Connected equitable domination in Fuzzy graphs

S.Revathi, C.V.R.Harinarayanan

***Abstract*— Let $G=(\sigma,\mu)$ be a fuzzy graph. A Subset S of V is an fuzzy equitable dominating set of a graph G if for every $v\in V-S$,there exists a vertex $u\in S$ such that degree of u and degree of v in G.An equitable dominating set S is said to be connected equitable dominating set if the subgraph $\langle S\rangle$ is induced by S is connected.The minimum of the cardinalities of the connected equitable dominating sets of G is called the connected equitable domination number and denoted by $\gamma_{fce}(G)$.**

Index Terms— **Equitable dominating set, Equitable domination number,Fuzzy connected equitable dominating set ,fuzzy connected equitable domination number.**

INTRODUCTION

The first definition of fuzzy graphs was proposed by Kafmann from the fuzzy relations introduced by Zadeh.A mathematical framework to describe the phenomena of uncertainity in real life situation is first suggested by Zadeh in 1965.The study of dominating sets in graphs was started by Orge and Berge. The Domination number was introduced by Cockayne and Hedetniemi[2].A.Somasundaram and S.Somasundaram[5] discussed domination in a fuzzy graph using effective edges.Nagoorgani and Chandrasekeran[3] discussed domination in a fuzzy graph using strong arcs.In this paper we introduce the connected equitable domination in a fuzzy graph and connected equitable domatic in a fuzzy graph.

I. BASIC DEFINITIONS

In this section , basic definitions relating to a fuzzy graph are given.

Fuzzy graph:

A fuzzy graph $G=(\sigma,\mu)$ is a pair of function $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ where for all $u,v\in V$,we have $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$.

Fuzzy Subgraph:

The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of $G=(\sigma,\mu)$ if $\tau(u)\le\sigma(u)\ for\,all\,u\in V$ and $\rho(u,v)\le\mu(u,v)$ for all $u,v\in V$.

Spanning fuzzy subgraph:

The fuzzy subgraph of $G=(\sigma,\mu)$ induced by τ is the maximal fuzzy subgraph of $G(\sigma,\mu)$ that has a fuzzy node set τ .Evidently,this is just the fuzzy subgraph (τ,ρ) where $\rho(u,v)\le\tau(u)\wedge\tau(v)\wedge\mu(u,v)$ for all $u,v\in V$

Connected fuzzy graph:

Two nodes that are joined by a path are said to be connected.

Fuzzy cardinality of a set:

For any subset S of V and let $S\subseteq V$ .Fuzzy cardinality of S is defined to be $\sum_{v\in S}\sigma(v)$

Order and Size:

The order P and size q of a fuzzy graph $G=(\sigma,\mu)$ are defined to be $p=\sum_{u\in V}\sigma(x)\ and\ q=\sum_{uv\in E}\mu(xy)$ respectively.

Degree:

Let $G:(\sigma,\mu)$ be a fuzzy graph. The degree of a vertex v is $d(v)=\sum_{u\neq v}\mu(u,v)$ .The minimum degree of G is $\delta(G)=\wedge\{d(v)/v\in V\}$ and the maximum degree of G is $\Delta(G)=\vee\{d(v)/v\in V\}$.

Neighborhood degree:

An edge e=uv of a fuzzy graph is called an effective edge if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$.

$N(u)=\{v\in V/\mu(u,v)=\sigma(u)\wedge\sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u)\cup\{u\}$ Is called the closed neighborhood degree of u, $d_N(u)=\sum_{v\in N(u)}\sigma(v)$ is called the neighborhood degree of u. The minimum neighborhood degree of G is $\delta_N(G)=\wedge\{d_N(v)/v\in V\}$ and the maximum neighborhood degree $\Delta_N(G)=\vee\{d_N(v)/v\in V\}$

Fuzzy domination number:

Let $G=(\sigma,\mu)$ be a fuzzy graph.A subset S of V is said to be a dominating set of G if for every $v\in V-S$ there exists $u\in S$ such that $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ .A dominating set S of a fuzzy graph G is called the minimal dominating set of G if for every node $v\in S$ .S-{v} is not a dominating set.

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The minimum scalar cardinality of S is called a domination number and it is denoted by $\gamma(G)$.The scalar cardinality of a fuzzy subset S of V is $|S|_f = \sum_{v \in S} \sigma(v)$

Connected domination Number:

A dominating set S is a connected dominating set of a fuzzy graph G if the fuzzy subgraph $\langle S \rangle$ is induced by S is connected .The minimum cardinality taken over all minimal connected dominating sets is called a connected domination number of a fuzzy graph G and it is denoted by $\gamma_c(G)$

Equitable dominating set:

A dominating set S is a equitable dominating set of a fuzzy graph G if for every $v \in V - S$ there exist a vertex $u \in S$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such an equitable dominating set is denoted by $\gamma_{fe}(G)$ and is called the equitable domination number of G.

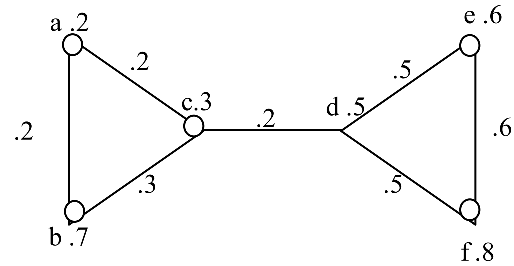
II.MAIN RESULTS

CONNECTED EQUITABLE DOMINATION IN FUZZY GRAPHS:

Definition:

Let $G = (\sigma, \mu)$be a connected fuzzy graph .An equitable dominating set S of a fuzzy graph G is called the connected equitable dominating set of a fuzzy graph if the induced subgraph $\langle S \rangle$ is connected,the minimum cardinality of a fced-set of fuzzy graph G is called the connected equitable domination number of G and is denoted by $\gamma_{fce}(G)$.

Example:



$S = \{c, d\}$

$V - S = \{a, b, e, f\}$is a connected equitable dominating set .

Theorem:

A connected equitable dominating set exist for a fuzzy graph G if and only if G is connected.

Theorem:

Any connected equitable dominating set of a fuzzy graph G is a equitable dominating set of a fuzzy graph.

Proof:

Let S be connected equitable set of a fuzzy graph for every $v \in V - S$ there exist a vertex $u \in S$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$ and the subgraph $\langle S \rangle$ is connected.Thus S is a equitable dominating set of a fuzzy graph.

Note:

The converse of the above theorem need not be true.

Theorem:

Any connected equitable dominating set S is a minimal connected equitable dominating set (MCES) if and only if for each $d \in S$ one of the following two conditions holds d is not adjacent to any vertex in S

(i) for some $f \in S$,$\sigma(d) < \sigma(f)$ or

(ii) there exists

avertex $c \in V - S$ suchthat $N(c) \cap S = \{b\}$

Proof:

Assume that S is an minimal connected equitable dominating set of G. Then for every vertex $d \in S$. $S - \{d\}$ is not a connected equitable dominating set .This means that some vertex c in $V - S \cup \{d\}$is not connected equitable dominating by any vertex in S-{d}.Then there are two cases

Case i:

If c=d which case d is adjacent to any vertex f in S such that $\sigma(d) < \sigma(f)$

Case ii:

$c \in V - S$,then c is not dominated by any vertex in S-{d} but it is dominated by some vertex f in S then f is adjacent to only vertex c in V-S.Thus $N(c) \cap S = \{f\}$.

Conversely suppose that S is a connected equitable dominating set and for each vertex $d \in S$ one of the two conditions holds. We now show that S is a minimal connected equitable dominating set. Suppose that S is not a minimal connected equitable dominating set and then there exists a vertex $d \in S$ such that S-{d} is a connected equitable dominating set. Hence d is adjacent to atleast one vertex in S-{d} and then condition (i) does not hold. Also if S-{d} is a connected equitable dominating set then every vertex of V-S is adjacent to atleast one vertex in D-S that is condition (i) or (ii) does not hold. This is a contradiction to our assumption that atleast one of the condition holds.

Remark:

Let G be a fuzzy graph without isolated vertices and let S be a connected equitable dominating set of G, then V-s need not be a connected dominating set of G.

Theorem:

For any fuzzy graph, $p - q \leq \gamma_{fce} \leq p - \delta_E$ where p,q and δ_E are the order, size and minimum effective incident degree of G respectively.

Proof:

Let S be a connected equitable dominating set and γ_{fce} be the VXV.Hence $p - q \leq \gamma$ .Now let u be the vertex with

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minimum effective incident degree δ_E .Clearly V-{u} is a connected equitable dominating set . $\gamma_{fce} \le p - \delta_E$
Hence $p - q \le \gamma_{fce} \le p - \delta_E$ is true.

Theorem:
If all the vertices have the same membership grade then

$$p - q \le \gamma_{fce} \le p - \Delta_E$$

Theorem:
For any connected fuzzy graphG
$\gamma_f(G) \le \gamma_{fe}(G) \le \gamma_{fce}(G)$
Proof:
From the definition of the connected equitable dominating set of a fuzzy graph G ,it is clear that for any fuzzy graph G ,any connected equitable dominating set S is also an equitable dominating set and every equitable dominating set is also a dominating set.

$$\therefore \gamma_f(G) \le \gamma_{fe}(G) \le \gamma_{fce}(G)$$

Similarly since every connected equitable dominating set for any connected fuzzy graph G is connected dominating set we have the following theorem.
Theorem:
For any connected fuzzy graph G, $\gamma_{fc}(G) \le \gamma_{fce}(G)$
The following theorem is straightforward from the definition of the connected equitable domination
Theorem:
For any connected fuzzy graph G with σ_i *vertices*
(i) $1 \le 1\gamma_{fce}(G) \le p_i$ (ii) $1\gamma_{fce}(G) = \sigma_i$ if and only if G has equitable isolated vertices
(iii) $\gamma_{fce}(G) = \sigma$ if there exist atleast one vertex $v \in G$ such that $\deg_e(v) = p - 1$
Theorem:
For any fuzzy graph G with $\delta_e(G) \ge 1$

$$\left\lceil \frac{p}{1 + \Delta_e(G)} \right\rceil \le \gamma_{fce}(G) \le 2q - p + 1$$

Proof:
Clearly from the definition of the fuzzy connected equitable dominating set we have $\gamma_{fce} \le p - 1$
then $\gamma_{fce}(G) = p - 1 \le 2q - p + 1$ and from theorem 3.5 we have $\gamma_f(G) \le \gamma_{fe}(G) \le \gamma_{fce}(G)$
and by theorem 3.6 ,we get $\left\lceil \frac{p}{1 + \Delta_e(G)} \right\rceil \le \gamma_{fce}(G)$.
Therefore $\left\lceil \frac{p}{1 + \Delta_e(G)} \right\rceil \le \gamma_{fce}(G) \le 2q - p + 1$
Theorem:
Let G be a connected fuzzy graph has no non –equitable edge and H is spanning subgraph of H .Then
$\gamma_{fe}(G) \le \gamma_{fe}(H)$
Proof:
Let G be a connected fuzzy graph and let H is the spanning subgraph of H. Suppose that S is the minimum equitable dominating set of H .Then S is also equitable dominate all the vertices in V(H)-S that is S is an equitable dominating set in a fuzzy graph. Hence $\gamma_{fe}(G) \le \gamma_{fe}(H)$.

Theorem:
Let G be a connected fuzzy graph has no non –equitable edge and H is connected spanning subgraph H.Then
$\gamma_{fce}(G) \le \gamma_{fce}(H)$.

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