BOUNDARY LAYER PHENOMENA OF MHD FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SHEET EMBEDDED IN A THERMALLY STRATIFIED MEDIUM

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ABSTRACT

In the present analysis MHD boundary layer flow and heat transfer towards an exponentially stretching sheet embedded in a thermally stratified medium subject to suction are described. Partial differential equations corresponding to the momentum and energy equations are converted into highly nonlinear ordinary differential equation, by using suitable transformations. Numerical solutions of these equations are obtained by shooting method. It is found that the heat transfer rate at the surface increases in presence of thermal stratification. Fluid velocity decreases with increasing magnetic parameter. Fluid velocity decreases with increase of suction parameter. It is noticed that the temperature decreases with increase of suction parameter. Temperature gradient increases considerably with increase of stratification parameter.

Key words: Boundary layer flow; exponentially stretching sheet; MHD flow; Suction; Thermally stratified medium.

Nomenclature

M	Magnetic parameter
η	Similarity variable
$\Pr$	Prandtl number
κ	The coefficient of thermal conductivity
S	Suction/injection parameter
μ	Dynamic viscosity
St	Stratification parameter
ν	Kinematic viscosity
T	Temperature of the fluid
ψ	Steam function
$T_w(x)$	Prescribed surface temperature
ρ	Density of the fluid
$T_\infty(x)$	Variable free-stream temperature
θ	Non dimensional temperature
u, v	Components of velocity in x and y directions
R	Radiation parameter

1. Introduction

The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid is of considerable interest because of its ever increasing industrial applications. Crane [1] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [2], Dutta et al. [3], Chen and Char [4], Andersson [5] extended the work of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations. On the other hand, Gupta and Gupta [2] stressed that realistically, stretching surface is not necessarily continuous. The study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin. This situation was dealt by Kumaran and Ramanaiah [6] in their work on boundary layer fluid flow. Recently, various aspects of such problem have been investigated by many authors such as Xu and Liao [7], Cortell [8,9], Hayat et al. [10] and Hayat and Sajid [11]. A few years later, Magyari and Keller [13] also focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet. Extension to that, Elbashbeshy [14] added new dimension to the study of Ali [12] on exponentially continuous stretching surface. Cannon [15] also considered the flow over a nonlinear stretching sheet. Khan [16] and Sanjayanand and Khan [17] studied the viscous-elastic boundary layer flow and heat transfer due to an exponentially stretching sheet. Later, Sajid and Hayat [18] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Recently, Bidin and Nazar [19] analysed the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet, which has been solved analytically by Sajid and Hayat [18]. Pal [20] reported mixed convection flow past an exponentially stretching surface in the presence of a magnetic field. Nadeem et al. [21] addressed the flow of Jeffrey fluid and heat transfer past an exponentially stretching sheet. Ishak [22] discussed the combined effects of magnetic field and thermal radiation on flow and heat transfer over an exponentially stretching sheet. Recently, Mukhopadhyay and Gorla [23] analysed the effects of partial slip on flow past an exponentially stretching sheet. Bhattacharyya [24] discussed the mass transfer in case of boundary layer flow over an exponentially stretching sheet with an exponentially moving free stream in a reactive species. Suction/injection (blowing) of a fluid through the bounding surface has significant effect on the flow field. In general, suction tends to increase the skin friction, whereas injection acts in the opposite manner. The process of suction/blowing also has its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery [25]. Suction is applied to chemical processes to remove reactants [26]. The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically by Yang et al. [27]. However, convective flow in a stratified

ISSN: 2278 – 7798

media has not received much attention. The study of magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are some examples of such fields [28]. In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role [29]. Kumaran et al. [30] reported that magnetic field makes the streamlines steeper which results the boundary layer thinner. The purpose of this present work is to extend the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet embedded in a stratified medium. Using suitable transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. Using shooting method, numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the stratification parameter in presence of suction at the wall.

2. Mathematical model

Consider the flow of an incompressible viscous electrically conducting fluid past a flat heated sheet coinciding with the plane $y=0$. The flow is confined to $y>0$. Two equal and opposite forces are applied along the $x-$ axis, so that the wall is stretched keeping the origin fixed (see Fig. 1). A variable magnetic field $B=B_0\, e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant [22]. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ where $T_w(x) > T_\infty(x)$.

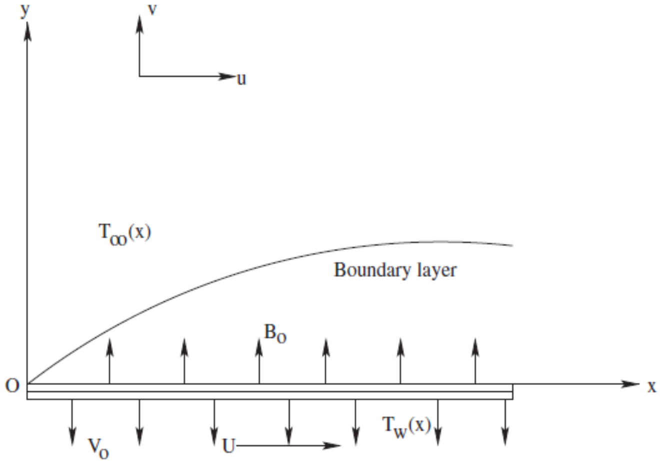

Figure 1 Sketch of the physical problem.

It is assumed that $T_w(x) = T_0 + be^{\frac{x}{2L}}$, $T_\infty(x) = T_0 + ce^{\frac{x}{2L}}$, where T_0 is the reference temperature, $b>0, c\geq 0$ are constants.

The continuity, momentum, and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

Where u and v are the components of velocity in the x and y directions respectively, $\nu=\frac{\mu}{p}$ is the kinematic viscosity, $q_r=\frac{-4\sigma_s \partial T^4}{3K_e \partial y'}$ is the radiative heat flux, expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms take the form $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$, c_p is the specific heat at constant pressure and κ is the thermal conductivity, ρ is the fluid density and μ is the coefficient of fluid viscosity. Detailed study about the horizontally stratified medium was given by Nielsen and Balling [31].

2.1. *Boundary conditions*

The appropriate boundary conditions for the problem are given by

$$u=U,\ v=-V(x),\ T=T_w(x) \text{ at } y=0 \qquad (4a)$$

$$u\to 0,\ T=T_\infty(x) \text{ as } y\to\infty \qquad (4b)$$

Here $U=U_0\, e^{\frac{x}{L}}$ is the stretching velocity, U_0 is reference velocity, $V(x)>0$ is velocity of suction and $V(x)<0$ is velocity blowing, $V(x)=V_0 e^{\frac{x}{L}}$, a special type of velocity at the wall is considered. V_0 is the initial strength of suction.

2.2. Method of solution

Introducing the suitable transformations as

$$\eta=\sqrt{\frac{U_0}{2\nu L}}\,e^{\frac{x}{2L}}y\,,\quad u=U_0e^{\frac{x}{L}}f'(\eta) \qquad (5a)$$

$$v=-\sqrt{\frac{\nu U_0}{2L}}\,e^{\frac{x}{2L}}\left\{f(\eta)+\eta f'(\eta)\right\} \qquad (5b)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_0}\,,\quad R=\frac{16\sigma_s T_\infty^3}{3K_e k} \qquad (5c)$$

and upon substitution of (5a), (5b) and (5c) in Equs .(2) and (3) the governing equations transforms to

$$f'''+ff''-2f'^2-Mf'=0 \qquad (6)$$

$$\theta''+\frac{\Pr}{1+R}(f\theta'-f'\theta)-\frac{\Pr}{1+R}stf'=0 \qquad (7)$$

and the boundary conditions take the following form:

$$f'=1,\ f=S,\ \theta=1-St \text{ at } \eta=0 \qquad (8)$$

$$f'\to 0,\ \theta\to 0 \text{ as } \eta\to\infty \qquad (9)$$

where the prime denotes differentiation with respect to η, $M=\frac{2\sigma B_0{}^2L}{\rho U_0}$ is the magnetic parameter, $S=\frac{V_0}{\sqrt{\frac{U_0\nu}{2L}}}>0$ (or <0) is the suction (or blowing) parameter, $St=\frac{c}{b}$ is the stratification parameter and $\Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number, $St>0$ implies a stably stratified environment, while $St=0$ corresponds to an unstratified environment.

3. Numerical method for solution

The above eqs. (6) and (7) along with the boundary conditions are solved by converting them to an initial value problem. We set

$$f'=z,\quad z'=p,\quad p'=2z^2+Mz-fp \qquad (10)$$

$$\theta'=q,\ q'=-\frac{\Pr}{1+R}\left(fq-z\theta-Stz\right) \qquad (11)$$

with the boundary conditions,

$$f(0)=S,\ f'(0)=1,\ \theta(0)=1-St. \qquad (12)$$

To integrate (10) and (11) as initial value problem, one requires a value for $p(0)$, and $q(0)$. As no such values are given at boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and integration is carried out. Comparing the calculated values for f' and θ at $\eta=10$ (say) with the given boundary conditions $f'(10)=0$ and $\theta(10)=0$ the estimated values were adjusted $f''(0)$ and $\theta'(0)$, and the fourth order Runge – Kutta method was applied with step size $h=0.01$. The procedure was repeated until the converged results were obtained.

4. Results and discussions

In order to analyze the results, numerical computations has been carried out for various values of suction parameter (S), stratification parameter (St), magnetic parameter (M), Prandtl number (Pr), radiation parameter (R). The results were analyzed by figures 2a – 6b.

Fig 2a and 2b depicts the effect of suction parameter S on velocity and shear stress profiles for exponentially stretching sheet. It is noticed that fluid velocity decreases with increase of suction parameter. It is very clear that shear stress incrreases initially, but after certain distance from the sheat shear stress decreases significantly. Fig 2c and 2d shows the influence of suction parameter on temperature and temperature gradient profiles. It is observed that the temperature decreases with increase of suction parameter. The temperature gradient increases with increase of of suction parameter initially but after a certain distance we can observe the reverse trend in the profiles. But far away from the wall appreciable change was not observed. ie suction at the boundary will reduce both the hydrodynamic and thermal boundary layer thickess.

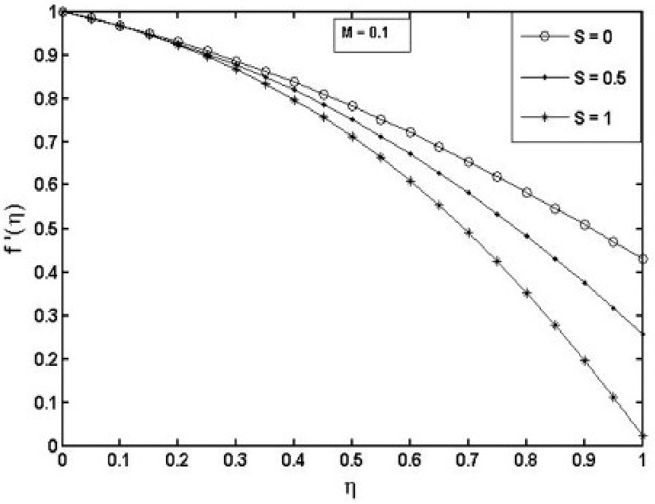

Fig – 2a: Variation of horizontal velocity $f'(\eta)$ with suction parameter

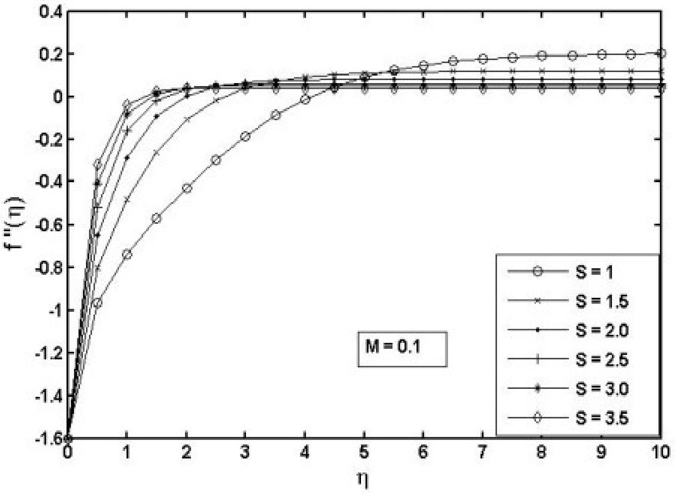

Fig – 2b: Variation of shear stress $f''(\eta)$ with suction parameter

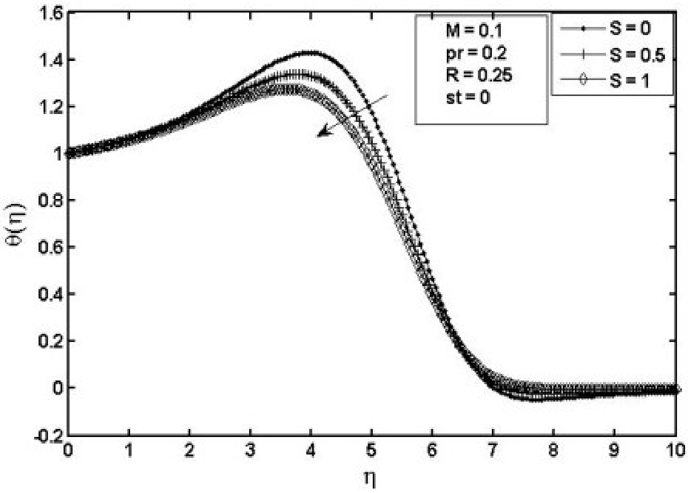

Fig – 2c: Variation of temperature $\theta(\eta)$ for several values of suction parameter

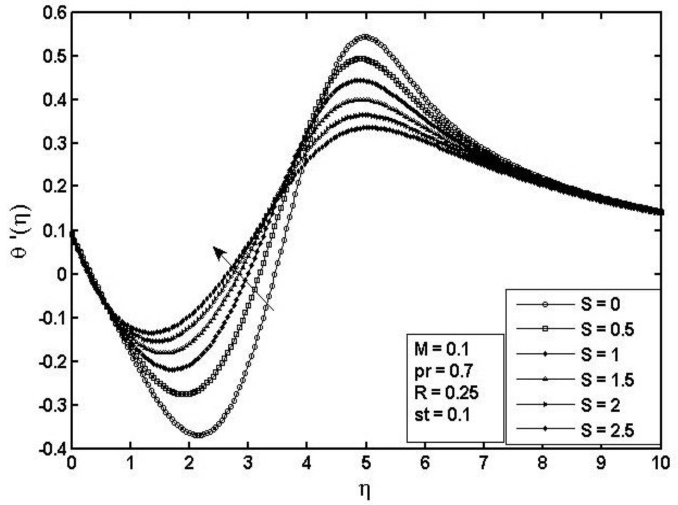

Fig – 2d: Variation of temperature gradient $\theta'(\eta)$ for several values of suction parameter.

Fig 3 represents velocity profiles for different values off magnetic parameter M. As the magnetic parameter increases fluid velocity is found to be decrease. Actually rate of transport decreases with the increase of the magnetic parameter M, because of the Lorentz force which opposes the motion of the fluid. As we move far away from the boundary the effect was nullified.

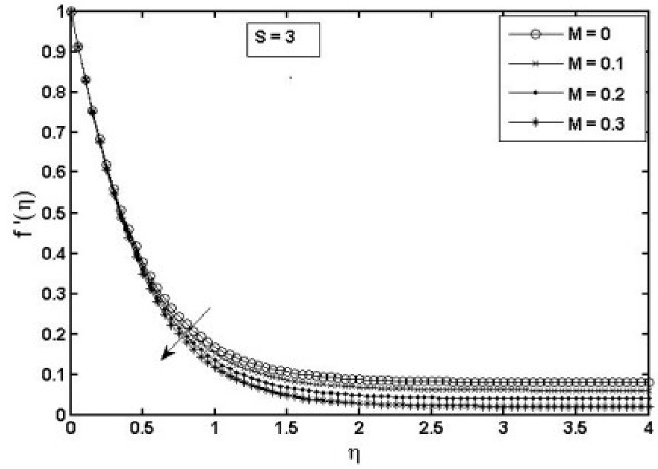

Fig – 3: Variation of horizontal velocity $f'(\eta)$ for several values of Magnetic parameter

The influence of stratification parameter on temperature and temperature gradient are presented in fig 4a and 4b. It is found that the temperature decreases as the the stratification parameter St increases. It is quite obvious, since increase in St means increase in free- stream temperature or decrease in surface temperature. Thermal boundary layer thickness is therfore also decreased with an increase in St. All profiles decay from maximum value at the wall and converge to zero at the outer edge of the boundary layer. Figure 4b exhibits that temperature gradient increases considerably with increase of stratification parameter. But the effect is almost zero far away from the wall.

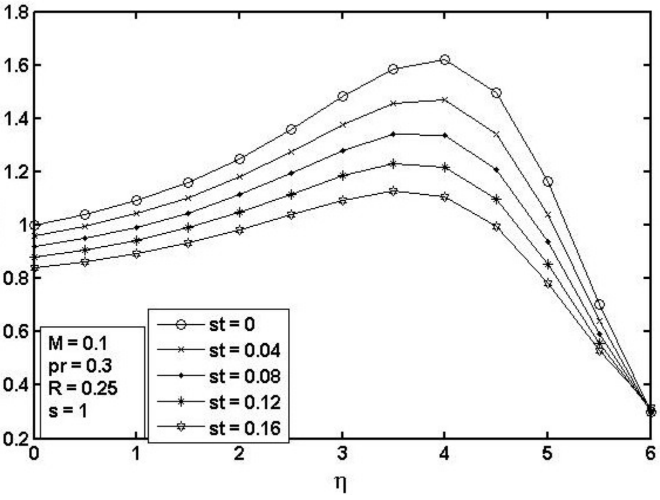

Fig – 4a: Variation of temperature $\theta(\eta)$ for several values of stratification parameter

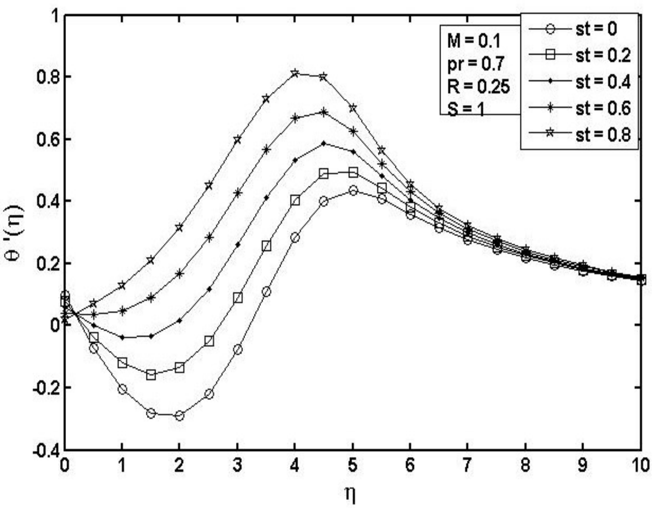

Fig – 4b: Variation of temperature gradient $\theta'(\eta)$ for several values of stratification parameter

Fig 5a and 5b shows the nature of temperature and temperature gradient profiles for different values of prandtl numbers. It is observed that increase of prandtl number leads to decrease of temperature of the fluid. However far away from the sheat the effect is almost zero. It shows that very nearer to the sheat temperature gradient seems to be decreases with increase of prandtl number. However a reverse trend was observed after a certain distance from the sheet.

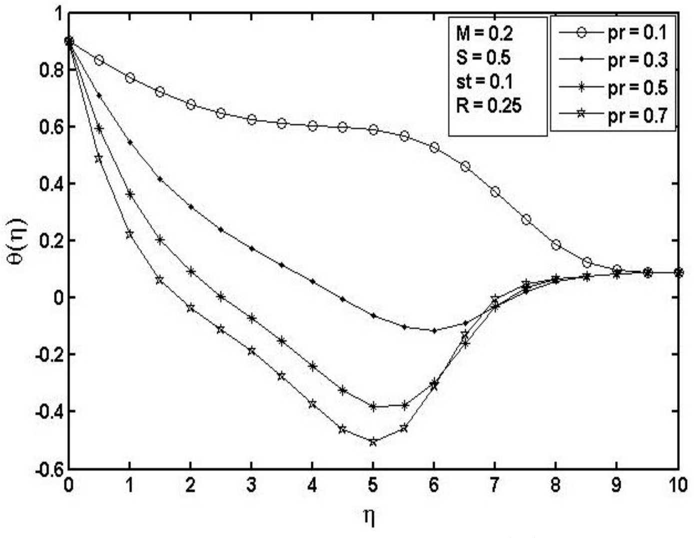

Fig – 5a: Variation of temperature $\theta(\eta)$ for several values of prandtl number

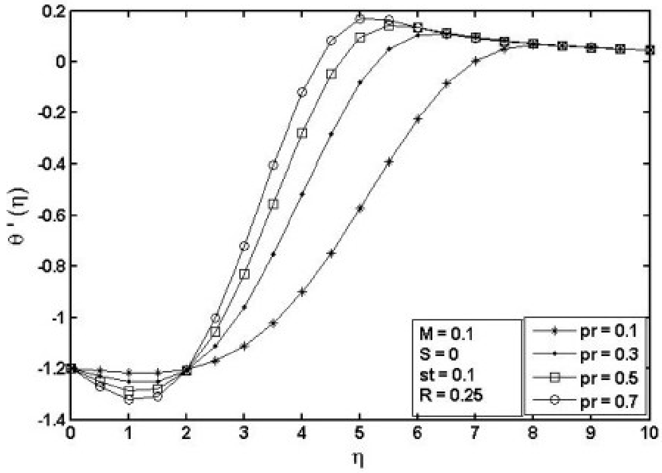

Fig – 5b: Variation of temperature gradient $\theta'(\eta)$ for several valules of prandtl number

Fig 6a and 6b analizes the the temperature and temperature gradient profiles under the influence of radiation parameter R. Fig 6a Shows that temperature of the fluid increases with increase of radiation parameter. Far away from the sheet the effect is zero. Fig 6b shows that increase of radiation parameter results in decrease of temperature gradient.

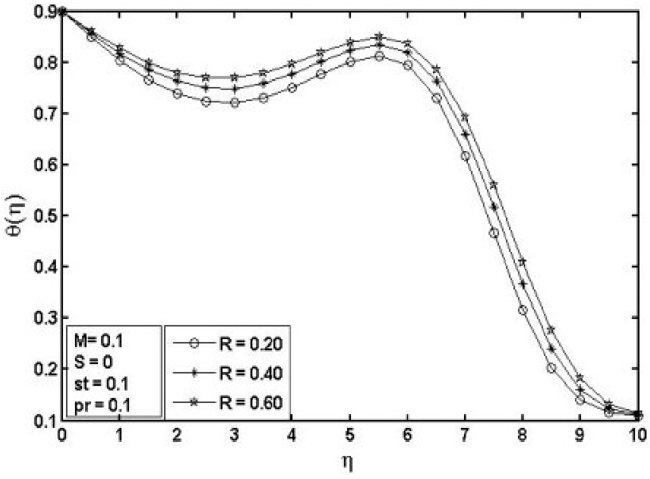

Fig – 6a: Variation of temperature for several values of radiation parameter

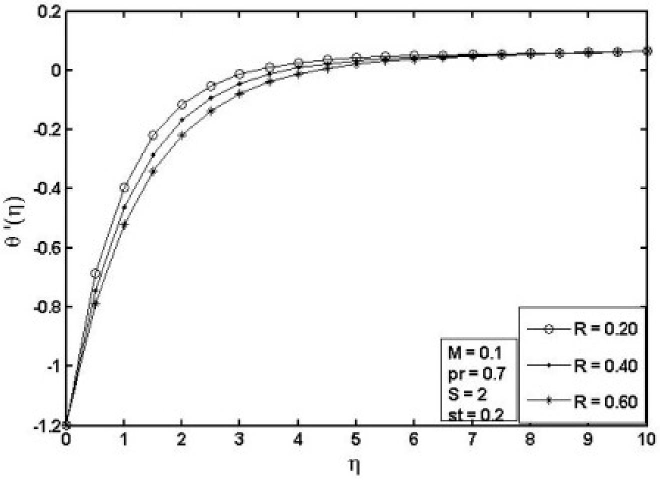

Fig – 6b: Variation of temperature gradient for several values of radiation parameter

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