Inverse Domination For Intuitionistic fuzzy Graphs

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Abstract - Let G be an intuitionistic fuzzy graph. Let D be a minimum fuzzy dominating set of G.We define inverse dominating set for intuitionistic fuzzy graphs. If V-D contains dominating set $D^{'}$ of G then $D^{'}$ is called an inverse dominating set of G with respect to D. In this paper we also define inverse dominating set, , inverse split dominating set, inverse non split dominating set and some properties for intuitionistic fuzzy graphs

Index Terms— Dominating set, intuitionistic fuzzy graph ,Inverse dominating set,inverse split and non split dominating set.

I INTRODUCTION

Kafmann introduced definition of fuzzy graphs.Rosenfeld introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theortic concepts such as paths,cycles,connectedness etc.The concept of domination in fuzzy graphs was investigated by A.Somasundaram and S. Somasundaram and A.Somasundaram presented the concepts of independent domination,total domination,connected domination of fuzzy graphs. The first definition of intuitionistic fuzzy graph was proposed by Atanssov.The concepts of domination in intuitionistic fuzzy graphs was investigated by R.Parvathi and G.Thamizhendhi .In this paper we develop the inverse dominating set for IFG

II. BASIC DEFINITIONS

Definition: 2.1

An intuitionistic fuzzy graph (IFG) is of the formG=(v,E), where $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$ and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, (i = 1,2, \ldots n)$

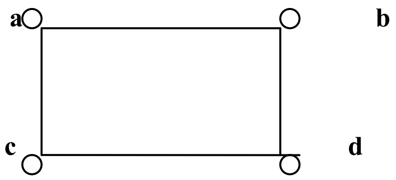
$ii) E \subseteq V \times V$ *where* $\mu_2 : V \times V \to [0,1]$ *and* $\gamma_2 : V \times V \to [0,1]$ *are suchthat*

$\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j)$,

$\gamma_2(v_i, v_j) \le \gamma_1(v_i) \wedge \gamma_1(v_j)$

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$

Example:



$$(\mu_1, \gamma_1)(a) = (0.7, 0.2), (\mu_1, \gamma_1)(b) = (0.8, 0.1)$$
$$(\mu_1, \gamma_1)(c) = (0.5, 0.4), (\mu_1, \gamma_1)(d) = (0.9, 0.1)$$
$$(\mu_2, \gamma_2)(a, b) = (0.7, 0.1), (\mu_2, \gamma_2)(b, d) = (0.8, 0.1)$$
$$(\mu_2, \gamma_2)(a, c) = (0.5, 0.2), (\mu_2, \gamma_2)(c, d) = (0.8, 0.1)$$

Definition:2.2

An arc $(v_{i,}, v_j)$ of an IFG G is called an strong arc if

$$\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j) \;, \gamma_2(v_i, v_j) \le \gamma_1(v_i) \wedge \gamma_1(v_j)$$

Definition 2.3

Let G =(V,E) be an IFG. Then the cardinality of G is defined to be

$$|G| = \left\{ \sum_{v_i \in V} \left[\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] + \sum_{v_i \in V} \left[\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right] \right\}$$

Definition:2.4

Let G=(V,E) be an IFG.The vertex cardinality of G is defined by

$$|V| = \left\{ \sum_{v_i \in V} \left[\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] \right\} \textit{for all}\, v_i \in V$$

The edge cardinality of G is defined by

$$|E| = \left\{ \sum_{v_i \in V} \left[\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right] \right\} \textit{ for all} (v_i, v_j) \in E$$

The vertex cardinality of an IFG is called the order of G and it is denoted by O(G).The cardinality of the edges in G is called the size of G, it is denoted by S(G).

III.INVERSE DOMINATING SET FOR IFG

Let S be a minimum dominating set of a IFG G.If V-S contains a dominating set $S^{'}$ of G then $S^{'}$ is called an inverse dominating set of G with respect to S.The minimum fuzzy cardinality taken over all inverse dominating sets of G is called the inverse domination number of IFG G and is denoted by $\gamma^{'}(G)$

Manuscript received july9,2014

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ISSN: 2278 – 7798

Remark:

i) For any complete IFG G, $K_\sigma, \gamma'(K_\sigma) \leq \gamma'(K_\sigma - v)$

ii) For any IFG G with atleast one inverse dominating set , $\gamma(G) \leq \gamma'(G)$

Theorem:3.1

For any IFG G with γ set S, $\gamma(G) + \gamma'(G) \leq p.$ Further, equality holds if V-S is independent and contains inverse dominating set S' with respect to S.

Proof:

Let S be a γ set of G .If S' is an inverse dominating set of G with respect to S then $S' \subseteq V - S.$

Therefore $|S'| \leq |V - S|$

Hence $\gamma'(G) \leq p - \gamma(G)$

Since V-S is independent and contains an inverse dominating set say S' with respect to S. Therefore V-S itself is inverse dominating set of the IFG G.Hence the proof

Corollary:

If G or $\overline{G}$ contains atleast one isolated vertex, then $\gamma'(G) + \gamma'(\overline{G}) \leq p.$

Theorem:3.2

For any IFG G with atleast one isolated vertex, $\gamma'(G) = 0$

Proof:

Let S be a γ set of G and $u \in S$ be an isolated vertex.Then $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $v \in V - S$

Then $\gamma'(G) = 0$

Theorem:3.3

For any IFG G $\gamma'(G) \leq \Gamma(G)$

Proof:

Let S be a γ set of G .We prove three cases:

Case i):V-S contains no dominating set. Then $\gamma(G) = \Gamma(G)$ *and* $\gamma'(G) = 0$

Case ii) V-S contains only one dominating set. This implies that $\gamma'(G) = \Gamma(G)$

Case iii) V-S contains atleast two dominating sets. Then minimum dominating set in V-S with minimum fuzzy cardinality is $\gamma'(G)$. *Therefore* $\gamma'(G) \leq \Gamma(G)$

Theorem:3.4

Let P be a path in a IFG G then $\gamma'(P) = \Gamma(P)$

Proof:

Since P contains only two dominating sets in G, the proof follows.

Theorem:3.5

For any IFG G $\gamma'(G) \leq \beta_0(G)$

Proof:

Let D be a γ set of G. Let S be a maximal independent set of $\langle V - D \rangle$. Then every vertex in V-D-S is adjacent to atleast one vertex in S. If every vertex in D is adjacent to atleast one vertex in S then S is an inverse dominating set. Otherwise, let $D' \subseteq D$ be a set of vertices in D such that no vertex in D' is adjacent the vertices of S. Since D is a minimum dominating set every vertex in D' must be adjacent to atleast one vertex in V-D-S. Let $S' \subset V - D - S,$ such that every vertex of D' is adjacent to atleast one vertex in S'.

Then there exists atleast one vertex $v \in S'$ such that both $N(v) \cap S \neq \phi$ *and* $N(v) \cap S \neq \phi$.

Therefore $(S \cup S' - (N(v) \cap S)$ is an inverse dominating set of G and

$|(S \cup S' - (N(v) \cap S)| \leq \beta_0(G)$

Hence $\gamma'(G) \leq \beta_0(G)$

Theorem:3.6

For any IFG G with atleast one inverse dominating set,

$$\gamma(G) \leq \frac{(p + \gamma'(G))}{3}$$

Proof:

For any IFG G with atleast one inverse dominating set, $\gamma(G) \leq \gamma'(G)$

Also $\gamma(G) \leq \frac{p}{2}$

Hence the result

Theorem:3.7

For any IFG G $= (\sigma, \mu), \gamma'(G) < p$

Proof:

We know that any IFG contains atleast one $\gamma - set$ *with* $\gamma(G) > 0$

$\gamma(G) + \gamma'(G) \leq p.$ *Thus* $\gamma'(G) < p$

Theorem:3.8

An inverse dominating set S of G is a minimal inverse dominating set iff for each $d \in S'$ one of the following conditions holds.

1.N(d) $\cap S' = \phi$

2.There is a vertex $c \in V - S'$ such that $N(c) \cap S' = \{d\}$

Proof:

Let S' be a minimal inverse dominating set and $d \in S'$.Then $S'_d = S' - d$ is not a dominating set and hence there exists $x \in V - S'_d$ such that x is not dominated by any element of S'_d

If x=d we get condition (1) and $x \neq d$ we get condition(2). Condition (2) is obvious

Theorem:3.9

If every nonend vertex of an IF tree T is adjacent to atleast one end vertex, then $\gamma(T) + \gamma'(T) = p$

Proof:

Suppose every non end vertex of an IF tree T is adjacent to atleast two end vertex.

Then the set of non end vertices S is the only minimum dominating set in IF tree T and the set of end vertices V-S is

ISSN: 2278 – 7798

the corresponding inverse dominating set in T.Thus
$\gamma(T)+\gamma^{'}(T)=|S|+|V-S|=p$
Suppose there are nonend vertices which are adjacent to exactly one end vertex. Let S and $S^{'}$ denote the minimum dominating and inverse dominating sets respectively. Let u be a nonend vertex adjacent to exactly one end vertex. Clearly if $u \in S\, and\, v \in S^{'}$ and if $v \in S\, and\, u \in S^{'}$. In any case $S+S^{'}=p$
Thus $\gamma(T)+\gamma^{'}(T)=p$

IV.Inverse split and nonsplit domination in IFG graphs

Definition 4.1
Let $D^{'}$ be a minimum inverse dominating set of IFG G with respect to D. Then $D^{'}$ an inverse split dominating set of G if the induced subgraph $\langle V-D^{'}\rangle$ is disconnected.

The inverse split domination number is denoted by $\gamma_s^{'}(G)$ and it is the minimum cardinality taken over all minimal inverse split dominating sets of G.

Definition4.2
Let $D^{'}$ be a minimum inverse dominating set of IFG G with respect to D. Then $D^{'}$ an inverse non split dominating set of G if the induced subgraph $\langle V-D^{'}\rangle$ is connected.

The inverse split domination number is denoted by $\gamma_{ns}^{'}(G)$ and it is the minimum cardinality taken over all minimal inverse non split dominating sets of G.

Result:
For any complete IFG K_n with $n \geq 2$ vertices
$\gamma_s^{'}(K_n)=0,\ \gamma_{ns}^{'}(K_n)\leq 1$

Theorem:4.3
For any IFG G $\gamma^{'}(G)\leq \gamma_s^{'}(G)\ and\ \gamma^{'}(G)\leq \gamma_{ns}^{'}(G)$

Proof:
Since every inverse split dominating set of G is an inverse dominating set of G.we have $\gamma^{'}(G)\leq \gamma_s^{'}(G)$.
Similarly, every inverse non split dominating set of G is an inverse dominating set of G,we have $\gamma^{'}(G)\leq \gamma_{ns}^{'}(G)$

Theorem:4.4
For any IFG G $\gamma^{'}(G)\leq \min\{\gamma_s^{'}(G)\ ,\gamma_{ns}^{'}(G)\}$

Proof:
Since every inverse split dominating set and every nonsplit dominating set of G are the inverse non split dominating sets of G,we have $\gamma^{'}(G)\leq \gamma_s^{'}(G)\ and\ \gamma^{'}(G)\leq \gamma_{ns}^{'}(G)$.
Hence $\gamma^{'}(G)\leq \min\{\gamma_s^{'}(G)\ ,\gamma_{ns}^{'}(G)\}$

Theorem:4.5
Let T be a IF tree such that any two adjacent cut vertices u and v with atleast one of u and v is adjacent to an end vertex then then $\gamma^{'}(T)=\gamma_s^{'}(T)$

Proof:
Let $D^{'}$ be a $\gamma^{'}$ set of T, then we consider the following two cases.
Casei) Suppose that atleast one of $u,v \in D^{'}\ then \langle V-D^{'}\rangle$ is disconnected with atleast one vertex. Hence $D^{'}$ is a $\gamma_s^{'}$ set of T. Thus the theorem is true.
Caseii) Suppose $u,v \in V-D^{'}$ since there exists an end vertex w adjacent to either u or v say u. This implies that $w \in D^{'}$. Thus it follows that $D^{''}=\{w\}\cup\{u\}$ is of $\gamma^{'}-set$ of T. Hence by casei) the the theorem is true.

Theorem:4.6
For any IF tree $\gamma_{ns}^{'}(T)\leq n-p$ where p is the number of vertices adjacent to end vertices

Theorem:4.7
For any IFG G
$\gamma_{ns}^{'}(G)\leq n-\delta(G)\ where\ \delta(G)\ is\ the$ minimum degree among the vertices of G.

Remark:
1.For any IF tree , $\delta(T)\leq 1$
2.If H is any connected IF spanning subgraph of G then $\gamma^{'}(G)\leq \gamma^{'}(H)$

Theorem:4.7
Let G be a graph which is not a cycle with atleast 5 vertices.Let H be a connected spanning subgraph of G then i) $\gamma_s^{'}(G)\leq \gamma_s^{'}(H)$ ii) $\gamma_{ns}^{'}(G)\leq \gamma_{ns}^{'}(H)$

Proof:
Since G is connected then any spanning tree T of G is minimally connected subgraph G such that
$\gamma_s^{'}(G)\leq \gamma_s^{'}(T)\leq \gamma_s^{'}(H)$
Similarly $\gamma_{ns}^{'}(G)\leq \gamma_{ns}^{'}(T)\leq \gamma_{ns}^{'}(H)$
Hence the proof

Theorem:4.8
If T is a IF tree which is not a star then
$\gamma_{ns}^{'}(T)\leq n-2\ for\, all\, n\geq 3$

Proof:
Since T is not a star,there exists two adjacent cut vertices u and v with degree u and degree v ≥ 2.This implies that $V-\{u,v\}$ is an inverse nonsplit dominating set of T.Hence the theorem.

Theorem:4.9
An inverse nonsplit dominating set $D^{'}$ of IFG G is minimal iff for each vertex $v \in D^{'}$ one of the following conditions is satisfied.
i)There exists a vertex $u \in V-D^{'}$ such that $N(u)\cap D=\{v\}$
ii)v is not an isolated vertex in $\langle D^{'}\rangle$
iii)u is not an isolated vertex in $\langle V-D^{'}\rangle$

Proof:
Suppose $D^{'}$ is a minimal inverse non split dominating set of G.Suppose the contrary.That is, if there exists a vertex

ISSN: 2278 – 7798

$v \in D^{'}$ such that v does not satisfy any of the given conditions, then there exists an inverse dominating set $D^{''} = D^{'} - \{v\}$ such that the induced subgraph $\langle V - D^{''} \rangle$ is connected. This implies that $D^{''}$ is an inverse nonsplit dominating set of G contradicting the minimality of $D^{'}$. Therefore the conditions is necessary. Sufficiency follows from the given conditions.

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ISSN: 2278 – 7798