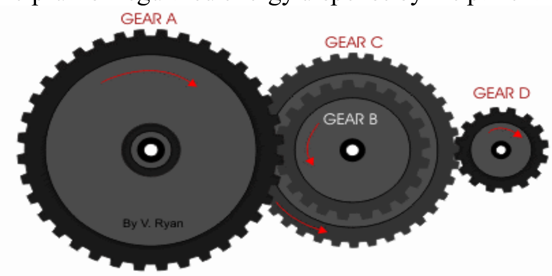


Fig.0- Gear Train Arrangement

***Working of Ratchet and Pallet Arrangement*:**

The torque transfer to ratchet through gear train produces rotational motion about the axis of the ratchet. This ratchet is attached to the pallet through two pins of pallet which follows the teeth trajectory of the ratchet. This pallet acts as an energy dispenser which regulates free motion of the ratchet, which in return offers rotational resistance to gears attached to the ratchet and other gear train. This combination in return gives restricted motion to the central shaft or it results in slowly unwinding of the torsion spring irrespective of its wounding speed.

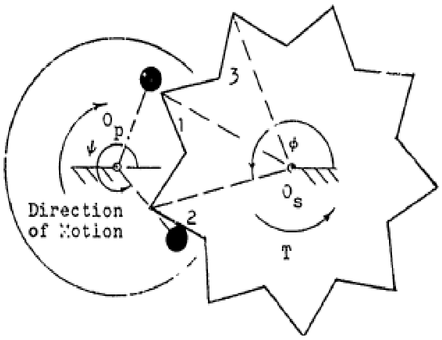


Fig.5 Ratchet and Pallet Arrangement**.**

IV Timer Kinematic & Dynamic Analysis

The objective of the kinematic & dynamic analysis is to develop a realistic and workable computer simulation of the dynamic behavior of the pin pallet runaway escapement which can be used in the analysis and synthesis of various safety and arming devices. The report describes the mathematical modeling of various regimes of motion which form the basis for the simulation. The simulation is applied to the time delay mechanism of the arming device and the influence of various parameter changes is explored.

Mathematical Model

The mathematical modeling consists of:

I. Coupled Motion of Escape Wheel & Pallet.

II. Free Motion of Escape Wheel & Pallet.

III. Impact of Pallet on Escape Wheel.

Coupled Motion

The present effort on the pin pallet escapement represents extension of the work of M. Anderson and S. L. Redmond New methods of contact kinematics for coupled motion, of contact sensing and of computational controls are developed. The following outlines the overall assumptions and derivations forth variou:3 regimes of motion. A detailed description of the actual-computer program and its controls is then given. Coupled motion, where the escape wheel is in constant contact with one of the pallet surfaces while It is being driven.

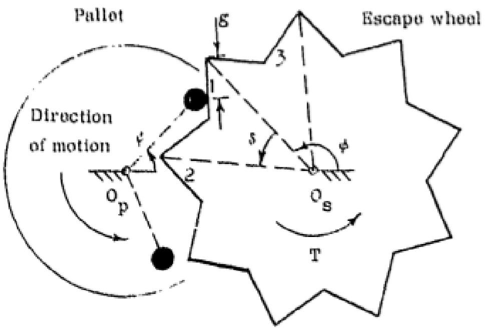


Fig.0- Coupled Motion

Fig. 6.1- shows a schematic representation of the pin pallet escapement indicates its basic geometric nomenclature.

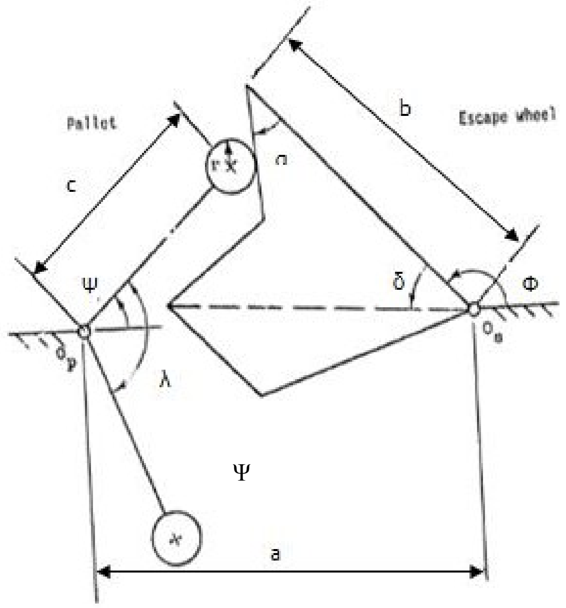


Fig.0.1- Escapement Nomenclature

Fig. 6. shows the upper pallet pin being driven in coupled motion by tooth no.1 of the escape wheel (which will be referred as the top contact). The escape wheel angle (Φ) is defined by the line from the escape-wheel pivot 'O_s' to the tip of the contacting tooth(or the one about to make contact) and the line connecting 'O_s' to the pallet pivot 'O_p'.
The quantity 'g', which represents the distance from the contact point to the tip of the escape-wheel tooth, is used to determine the end of coupled motion. Following is the derivation of differential equation of coupled motion in terms of the escape wheel angle(Φ) and kinematic quantities associated with coupled motion.

Equation of Motion

Differential Equation of Coupled Motion

Escape wheel $\ddot{\phi} + W\dot{\phi}^2 = Y$

Pallet $\ddot{\varphi} = U\ddot{\phi} + V\dot{\phi}^2$

Where,

$$W = \frac{\frac{I_p V}{E}}{\frac{I_p U}{E_1} + \frac{I_s}{F_1}}$$

$$Y = \frac{\frac{TORK}{F_1}}{\frac{I_p U}{E_1} + \frac{I_s}{F_1}}$$

$$\dot{g} = \frac{aP}{S}\dot{\varphi}$$

$$P = b\sin(\varphi) + g\sin(\varphi - \alpha) + r\cos(\varphi - \alpha)$$

$$S = g + b\cos(\alpha) + a\cos(\varphi - \alpha)$$

$$Q = b\cos(\varphi) + g\cos(\varphi - \alpha) - r\sin(\varphi - \alpha)$$

$$A_1 = b\cos(\alpha) + g$$

$$B_1 = b\sin(\alpha)$$

$$C_1 = -[r + c\sin(\varphi - \alpha - \psi]$$

$$D_1 = c\cos(\varphi - \alpha - \psi)$$

$$U = \frac{Q + \frac{aP}{S}\sin(\varphi - \alpha)}{c\cos(\varphi)}$$

$$E_1 = D_1 - \mu\, C_1 \frac{\dot{g}}{|\dot{g}|}$$

$$F_1 = A_1 + \mu\, B_V \frac{\dot{g}}{|\dot{g}|}$$

$$V = \frac{1}{c\cos(\varphi)}[\frac{2aP}{S}\cos(\varphi - \alpha) - P]$$
$$+ \frac{1}{c\cos(\varphi)}[\frac{2a^2P}{S^2}\sin^2(\varphi - \alpha)]$$
$$+ \frac{1}{c\cos(\varphi)}[\frac{aQ}{S}\sin(\varphi - \alpha)]$$
$$- \frac{1}{c\cos(\varphi)}[\frac{a^2P^2}{S3}\sin(\varphi - \alpha)]$$
$$+ \frac{\tan(\psi)}{c^2\cos^2(\psi)}[Q + \frac{Pa}{S}\sin(\varphi - \alpha)]^2$$

Free Motion

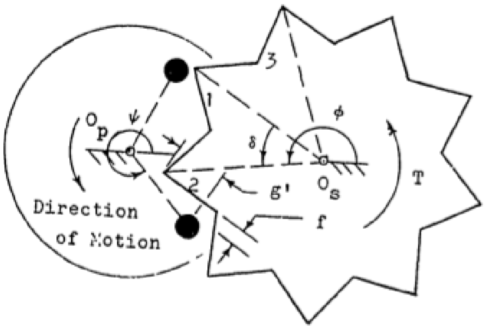


Fig. 7- Free Motion

When coupled motion is finished (i.e, g = 0) or when separation of contact occurs after impact, the escape wheel and pallet move independently of each other in free motion. Fig.7 shows this free motion for bottom phase of the action.

Differential Equation of Free Motion

Escape Wheel $\ddot{\phi} = \dfrac{TORK}{I_s}$

Pallet $\ddot{\psi} = 0$

Impact Motion

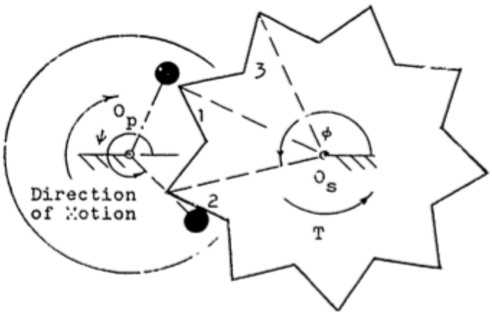


Fig 8- Impact Motion

Impact follows free motion whenever 'f' equals zero, g' is smaller than zero, and the relative velocity between the contacting surfaces warrants it.

Fig. 8.1 shows impending motion for the subsequent top phase of the motion, i.e. the top pallet pin is about to make contact with tooth no. 3 of the escape wheel. All kinematic and dynamic expressions are derived in such a manner that they are valid for top and bottom action

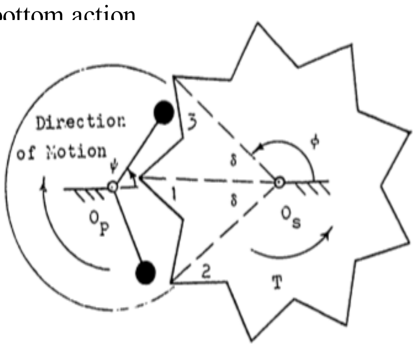


Fig. 8.1 Impending Top Contact

Impact Equations

Normal Velocities at the impact point

Escape Wheel $V_S = \dot{\varphi}(b\cos(\alpha) + g)$

Pallet $V_P = \dot{\psi}\, c\cos(\varphi - \alpha - \psi)$

Based on above equations and requirement from costumer the pallet has been designed with brass material for time variable. With these inputs, simulation results, as brought out in subsequent paragraph V were obtained.

V SIMULATION RESUITS

- ***Input Data***:-

The following table lists the input conditions considered for the simulation of timer.

S.No	Parameter	Value	Unit
1	Angular Twist	330	Deg
2	Mass of Torsion Spring	1.52	g
3	Mass of Main Shaft	0.55	g
4	Mass of Ratchet Spring	0.11	g
5	Mass of Gear-1	0.46	g
6	Mass of Ratchet-1	0.22	g
7	Mass of Last Gear	0.34	g
8	Mass of Middle CAM	0.95	g
9	Radius of Last Gear	6.5	mm
10	Radius of Pin	0.2	mm
11	Radius of Pallet	6	mm
12	No. of Tooth – N1	76	
13	No. of Tooth – N1p	8	
14	No. of Tooth – N2	60	
15	No. of Tooth – N2p	8	
16	No. of Tooth – N3	60	
17	No. of Tooth – N3p	8	
18	No. of Tooth – N4	60	
19	No. of Tooth – N4p	8	
20	No. of Tooth – N5	25	
21	Angle between Pin-1 & Pin-2	136.07	Deg
22	Tooth Angle of Last Gear	14.4	Deg
23	Escape Wheel Tooth Half Angle	56.1	Deg
24	Distance between Pivot Points of Last Gear & Pallet	7.58	Mm
25	Radius of Escape Wheel	7.04	mm
26	Distance of top pin from center of pallet	2.59	mm
27	Distance of bottom pin from center of pallet	2.08	mm
28	Radius of Last Gear	6.5	mm
29	Coefficient of Friction	0.4	
30	Coefficient of Restitution	0.65	
31	Gear Efficiency	0.65	
32	Input Torque	0.019	N-m

Table .1.Lists of Input Parameter

- ***Torque & Bending Moment Calculations***

The following table gives the bending moment calculation for the gear shafts.

Diameter of Gear Shaft (m)	1.00E-03
Length of the Shaft (m)	1.25E-02
Diameter of Insert (m)	5.00E-04
Weight of the Gear (kg)	4.00E-04
Radius of the Gear (m)	7.00E-03
Bending Moment (N-m)	2.45E-05

Table -2 Torque & Bending Moment Values**.**

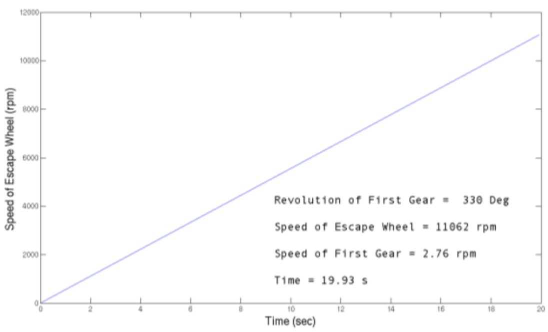
Sr. No	Comp-onent	Torque	Equivalent Twisting Moment	Shear Stress	Normal Force	Resisting Torque
		(N-m)	(N-m)	(Mpa)	(N)	(N-M)
1	Torsion Spring	1.90E-02				
2	Gear 1	1.88E-02	2.26E-02	1.15E+02	9.77E-01	4.89E-04
3	Gear 2	1.93E-03	2.31E-03	1.18E+01	1.00E-01	5.01E-05
4	Gear 3	1.98E-04	2.41E-04	1.23E+00	1.03E-02	5.14E-06
5	Gear 4	2.03E-05	5.04E-05	2.57E-01	1.05E-03	5.27E-07
6	Gear 5	2.08E-06	4.42E-05	2.25E-01	1.08E-04	5.40E-08

Table -3 Result for Torque & Bending Moment Calculation

From the moment & torque calculations, equivalent twisting moment is calculated and the shear stress is predicted. The above
table also list the various torques and resisting torques at each gear assembly.

Case 1: Timer without Pin Pallet Escapement

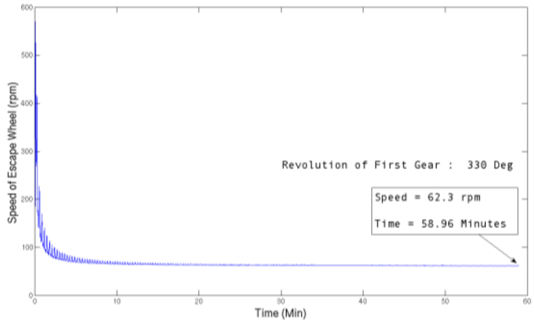
As shown in Graph 1, without pin pallet escapement, the timer takes 19.93 seconds to complete 330 degree rotation of the first gear. The speed of the escape wheel is in the order of 11000



rpm.

Graph.1- Time Vs Speed of Escape Wheel (without Pin Pallet Engagement)

Case 2: Timer with Pin Pallet Escapement



Graph 2- Time Vs Speed of Escape Wheel (with Pin Pallet Engagement)

With pin pallet engagement, the speed of the escape wheel reduces and reaches to a steady state as shown in Graph 2 The steady state rpm is around 62. The timer takes 60 minutes to complete 330 degree rotation of the first gear.

VI Conclusion

The mathematical model of the mechanism for mechanical timer with runway escapement was carried out as the first step to establish the tools for the computer simulation and optimization of the function of mechanical timer.
The mathematical model consist of differential equation of motion that are expressed depending on kinematics, geometrical and physical parameter of each component.

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