Experimental Investigation of Shafts on Whirling of Shaft Apparatus

Mr. Balasaheb Keshav Takle

***Abstract*— Present paper deals with study and analysis of theoretical and practical frequencies of different shaft diameters. The design of various components of experiment is discussed in detail. Design validation of experimental values is done by comparison with theoretical outputs. Whirling is usually associated with fast-rotating shafts. Frequencies at different speeds for various shaft diameters are evaluated and compared. A relation between natural frequencies and shaft diameters is established using graphical approach. The governing equations of both theoretical & actual natural frequencies are deduced. Effect of self weight of shaft and its effect on natural frequency is also discussed.**

***Index Terms*—Whirling, Shafts, Natural Frequency, Design Validation, Mass on shaft, self-weight**

Symbols

M= Mass (kg)
E = Young's Modulus (N/mm^2)
N_C = critical speed (rev/s)
I = second Moment of Area (m^4)
G = acceleration due to gravity (m/s^2)
y = deflection from δ (m)
O = centroid location
G = Centre of Gravity
L = Length of shaft (m)
δ = static deflection (m)
ω = angular velocity of shaft (rad/s)

I. Introduction

All rotating shafts, even in the absence of external load, deflect during rotation due to self weight. The combined weight of a shaft and shaft-mountings can cause deflection that will create resonant vibration at some speed. These speeds are commonly known as critical speed. Shaft deflection depends upon the followings:-

a) Material Stiffness and number of supports
b) Self Weight and mountings
c) Unbalanced centrifugal forces
d) Lubricant viscosity

Therefore, calculation of critical speed for any shaft is necessary in order to avoid the resonance. There are two methods used to calculate critical speed, Rayleigh-Ritz and Dunkerley's method. Both these Ritz and Dunkerley's equations are an approximations to the first natural frequency of vibration, which is assumed to be the first natural frequency of vibration, which is assumed to be nearly equal to critical speed of rotation.

In general Ritz and Dunkerley's equation overestimates and the Dunkerley's equation underestimates the natural frequency. The equation illustrated below is the Ritz and Dunkerley's equation, good practice suggests that the maximum operation speed should not exceed 75% of the critical speed. The critical speed equation is demonstrated in equation (1) below;

$$N_C = \frac{30}{\pi}\sqrt{g/\delta_{st}} \qquad (1)$$

Discontinuities are unavoidable, due to assembly, manufacturing and application considerations, which all ensure that the centre of gravity of the shaft cannot coincide with the axis of rotation. The centrifugal forces in such rotating shafts were first cited by Dunkerley. Elastic properties of the shaft material were the root cause of the restoring force which affected its natural frequency and critical speed was also cited by Dunkerley. Further research shows that the speed at which the shaft would suffer an infinite deflection i.e. whirl can also be evaluated using the above approach.

When the speed of rotation is increased the centrifugal force also increases and so does the restoring force. At low critical speeds, there is increase the restoring forces with the increase in shaft deflection. Shaft deflection is unchecked and the shaft behaves as a flexible element. Above the critical speeds, the shaft rotates about the centre of mass of the assembly.

1.1 Shaft Carrying a Mass with Eccentric Centre of Gravity.

If we examine the simplest case of a single, heavy rotor rigidly attached to a light (inertia less) spindle, then the physical situation can be expressed in Figure 1.

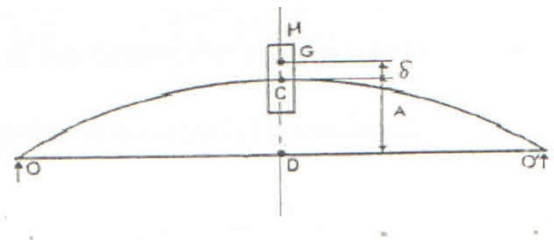

Figure 1. Whirling of shaft

The system consists of a disc of mass M located on a shaft simply supported by two bearings. The centre of gravity G of the disc is at a radial distance δ from the geometric centre, C. the centerline of the bearings OO' intersect the plane of the disc at D, at which point the disc centre C is deflected a distance A.

The centre of gravity G thus revolves around point D, describing a circle radius ($A+\delta$) and the centrifugal reaction thus produced is; $M\omega$ ($A+\delta$) for any given speed ω. This force, according to Dunkerley's, is balanced by the elastic restoring force of the shaft at point D equal to KA where K is stiffness. We further get

$M\omega^2 (A+\delta) = KA$

Then,

$$A = \frac{M\omega^2\delta}{K - M\omega^2} \qquad (2)$$

This equation will become infinite when $K - M\omega^2 = 0$ or

$$\omega^2 = \frac{K}{M} \qquad (3)$$

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Therefore, if we denote the critical whirling speed by

$$\omega_c \frac{K^{0.5}}{M} \tag{4}$$

and using in (1), we get:

$$A = \frac{\omega^2 \delta}{\omega_c^2 - \omega^2} \delta \tag{5}$$

Therefore, at $\omega < \omega_c$ then A and δ have the same sign i.e. the centre of gravity G is situated as shown in fig,2. At $\omega < \omega c$ the deflection of A becomes infinite as described above. At $\omega > \omega c$ A and δ are of opposite signs and hence the centre of gravity now lies between C and D, inferring that the disc has rotated through 180^0 from its rest position. For very high speeds where $\omega >> \omega c$ the amplitude A tend to $-\delta$, hence the disc rotates about G with perfect stability.

If equation 2 is compared with the equation of motion for a single load W, undergoing a simple harmonic motion, similar motion is observed. Analysis of the problem shows that at the whirling speed, A, the radius of the shaft rotation about bearing centerline, and δ, the radius of G from the geometric centre of the disc, are perpendicular which is analogous to the resonant conditions which exist for a forced vibration where the distributing force vector is 90 degrees in advance of the displacement vector.
Dunkerley's deduced that the whirling speeds were equal to the natural frequencies of transverse vibration, there being the same number of whirling speeds as natural frequencies for a given system. Thus a theoretical value for the critical speed may be obtained from the formula for the fundamental frequency of transverse vibrations:

$$F = \left(\frac{EIg}{WL^4}\right)^{0.5} C \tag{6}$$

Where f = natural frequency of transverse vibration (Hz)
E = Young's Modulus
I = second moment of area of shaft
W = weight per unit length of shaft
g = acceleration due to gravity
C = constant dependent upon the end conditions.
For various end conditions; the values are shown in Table 1

Table 1 Values of Constants for different end conditions

Case	Ends	C_1	C_2
1	Free -free	1.572	3
2	Fixed	3.75	8.82
3	Cantilever	0.56	-
4	Fixed-free	2.459	7.96

The value of C_1 is the constant for use in calculating the first natural frequency and C_2 is that necessary for the second mode. For rotating shaft there is a speed at which, for any small initial deflection, the centripetal force is equal to re-instable force. At this point the deflection increases such that shaft is said to "whirl". Below and above this speed this effect is very much reduced. This critical (whirling speed) is depends on the shaft diameter, material and different loads which act on it. The critical speed N_c of a shaft is given by equation (7)

$$N_C = \frac{\sqrt{k/m}}{2\pi} \tag{7}$$

Where m = the mass of the shaft assumed concentrated at single point. K is the stiffness of the shaft to transverse vibrations

For a horizontal shaft it can be expressed as given in equation (8)

$$N_C = \frac{\sqrt{g/y}}{2\pi} \tag{8}$$

Where y = the static deflection at the location of the concentrated mass

2.0 EXPERIMENTAL PROCEDURE:

In this, boundary situation (rigidly fixed at both ends) is tested for the specimen described in earlier section. The test process is described below in following steps:

1. The dimension of the specimen is measure using ruler and caliper.

2. The shaft is mounted on the machine by tightening it in the chuck by means of the setscrew provided with the chuck, with the shaft running throughout the guides G, positioned evenly along its length. The adjustment support U , containing retainer N, is then be brought up to locate the threaded portion of the test shaft in the central hole of the retainer by a locknut, which runs on the threaded portion of the shaft. Both supports at D are sliding into the position.
3. Mounting the shaft on the machine: At this point, it is thus significant to make sure that the setscrew is tightened and that the guides and supports are rigidly fixed to the main frame, by tightening the hand wheels located beneath each. Most shaft failures are produced because of inadequate support, which results from insufficient tightening up of the apparatus prior to testing.
4. Next step is to switch on the speed control and rotate the control knob slowly in a clockwise direction until the first natural frequency is observed, that is indicated by the formation of a bow as shown in figure 2. When the speed is increased further the shaft begins to vibrate violently as it nears the critical speed. Once the critical speed is passed the shaft re-stabilizes and on further increase on speed the second natural frequency is reached which is indicated by the double bow as shown in figure 3.

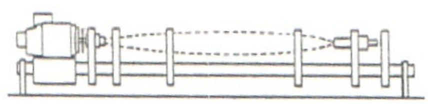

Figure 2: First Mode of Whirl

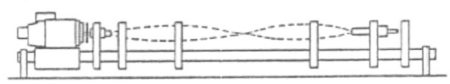

Figure 3: Second Mode of Whirl

5. Now further the speed of rotation of the shaft is measured at its first and second natural frequencies using contact type tachometer.
6. To get the average value of shaft speed, the measurement is carried thrice.
7. The average value thus obtained is used to evaluate the natural frequency of shaft with varying diameters.

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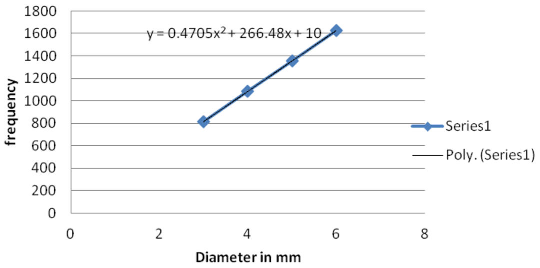
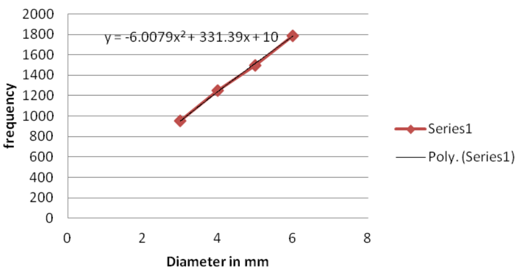
Sr.No.	Shaft Diameter (mm)	Voltage (V)	Practical Frequency (rpm)	. Theoretical Frequency (rpm)
1	3	90	812.91	950
2	4	100	1084.134	1250
3	5	108	1354.4264	1500
4	6	115	1625.510	1789



Graph 1 Diameter vs. Frequency (Practical)



Graph 2 Diameter vs. Frequency (Theoretical)

$y = 0.4705x^2 + 266.48x + 10$ (9) for Practical frequency

$y = -6.0079x^2 + 331.39x + 10$ (10) for Theoretical frequency

From above two graphs we got two polynomial equations, from which we can found out practical and theoretical values of frequency for various diameters, where y is the frequency of the shaft while x is the diameter of shaft.

CONCLUSIONS

Experimental verification of natural frequencies of different shaft diameters is thus achieved. The speed which needs to be avoided is found out using theoretical frequency equations for the shafts. The whirling phenomenon and its insight are thus thoroughly described. As the whirling of shaft apparatus is manufactured using conventional arrangements, the difference in theoretical and experimental values is justified. Close correlation between theoretical and experimental frequencies is observed for 4 and 6 mm diameter shafts. This process can be applied to test shafts used in automobile propeller shaft, ships, airplanes and other similar applications.

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