Inelastic Interaction of Nonequilibrium Carriers with Acoustic Phonons in Semiconductor Inversion Layer

A. K. Ghorai

Abstract — **A theory of the rate of increase in intravalley acoustic phonons due to inelastic interaction of the nonequilibrium electrons with the deformation potential acoustic phonons is developed in a two-dimensional electron gas (2DEG) formed in semiconductor interfaces under the condition of low lattice temperature when the approximations of the well-known traditional theory are not valid. The rate is estimated for an n-channel (100) oriented Si inversion layer and the significance of the phonon energy in the study of electron transport in 2DEG is discussed.**

Index Terms — **Semiconductor, Inversion Layer, Phonon.**

I. INTRODUCTION

The study of the electrical transport in the two-dimensional electron gas (2DEG) formed in semiconductor inversion and accumulation layers of metal-oxide-semiconductor field-effect transistors (MOSFET) has been largely incited by its fundamental physical importance as well as possible device applications [1-4].

The electron transport in a 2DEG is controlled by various scattering mechanisms like electron-lattice scattering, impurity and surface roughness scattering near the oxide-semiconductor interface under the prevalent range of the lattice temperature T_L and carrier concentration N_i. At lower temperatures the intravalley acoustic phonon and impurity scatterings dominate. The optical and intervalley phonon scatterings can be important only at high temperatures when an appreciable number of corresponding phonons is excited or in the presence of a high electric field when the nonequilibrium electrons can emit high-energy phonons. It is well known that there is a range of lattice temperature ($T_L < 20\text{ K}$) when the free carriers interact dominantly with intravalley deformation potential acoustic phonons and the acoustic phonon scattering plays an important role in controlling the transport characteristics if the content of impurity atoms in the system under study is relatively low [1-13]. It should be borne in mind that the possibility of obtaining the materials of higher and higher purity is not beyond the scope of the present day advanced semiconductor technology. Again at such low temperatures the electrons become hot in relatively weak fields of the order of only a few volts per centimeter. The electron transport under these conditions is limited by the acoustic scattering of the nonequilibrium carriers [14, 15]. Thus the study of the problem of electrical transport in semiconductors at low lattice temperatures has become interesting.

Useful results on the study of the transport in 2DEG at low lattice temperatures have already been reported. The 2DEG in GaAs has been realized by Störmer et al. [9], who employed a GaAs-Ga$_x$Al$_{1-x}$As heterostructure and observed Shubnikov-de Haas oscillations around 4.2 K and reported the mobility values at the same temperatures. A theory of intravalley acoustic phonon scattering of the free carriers has been developed in 2DEG at low temperatures and the corresponding scattering rates are used to obtain the zero-field mobility characteristics in Si inversion layers with the help of Monte Carlo simulation of velocity autocorrelation function [11].

The purpose of this article is to calculate the rate of increase of intravalley acoustic phonon due to scattering of the nonequilibrium electrons with acoustic mode lattice vibrations in a 2DEG system under the condition of low lattice temperature. This rate has already been calculated in bulk semiconductor with the help of traditional theory [14] as well as under the condition of low temperature when the phonon energy cannot be neglected in comparison to the carrier energy [16].

II. THEORY

In an oxide-semiconductor interface, we consider carrier transitions between two states $\vec{k}$ and $\vec{k}+\vec{q}$ in the course of a collision accompanied with either emission or absorption of a phonon of wave vector $\vec{q}$, resulting in an increase in $N_{\vec{q}}$, the number of phonons. The rate of increase in the number of phonons can be written using the perturbation theory as [14]

$$\begin{aligned}\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{2\pi}{\hbar}\sum_{\vec{k}} \Big[& \left|\langle \vec{k}, N_{\vec{q}}+1 \middle| H'_{ac} \middle| \vec{k}+\vec{q}, N_{\vec{q}}\rangle\right|^2 \\ & \times \delta\left(\epsilon_{\vec{k}}, N_{\vec{q}}+1-\epsilon_{\vec{k}+\vec{q}}, N_{\vec{q}}\right) f_0\left(\vec{k}+\vec{q}\right) \\ & - \left|\langle \vec{k}+\vec{q}, N_{\vec{q}}-1 \middle| H'_{ac} \middle| \vec{k}, N_{\vec{q}}\rangle\right|^2 \\ & \times \delta\left(\epsilon_{\vec{k}+\vec{q}}, N_{\vec{q}}-1-\epsilon_{\vec{k}}, N_{\vec{q}}\right) f_0\left(\vec{k}\right)\Big], \qquad (1)\end{aligned}$$

where the square of the matrix element of the electron-lattice scattering is given by [6]

$$\left|\langle \vec{k}+\vec{q} \middle| H'_{ac} \middle| \vec{k}\rangle\right|^2 = \left(\frac{\varepsilon_a^2 \hbar q^2}{2sd\rho_v\omega_q}\right)\left(N_{\vec{q}}+\frac{1}{2}+\frac{1}{2}\delta N_{\vec{q}}\right),$$

where $\delta N_{\vec{q}} = +1$ for emission,

$\delta N_{\vec{q}} = -1$ for absorption.

Manuscript received September, 2014.

A. K. Ghorai, Physics Department, Kalimpong College, Darjeeling - 734301, West Bengal, INDIA.

Here $\mathcal{E}_a$ is the effective deformation potential constant which assumes a value larger than that for the bulk material for higher-order subbands [6]. Vass *et al* [17] developed a theory to determine the surface deformation potential constant $\mathcal{E}_a$ in terms of a bulk value of the deformation potential $\mathcal{E}_1$ and carrier concentration N_i as

$$\mathcal{E}_a = \mathcal{E}_1 + 2.5 \times 10^{-8} \times N_i^{2/3} \quad \text{eV}.$$

The parameter s is the surface area, d is the width of the layer of lattice atoms with which the electrons can interact, and ρ_v is the mass density. The frequency of the lattice vibration $\omega_q = u_l q$, u_l is the acoustic velocity. The hot electron distribution function $f_0(\vec{k})$ is given by the Maxwell-Boltzmann distribution at an effective electron temperature T_e as [14]

$$f_0(\vec{k}) = \frac{N_i}{N_C^{2D}} e^{-\epsilon_{\vec{k}}/k_B T_e}.$$

Here k_B is the Boltzmann constant and $\epsilon_{\vec{k}}$ is the energy of the electron which can be given for spherical constant energy surfaces as [6]

$$\epsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m_\parallel^*} ,$$

where $\hbar$ is the Dirac constant, and $m_\parallel^*$ is the effective mass of the electron parallel to the interface. The effective density of state N_C^{2D} in 2DEG can be given as[18]

$$N_C^{2D} = \frac{m_\parallel^*}{\pi\hbar^2} k_B T_e.$$

The summation over two-dimensional lattice wave vector $\vec{k}$ in Eq.(1) can be transformed into integral as

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{\mathcal{E}_a^2 m_\parallel^*}{2\pi d \rho_v u_l \hbar^2} \int_k \left\{ \frac{(N_{\vec{q}}+1) f_0(\vec{k}+\vec{q}) dk}{\left[1 - \left(\frac{q}{2k}\right)^2 \left(1 + \frac{2m_\parallel^* u_l}{\hbar q}\right)^2\right]^{1/2}} - \frac{N_{\vec{q}} f_0(\vec{k}) dk}{\left[1 - \left(\frac{q}{2k}\right)^2 \left(1 - \frac{2m_\parallel^* u_l}{\hbar q}\right)^2\right]^{1/2}} \right\}. \qquad (2)$$

Under the condition of low temperature since the phonon energy cannot be neglected, the limits of integration over $\vec{k}$ as ascertained from the energy and momentum balance equations may be taken to be $(q/2 - m_\parallel^* u_l/\hbar)$ and ∞. If $f_0(\vec{k}+\vec{q})$ is expanded in a Taylor's series around $\vec{k}$ then one can obtain from Eq.(2), the rate of increase in the number of phonons as

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{\mathcal{C}_a N_i}{\sqrt{T_n}} \left[(N_{\vec{q}}+1)\, e^{-\frac{\alpha}{k_B T_L T_n}} \left\{ 1 + \sum_{j=1}^{\infty} \frac{(-x/T_n)^j}{j!} \right\} \Gamma\left(\tfrac{1}{2}, p\right) - N_{\vec{q}}\, e^{-\frac{\beta}{k_B T_L T_n}}\, \Gamma\left(\tfrac{1}{2}, q\right) \right]. \qquad (3)$$

Here

$$\mathcal{C}_a = \frac{\varepsilon_a^2}{2\hbar d \rho_v u_l^2}\left(\frac{\varepsilon_s}{k_B T_L}\right)^{1/2}, \; \varepsilon_s = \frac{1}{2} m_\parallel^* u_l^2, \quad x = \hbar q u_l / k_B T_L,$$

$$T_n = T_e/T_L\,, \quad \alpha = \frac{a(k_B T_L)^2}{16\varepsilon_s} x^2, \; \beta = \frac{b(k_B T_L)^2}{16\varepsilon_s} x^2,$$

$$a = \left(1 + \frac{4\varepsilon_s}{k_B T_L x}\right)^2, \quad b = \left(1 - \frac{4\varepsilon_s}{k_B T_L x}\right)^2,$$

$$p = \frac{1}{T_n}\left[\frac{(1-a)k_B T_L}{16\varepsilon_s} x^2 - \frac{1}{2}x + \frac{\varepsilon_s}{k_B T_L}\right],$$

$$q = \frac{1}{T_n}\left[\frac{(1-b)k_B T_L}{16\varepsilon_s} x^2 - \frac{1}{2}x + \frac{\varepsilon_s}{k_B T_L}\right],$$

$\Gamma(m\,, n)$ is incomplete Γ –function.

Under the condition of high lattice temperature ($T_L > 20$ K) when the phonon energy can indeed be neglected and the phonon ensemble obeys the equipartition law then the rate of increase in the number of acoustic phonons can be given as

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{\mathcal{C}_a \sqrt{\pi}\, N_i}{\sqrt{T_n}} e^{-\frac{\gamma}{k_B T_L T_n}}\,, \qquad (4)$$

where $\gamma = \frac{(k_B T_L)^2}{16\varepsilon_s} x^2$.

III. RESULT AND DISCUSSION

To observe the effect of phonon energy on the rate of increase in the phonon number due to the interaction of the free carriers with the acoustic phonon in quantized surface layers we apply the above theory for an n-channel (100) oriented Si inversion layer with the material parameter values [11] : $\mathcal{E}_1 = 9.8$ eV , $u_l = 9.037 \times 10^3$ m s^{-1} , $\rho_v = 2.329 \times 10^3$ kg m^{-3} , $\epsilon_{sc} = 11.9$, longitudinal effective mass $m_l^* = 0.96 m_0$, transverse effective mass $m_t^* = 0.19 m_0$, m_0 being the free electron mass. At low lattice temperatures one may consider presumably the electrons occupy only the lowest subband when the layer thickness d is given by $(\hbar^2 \epsilon_{sc}/2m_\perp^* e^2 N_i)^{1/3}\gamma_0$. Here γ_0 is the zeroth root of the Airy's function. For the (100) surface of Si the six valleys are not equivalent. The two equivalent valleys for which $m_\parallel^* = m_t^*$, $m_\perp^* = m_l^*$ occupy the lowest subband [1,4,6].

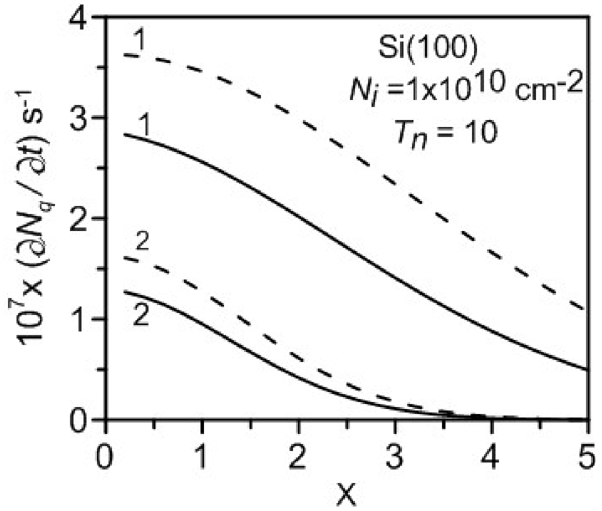

Fig.1 : Rate of increase of phonon number in Si(100) layer due to interaction of nonequilibrium electrons with the intravalley acoustic phonons. Curve 1 and 2 are for lattice temperatures of 4 and 20 K respectively, of which the solid curves are obtained from the theory where the phonon energy is taken into account [Eq.(3)] and the dashed curves are obtained from the traditional theory where the phonon energy is neglected [Eq.(4)].

From the Fig.1 it is obvious that under the condition of low temperature when the deformation potential acoustic phonon energy is taken into account in the energy balance equation with sufficient precision, the number of phonons increases at a slower rate compared to what predicted from the traditional theory where the acoustic phonon energy is neglected in the calculation even at lower temperatures. It is also observed that $(\partial N_{\vec{q}}/\partial t)$ is more with the lowering of lattice temperature at a particular T_n whether the phonon energy is taken into account or not. Again as expected the discrepancy between the results of the two theories is more when the lattice temperature is lower as shown by curves marked 1 for the lattice temperature of 4 K in comparison to the curves marked 2 for the lattice temperature of 20 K.

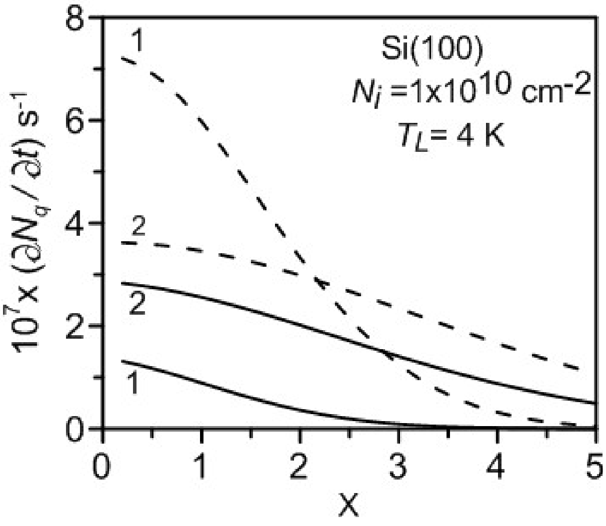

Fig.2 : Dependence of the rate of increase of the number of phonons upon the phonon wave vector x in Si(100) layer for different values of T_n . Curves 1 and 2 are for $T_n = 2.5$ and $T_n = 10$ respectively. The solid curves are obtained from the theory where the phonon energy is taken into account [Eq.(3)] and the dashed curves are obtained from the traditional theory where the phonon energy is neglected [Eq.(4)].

In Fig.2 the dependence of the increase in phonon number on phonon wave vector has been compared for different electron temperature at a particular lattice temperature. It may be noted that $(\partial N_{\vec{q}}/\partial t)$ decreases with phonon wave vector q at a faster rate with the lowering of T_n. But the rate of increase in phonon number is more for higher electron temperature for a particular q. This apart, it is observed that the difference in results obtained by the two theories is more for lower values of electron temperature.

IV. CONCLUSION

The results from the figures reveal how significantly the finite value of the acoustic phonon energy affects the rate of increase in the phonon number at any T_n for different values of the lattice temperatures in comparison to what follows from the traditional theory where the phonon energy is indeed neglected even at lower temperatures. So care should be taken to account for the finite energy of the acoustic phonon or their true energy distribution whenever any study relating to electron-phonon interaction is done particularly at low temperatures.

REFERENCES

[1] F. Stern, and W. E. Howard, “Properties of semiconductor surface inversion layers in the electric quantum limit,” *Phys. Rev.* vol.163, no.3, pp.816-835, Nov. 1967.

[2] Electronic Properties of Two-Dimensional Systems, Proceedings of the International Conference on the Electronic Properties of Two-Dimensional Systems, Berchtesgaden, Germany, 1977, Eds. G. Dora and P. J. Stiles (North-Holland, Amsterdam,1978).

[3] F. Stern, “Self-consistent results for *n*-type Si inversion layers,” *Phys. Rev. B* vol.5, no.12, pp.4891-4899, Jun. 1972.

[4] T. Ando, A. B. Fowler, and F. Stern, “Electronic properties of two-dimensional systems,” *Rev. Mod. Phys.* vol.54, no.2, pp.437-672, Apr.1982.

[5] C. T. Sah, T. N. Ning, and L. T. Tschopp, “The scattering of electrons by surface oxide charges and by lattice vibrations at the silicon-silicon dioxide interface,” *Surf. Sci.* vol.32, no.3, pp.561- 575, Sep. 1972.

[6] C. Y. Wu, and G. Thomas, "Two-dimensional electron-lattice scattering in thermally oxidized silicon surface-inversion layers," *Phys. Rev. B*. vol.9, no.4 pp.1724-1732, Feb. 1974.

[7] S. Kawaji, “The two-dimensional lattice scattering mobility in a semiconductor inversion layer,” *J. Phys. Soc. Japan* vol. 27, no.4, pp.906-908, May 1969.

[8] D. K. Ferry, "Optical and intervally scattering in quantized inversion layers in semiconductors," *Surf. Sci.* vol.57, no.1, pp.218-228, Jul. 1976.

[9] H. L. Störmer, R. Dingle, A. C. Gossard, W. Weigmann, and M. D. Sturge, “Two-dimensional electron gas at a semiconductor-semiconductor interface,” *Solid. State. Commun*. vol.29, no.10, pp.705-709, Mar. 1979.

[10] A. K. Ghorai, and D. P. Bhattacharya, “Lattice-controlled mobility in quantized surface layers at low temperature,” *J. Appl. Phys*. vol.80, no.5, pp.3130-3132, Sep. 1996.

[11] A. K. Ghorai, and D. P. Bhattacharya, “Lattice-controlled electron transport characteristics in quantized surface layers at low temperature,” *Surf. Sci.* vol.380, no.2-3, pp.293-301, May 1997.

[12] A. K. Ghorai, “Effect of Screening on Acoustic Phonon Limited Electron Mobility in Quantized Surface Layer in Semiconductor,” *IJSETR,* vol.2, issue 7, pp.1453-1457, Jul. 2013.

[13] C. Jacoboni, and L. Reggiani, “The Monte Carlo method for the solution of charge transport in semiconductors with applications to covalent materials,” *Rev. Mod. Phys*. vol.55, no.3,pp.645-705, Jul. 1983.

[14] E. M. Conwell, in Solid State Physics, ed. F. Seitz and D. Turnbull (Academic, New York, 1967), Suppl.9.

[15] C. Canali, C. Jacoboni, F. Nava, G. Ottaviani, and A. Alberigi Quaranta, “Electron drift velocity in silicon,” *Phys. Rev*. B vol.12, no. 6 , pp.2265, Sep. 1975.

[16] N. Chakrabarty and D. P. Bhattacharya, “On the electron-phonon interaction in high purity semiconductors at low lattice temperatures,” J. Phys. Chem. Solids vol.57, no.5, pp.653-654,1996.

[17] E. Vass, R. Lassnig, and E. Gormic, “*Proc of workshop on IVth international conference of electronic properties of 2D systems*,” New London, USA, 24-28 Aug. 1981.

[18] E. F. Schubert, Doping in III-V Semiconductors, (Cambridge University Press, 2005).