Modeling and Controller Designing of Rotary Inverted Pendulum (RIP)-Comparison by Using Various Design Methods

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*Abstract*__ **Inverted pendulum control is one of the fundamental and interesting problems in the field of control theory. This paper describes the steps to design various controllers for a rotary motion inverted pendulum which is operated by a rotary servo plant, SRV 02 Series. Nonlinear model of the rotary inverted pendulum and linear model (Upright) in the neighborhood of an equilibrium state are described. In this paper the controller consists of three parts: a swing-up controller, a catch controller, and a state feedback stabilizing controller. Designing the control system using PID is quiet challenging task for the rotary inverted pendulum because of its highly nonlinear and open-loop unstable characteristics. Modern control techniques are analyzed to design the controllers for linear model of rotary inverted pendulum, are called stabilization controllers. The paper describes the two Modern Control techniques those are Full State Feedback (FSF) controller and Linear Quadratic Regulator controller (LQR). Designed a swing up controller that raises the pendulum to the inverted position in a controlled manner, where the stabilization controller can stabilize it (Self-erecting pendulum). Here FSF and LQR control systems are tested both for the Upright and Swing-Up mode of the Pendulum. Digitalization of the plant is done by designing discrete 2DOF(Two Degree of Freedom) pid controller using root locus technique. MATLAB based simulation results are described and compared based on the above control methods which are designed to control the Rotary Inverted Pendulum.**

Index Terms—Full state feedback, Linear Quadratic Regulator, Two Degree of Freedom

I.INTRODUCTION

To verify a modern control theory the inverted pendulum control can be considered as a very good example in control engineering. It is a very good model for the attitude control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc.

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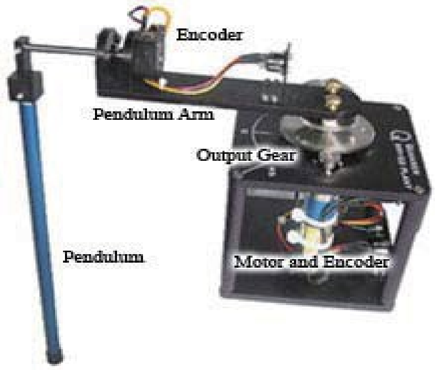
This model also can be an initial step in stabilizing androids. The inverted pendulum is highly nonlinear and open-loop unstable system that makes control more challenging. It is an intriguing subject from the control point of view due to its intrinsic nonlinearity. Common control approaches such as, FSF control and LQR requires a good knowledge of the system and accurate tuning in order to obtain desired performances. However, an accurate mathematical model of the process is often extremely complex to describe using differential equations. Moreover, application of these control techniques to a humanoid platform, more than one stage system, may result a very critical design of control parameters and stabilization difficulty.

The analysis and design of feedback control system are carried out using transfer functions along with various tools such as root-locus plots, Bode plots, Niquist plots, Nichol's chart etc. These are the techniques in classical control theory where the classical design methods suffer from certain limitations because; the transfer function model is applicable only for linear time-invariant system and generally restricted to SISO system. The transfer function technique reveals only the system output for a given input and it does not provide any information of internal behavior of the system. These limitations of the classical method have led to the development of state variable approach, direct time domain approach, which provides a basis of modern control theory .It is a powerful technique for the analysis and design o linear and nonlinear, time-invariant or time-varying multi input, multi-output (MIMO) system response.

FSF control also known as Pole Placement, is a method which is employed in state feedback control theory to place the closed-loop pools of a plant in pre-determined locations in the s-plan. Placing poles is desirable because the location of the pools determines the Eigen values of the system, which controls the characteristics of the system response. The FSF algorithm is actually an automated technique to find an appropriate state-feedback controller. Another alternative technique, LQR is also a powerful method to find a controller over the use of the FSF algorithm. The rotary motion inverted pendulum, which is shown in Figure 2, is driven by a rotary servo motor system (SRV-02).



Fig(1) Rotary inverted pendulum model SRV-2.

II. MATHEMATICAL MODELING OF ROTARY MOTION INVERTED PENDULUM

Figure (2) (a) shows the rotational direction of rotary inverted pendulum arm. Figure 2 (b) depicts the pendulum as a lump mass at half of the pendulum length. The pendulum is displaced with an angle α while the direction of θ is in the x-direction of this illustration. So, mathematical model can be derived by examining the velocity of the pendulum center of mass. The following assumptions are important in modeling of the system:

1) The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.

2) The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.

3) A small disturbance can be applied on the pendulum. As the requirements of the design, the settling time, Ts, is to be less than 0.5 seconds, i.e. Ts < 0.5secs. The system overshoot value is to be at most 10%, i.e. Ts = 10%. The following system parameters are the list of the terminology used in the derivations of system model.

The system parameters are as follows:
Armature resistance, R=2.6Ω
Motor voltage constant, K_m=0.00767 V-s/rad
Motor torque constant, K_τ=0.00767 N-m
Armature inertia, $J_m = 3.8710\times 10^{-7}\ Kgm^2$
Tachometer inertia $J_{tac} = 0.7\times 10^{-7}\ Kgm^2$
High gear ratio, $K_g = (14)(5)$
Equivalent viscous friction referred to the secondary gear $B_{eq} = K^2{}_g B_m + B_L = 4\times 10^{-3} Nm/(rad/s)$
Motor efficiency due to rotational loss $\eta_{gb} \cong 0.87$
Gearbox efficiency, $\eta = \eta_{mr}\eta_{gb} = (0.87)(0.85) = 0.7395$

Arm radius = r =21.5cm
Pendulum length= L=16.75cm
Pendulum mass =m=0.125Kg
Gravitational acceleration=g=9.8m/s^2
Distance of Pendulum Center of mass from ground=h
θ =Servo gear angular displacement
ω =Servo gear angular velocity
α =Pendulum angular deflection
υ =Pendulum angular velocity

There are two components for the velocity of the Pendulum lumped mass. So,

$$V_{Pen.center\ \ of\ mass} = -Lcos\alpha(\dot{\alpha})\hat{x} - Lsin\alpha(\dot{\alpha})\hat{y} \quad (1)$$

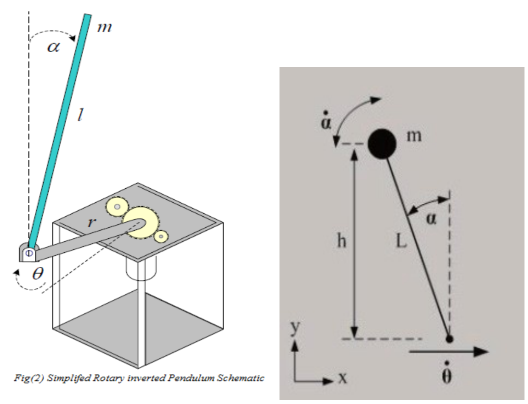
The pendulum arm also moves with the rotating arm at a rate of:

$$V_{arm} = r\dot{\theta} \quad (2)$$

The equations (1) and (2) can solve the x and y velocity components as,

$$V_x = r\dot{\theta} - Lcos\alpha(\dot{\alpha}) \quad (3)$$

$$V_y = -Lsin\alpha(\dot{\alpha}) \quad (4)$$



(a) (b)

Fig (2) Pendulum motion and lump mass

A. Deriving the system dynamic equations

Having the velocities of the pendulum, the system dynamic equations can be obtained using the Euler-Lagrange formulation.

1) Potential Energy: The only potential energy in the system is gravity. So,

$$V = P.E_{pendulum} = mgh = mgLcos\alpha \quad (5)$$

2) Kinetic Energy: The Kinetic Energies in the system arise from the moving hub, the velocity of the point mass in the x-direction, the velocity of the point mass in the y direction and the rotating pendulum about its center of mass.

$$T = K.E_{hub} + K.E_{V_x} + K.E_{V_y} + K.E_{pendulum} \quad (6)$$

Since the modeling of the pendulum as a point at its center of mass, the total kinetic energy of the pendulum is the kinetic energy of the point mass plus the kinetic energy of the pendulum rotating about its center of mass. The moment of inertia of a rod about its center of mass is,

$$J_{cm} = (1/12)MR^2 \quad (7)$$

Since L is defined as the half of the pendulum length, R in this case would be equal to 2L. Therefore the moment of inertia of the pendulum about its center of mass is,

$$J_{cm} = \left(\frac{1}{12}\right)MR^2 = \left(\frac{1}{12}\right)M(2L)^2 = (1/3)ML^2 \qquad (8)$$

So, the complete kinetic energy, T, can be written as

$$T = (1/2)J_{eq}\dot{\theta}^2 + \left(\frac{1}{2}\right)m(r\dot{\theta} - Lcos\alpha(\dot{\alpha}))^2 + \left(\frac{1}{2}\right)m(-Lsin\alpha(\dot{\alpha}))^2 + J_{cm}\,\dot{\alpha}^2 \qquad (9)$$

After expanding the equation and collecting terms, the Lagrangian can be formulated as,

$$L = T - V = \left(\frac{1}{2}\right)J_{eq}\dot{\theta}^2 + ((2/3)mL^2\dot{\alpha}^2) - mLrcos\alpha(\dot{\alpha})(\dot{\theta}) + \left(\frac{1}{2}\right)mr^2\dot{\theta}^2 - mgLcos\alpha \qquad (10)$$

The two generalized co-ordinates are θ and α. So, another two equations are,

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\theta}}\right) - \frac{\delta L}{\delta\theta} = T_{output} - B_{eq}\dot{\theta} \qquad (11)$$

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\alpha}}\right) - \frac{\delta L}{\delta\alpha} = 0 \qquad (12)$$

From the above definition let

$$\dot{\theta} = \omega \qquad (13)$$

$$\dot{\alpha} = \vartheta \qquad (14)$$

Solving the equations we get the governing differential equations of the system are as follows

$$(J_{eq} + mr^2)\dot{\omega} - \left(\frac{1}{2}\right)mLrcos\alpha\dot{\vartheta} + (1/2)mLrsin\alpha\vartheta^2 = T_{output} - B_{eq} \qquad (15)$$

$$\left(\frac{1}{3}\right)mL^2\dot{\vartheta} - \left(\frac{1}{2}\right)mLrc\$os\alpha\dot{\omega} - (1/2)mgLsin\alpha = 0$$
(16)

The output Torque of the motor which act on the load is defined as,

$$T_{output} = K_1V_i(t) - K_2\omega(t)$$
(17)

Where $K_1 = \frac{\eta K_m K_g}{R_a}$

$$K_2 = \frac{\eta {K_m}^2 {K_g}^2}{R_a}$$

We can write

$$a\dot{\omega} - bcos\alpha\dot{\upsilon} + bsin\alpha\upsilon^2 + B_{eq}\,\omega = K_1V_i(t) - K_2\omega(t)$$

$$c\dot{\upsilon} - bcos\alpha\dot{\omega} - dsin.\alpha = 0 \qquad (18)$$

And also

$$\dot{\upsilon} = \frac{1}{f(u)}[adsin\alpha - (pbcos\alpha)\omega - (b^2cos\alpha sin\alpha)\upsilon^2 + (bK_1cos\alpha)V_i(t)]$$

$$\dot{\omega} = \frac{1}{f(u)}[dbcos\alpha sin\alpha - (cp)\omega - (cbsin\alpha)\upsilon^2 + (cK_1)V_i(t)] \qquad (19)$$

where

$$a = (J_{eq} + mr^2), b = \left(\frac{1}{2}\right)mLr, c = \left(\frac{1}{3}\right)mL^2$$

$$d = (1/2)mgL, p = B_{eq} + K_2, f(u) = ac - b^2cos^2\alpha$$

In order to design a linear regulator state feedback, we need to linearise the model.

Equations (19) can be linearised by considering the equilibrium state of the system. If we assume α is small (i.e., when the Inverted Pendulum is near its equilibrium point), we can linearised these equations. For small α, $\sin\alpha \approx \alpha$ and cosα≈1. Also, for small α, υ^2 is negligible, and we get the following linearised equation

$$a\dot{\omega} - b\dot{\upsilon} + B_{eq}\,\omega = K_1V_i(t) - K_2\omega$$

$$c\dot{\upsilon} - b\dot{\omega} - d\alpha = 0$$

And

$$\dot{\upsilon} = \frac{ad}{e}\alpha - \frac{pb}{e}\omega + \frac{bK_1}{e}V_i(t)$$

$$\dot{\omega} = \frac{bd}{e}\alpha - \frac{pc}{e}\omega + \frac{cK_1}{e}V_i(t) \qquad (20)$$

$$e = ac - b^2, p = B_{eq} + K_2$$

The linearized model is used to design the stabilizing controller. We now obtain an state-space model

$\dot{\boldsymbol{x}}$(t)=$\boldsymbol{Ax}$(t)+$\boldsymbol{B}u$(t)

y(t)= $\boldsymbol{Cx}$(t)

For the combined servomotor and the Inverted Pendulum module, we choose the state variables as $\boldsymbol{x}$(t)

$\boldsymbol{x}$(t)= $[\theta\ \alpha\ \omega\ \upsilon]^T$

By combining Equations(20) we obtain the following linear state space model of the inverted pendulum.

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \dot{\omega}\\ \dot{\upsilon}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{bd}{e} & \frac{-pc}{e} & 0\\ 0 & \frac{ad}{e} & \frac{-pb}{e} & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \omega\\ \upsilon\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{cK_1}{e}\\ \frac{bK_1}{e}\end{bmatrix}V_i(t)$$

III. CONTROLLER DESIGN

A. Designing stabilization controller for linearised model (Up right mode) of RIP by using modern controller design techniques.

1. FSF controller
2. LQR controller

B. For controlling the nonlinear model (Swing up mode) of the RIP, design following controllers so that the RIP getting into linearization area .Then apply stabilization controller.

1. Swing up controller
2. Catch controller

C. Designing 2DOF discrete controller using root locus method for linearised model of RIP

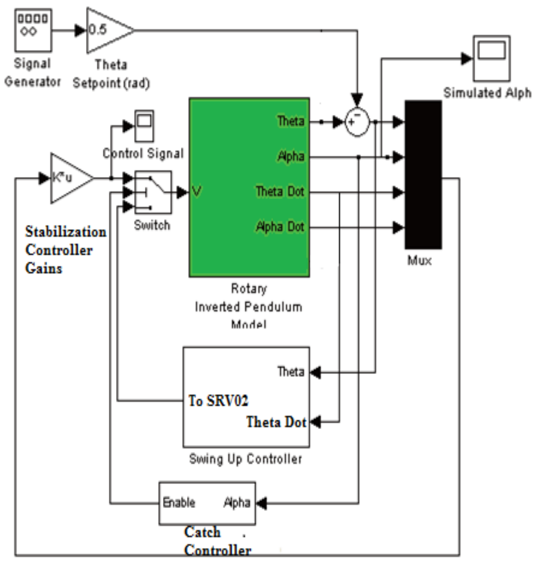


Fig (3) Block diagram Controller for rotary inverted pendulum model (MATLAB Simulink).

A. Stabilization controller design

1. *FSF controller design based on Ackerman's formula*

In modern control system, state x is used as feedback instead of plant output y and k indicates the gain of the system. To design a FSF controller Ackerman's formula is used which is an easy and effective method in modern control theory to design a controller via pole placement technique. Figure 4 shows a basic block diagram of a FSF controller of a system.

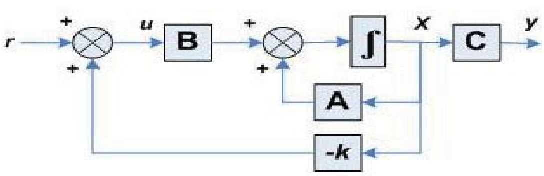


Fig (4) Block diagram of a FSF controller of a system

Ackerman's formula is represented as,

$$K = [0 \dots .0\ 1] M_c^{-1} \phi_d(A)$$

$$M_c = [B\ AB \dots\ A^{(n-1)}B]$$

Where MC indicates the controllability matrix and $\phi_d(A)$ is the desired characteristic of the closed-loop poles which can be evaluated as s=A. The close loop transfer function is selected based on ITAE table (figure 5) and the value of frequency is taken as10rad/s

As the denominator of the transfer function of $\theta(s)/V_i(s)$ is a fourth order polynomial, from the ITAE table the characteristic equation will be,

$$S^4 + 24.1S^3 + 493S^2 + 5140S + 10000 = 0$$

$$s^2 + 3.2\omega_0 s + \omega_0^2$$
$$s^3 + 1.75\omega_0 s^2 + 3.25\omega_0^2 s + \omega_0^3$$
$$s^4 + 2.41\omega_0 s^3 + 4.93\omega_0^2 s^2 + 5.14\omega_0^3 s + \omega_0^4$$
$$s^5 + 2.19\omega_0 s^4 + 6.50\omega_0^2 s^3 + 6.30\omega_0^3 s^2 + 5.24\omega_0^4 s + \omega_0^5$$
$$s^6 + 6.12\omega_0 s^5 + 13.42\omega_0^2 s^4 + 17.16\omega_0^3 s^3 + 14.14\omega_0^4 s^2 + 6.76\omega_0^5 s + \omega_0^6$$

Fig (5) ITAE table

The controller matrix gain can be calculated using the MATLAB code. So the gain matrix becomes,

- K =[-8.9144 32.6339 -5.1506 5.7392]

Using this K and the control law u = -Kx, the system is stabilized around the linearized point (pendulum upright).The state feedback optimal controller is only effective when the pendulum is near the upright position. In the plant model of the rotary inverted pendulum, the output theta (θ) is regulated by the input voltage (V). Here theta has the responsibility to keep the inverted pendulum vertically upright where alpha (α) will be zero. Figure 3 shows the controller block diagram to control the pendulum.

The pendulum starts from down words position and whenever it comes to the upright mode the full state feedback controller will maintain the pendulum in that position and make it stable.

2. *Controller design using LQR technique*

Linear Quadratic Regulator (LQR) is design using the linearized system. In a LQR design process, the gain matrix K, for a linear state feedback control law *u* = -*Kx*, is found by minimizing a quadratic cost function of the form as,

$$J = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) dt \qquad (21)$$

Here Q and R are weighting parameters that penalize certain states or control inputs. In the design the weighting parameters of the optimal state feedback controller are chosen as,

$$Q = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R = 1$$

In this design, the controller gain matrix, K, of the linearised system is calculated using MATLAB function.

Basically this method is another powerful technique to calculate the gain matrix which is applied in the same model using in FSF controller design.

K =[-2.4495 27.5815 -2.5505 3.9197]

Here the selected diagonal matrix, Q is chosen where the values of q11, q22, q33 and q44 are 6, 1, 1 and 0 respectively. The diagonal values are selected based on iterative method. It is found that the diagonal

values for q11 and q22 are more sensitive than others. Output performances are tested based on four different values of the matrix Q. The test cases are as follows
,

$$Q_1 = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} Q_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Q_3 = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} Q_4 = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 13(a) and 13(b) show the simulated output of theta and alpha based on the Q1 matrix where diagonal values are6, 1, 1 and 0 (test 1). From the results of other test cases, it is clearly identified that the settling time of theta for test Q1 is less. Though the alpha overshoot and settling time for test Q2 is smaller to test Q1, still the performance of the test Q1 is better than other because of the settling time. In practical case the test Q1 result also shows the better performance

B. Controlling the nonlinear model (Swing up mode) of the RIP:

Control strategy:

The controller consists of three parts: a *swing-up controller*, a *catch controller*, and a state *feedback stabilizing controller*.

Manual Mode – For this mode, the pendulum is raised manually by hand towards the upright position (no swing up controller) until it is captured by the catch-controller which turns on the state feedback stabilizing controller.

Self-erecting pendulum **-** Initially, the pendulum is in the downward position and the swing-up controller is used to bring it to the top position. Once it is sufficiently close to the top equilibrium position, it is caught and switched to the state feedback stabilization controller.

1. *Swing-up Controller*

As you have found out the linearised model is accurate when the pendulum angle α is within a small range$\pm 30^0$.Therefore, the state feedback stabilization controller is accurate when pendulum is within this region. In the manual mode, the pendulum is brought up to the vertical position by hand. Alternatively a Swing-up controller can be designed to swing the pendulum to the upright position from the stable ‘rest’ position. This controller is responsible for the swinging up the pendulum to a position in which the stabilization mode can takeover. First the servomotor is placed under position control then an algorithm is prescribed for the driving force. As in position control we use a rate feedback and a position feedback given by

$$V_i(s) = K_p[\theta_i(s) - \theta_0(s)] - K_D\Omega_0(s) \qquad (21)$$

This feedback loop is shown in Figure 3.10. The purpose of this system is to have the output angle$\theta_0(t)$, follow the desired position$\theta_i(t)$. Design the inner loop for position control to meet the following time-domain specifications:

- Step response damping ratio ζ=0.8
- Step response peak time t_p= 0.115second

$$\frac{\theta_0(s)}{\theta_i(s)} = \frac{K_p a_m}{S^2 + (K_D a_m + b_m)S + K_p a_m} \qquad \textbf{(22)}$$

$$a_m = \frac{\eta K_m K_g}{R_a J_{eq}} \quad b_m = \frac{B_{eq}}{J_{eq}} + \frac{\eta K_m{}^2 K_g{}^2}{R_a J_{eq}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \frac{\theta_0(s)}{\theta_i(s)} = \frac{\omega_n{}^2}{S^2 + 2\zeta\omega_n S + \omega_n{}^2} \qquad \textbf{(23)}$$

Where $K_p = \frac{\omega_n{}^2}{a_m} \quad K_D = \frac{2\zeta\omega_n - b_m}{a_m}$

From the given specifications computeω_n, and use to determine, K_p and K_D

Many schemes can be devised to prescribe a driving force for a suitable trajectory in a controlled manner that energy is gradually added to the system to bring the pendulum to the inverted position.

2. *Catch Controller*

Catch Controller purpose is to track the pendulum angle α, and facilitate switching between the swing-up and stabilization modes. This controller is to be enabled when α is in the neighborhood of zero, within± 5 and for as long as$|\alpha| < 25^0$

Switch for activating the stabilization controller

This switch is triggered by the output of the Catch controller. Initially when the pendulum is hanging down, the Switch terminal 2 (See Figure 3) is at zero state and the control signal is passed through terminal 3 (lower terminal). If the pendulum is brought to the upright position (either manually or by means of the swing-up controller) as α reaches in the neighborhood of $\pm 5^0$ the Catch controller is enabled. The Switch control input state becomes 1, the signal is passed through terminal 1 (top terminal) and stabilization controller takes over.

C.2DOF Discrete controller design using root locus method:

Transfer functions representing above linear model by substituting system parameters are,

$$G_{\alpha,V_i}(s) = \frac{\alpha(s)}{V_i(s)} = \frac{33.51S^2}{S^4 + 22.52S^3 - 91.17S^2 - 945.6S}$$

$$G_{\theta,\alpha}(s) = \frac{\theta(s)}{\alpha(s)} = \frac{39.1S^2 - 1642}{33.51S^2} \qquad (24)$$

Selected 4 times the highest damped natural frequency as sampling frequency

Controller specification:
Mp<5%
Settling Time<0.5sec
Dominant pole location in s-domain
$S = -\zeta\omega_{n} + i\omega_n\sqrt{1-\zeta^2}$
Dominant closed loop pole location in Z-domain
$$z = e^{sT} = 0.9199 \pm 0.0774$$
With Sampling time=0.01ms we can write
$$G_{\alpha,V_i}(z) = \frac{\alpha(z)}{V_i(z)} = \frac{0.001558\,z^2 - 0.0001126\,z - 0.001445}{z^3 - 2.807\,z^2 - 2.605\,z - 0.7984}$$

$$G_{\theta,\alpha}(z) = \frac{\theta(z)}{\alpha(z)} = \frac{1.167\,z^2 - 2.335\,z + 1.165}{(z-1)^2} \qquad (25)$$

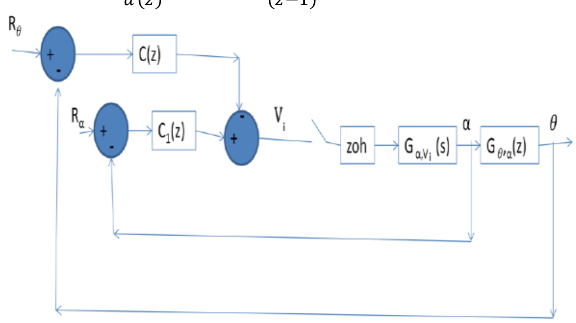


Fig (6) Block diagram of RIP

We have one pole outside of unit circle, and one zero on unit circle.

Since a root locus always starts from an open loop pole and ends at an open loop zero or infinity, the root locus will always lie between the above mentioned pole and zero, irrespective of the gain of the system, this rules out a P controller. To bring the unstable pole inside the unit circle, we have to make multiple roots branch from the root locus which is lying outside the unit circle. To accomplish this, we place one pole at (1,0). Now, we place two zeros inside the unit circle so that the root locus between them will branch in from the pole which is lying outside the unit circle. Hence, we have selected a PID controller for controlling the system with only one feedback. The basic controller form for this type of controller is:

$$C_1(z) = K_p + \frac{K_v}{T_s}(1 - z^{-1}) + \frac{K_i T_s}{(1 - z^{-1})} \qquad (26)$$

which can be written as;

$$C_1(z) = K\frac{(z-\alpha)(z-\beta)}{z(z-1)} \qquad (27)$$

The pole and zero and unit circle are not considered in calculation as, angle contribution from zero will be cancelled by pole at the same location.

Angle deficiency=2.4-(4.81-29.28-69.33-148.68)
= -249.33

Angles to be contributed by zeros
$= -(-249.33+180) \approx 69.7\ deg$

The tentative locations of zeros are calculated to be 0.8 and 0.9

$$C_1(z) = \frac{V_{i\alpha}(z)}{\alpha(z)} = 80\frac{[(z-0.8)(z-0.9)]}{z(z-1)} \qquad (28)$$

In order to achieve the secondary task of controlling the motor angle θ and maintaining it at a specified reference value, we try to get additional feedback for θ as well.

Here, design constraint on setting time of theta and overshoot of theta are not stringent. We will just follow, Theta Ts> Alpha Ts We are doing this because we do not want to saturate motor voltage. Our main objective is to keep pendulum in upright position.

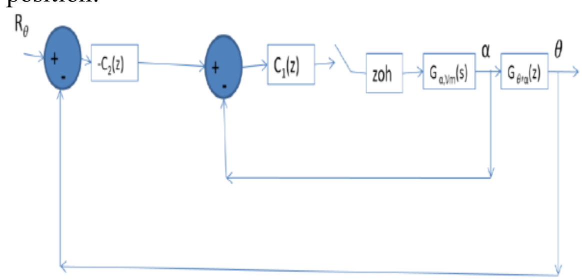


Fig (7) Reduced block diagram of RIP

In the above block diagram we have inner loop which is controlling pendulum angle and the outer loop is controlling the motor swing angle.

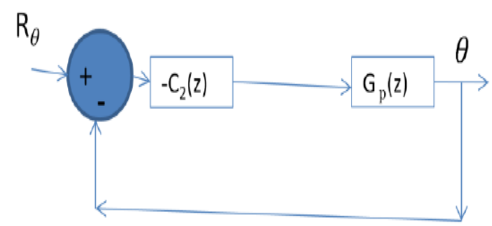


Fig (8) Final block diagram of RIP

$$G_p(z) = \frac{G_{\alpha,V_i,op}(z)}{1 + G_{\alpha,V_i,op}(z)} \times G_{\theta,\alpha}(z) \qquad (29)$$

At this point, it is very difficult to analyze the open loop transfer function with root locus analysis. We will go for bode diagram to see the type of controller required.

From above bode plot, our system is unstable on its own because of negative phase margin. We can make our system stable with a lead compensator. A lead compensator increases systems bandwidth and improves phase margin. Hence, the form of controller is PD

$$C_2(z) = K_p + \frac{K_v}{T_s}(1 - z^{-1}) \qquad (30)$$

We can write as $C_2(z) = K\frac{(z-z_1)}{z}$

$$C_2(z) = \frac{V_{i\theta}(z)}{\theta(z)} = (-15) \times \frac{(z-0.94)}{z} \qquad (31)$$

We have achieved reasonable phase margin and gain margin to have system stable.

The response takes little longer to settle down. This is beneficial and necessary since in a cascade control, we require the outer loop to have a larger settling time than the inner loop. As long as we are able to achieve the desired motor swing angle, our controller is satisfactory

IV. COMPARISON OF RESPONSE FOR PENDULUM ANGLE,

TABLE 1

	Rising time	Settling time	Overshoot range	Steady-state error
FSF Controller	0.16	1.24	-16.6 to -16.6	-0.004
LQR Controller	0.14	0.39	2.5 to 0.86	-0.0003
2DOF Discrete PID Controller	0.04	0.491	0.00621 to -0.00278	-0.0059

MATLAB Simulation Results

1).Stabilization Controller Output Results:

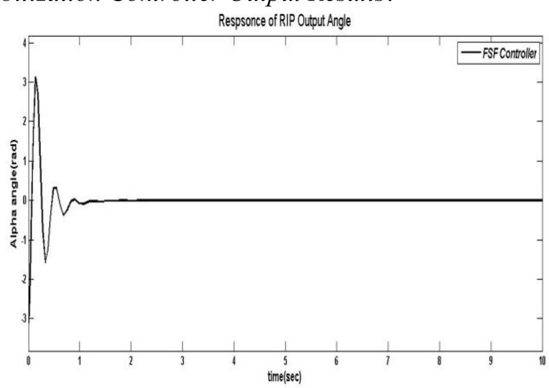


Fig (9) FSF Controller Output Response

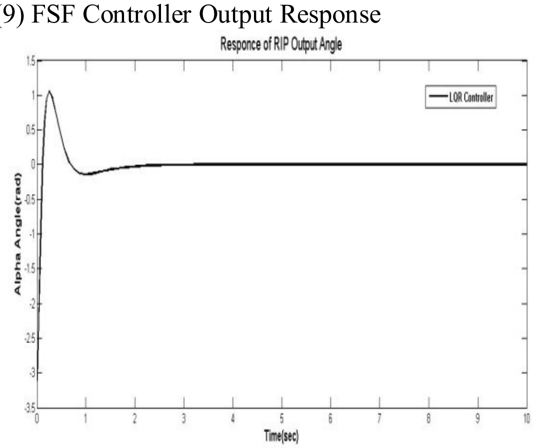


Fig (10) LQR Controller Output

2).2DOF Discrete Controller Output Results

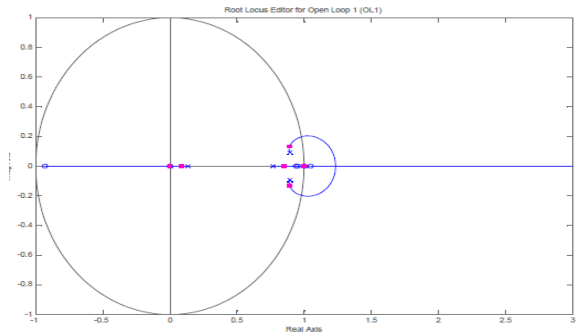


Fig (11) Root locus diagram of RIP final system

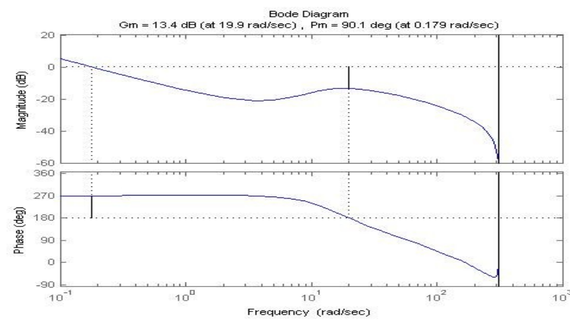


Fig (12) Bode Diagram after Compensation

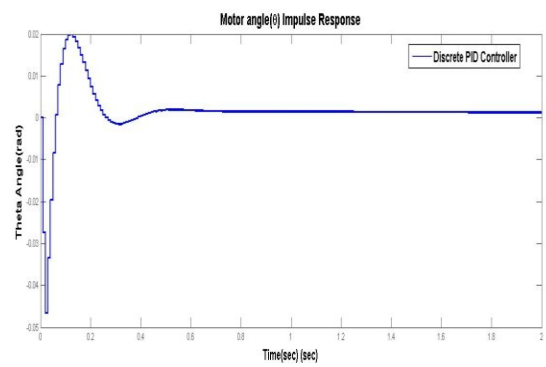


Fig (13) Motor output angle Response

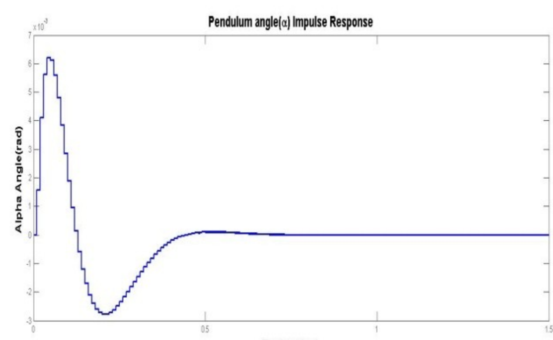


Fig (14) RIP Output Angle Response

V. CONCLUSION

Based on the result, shown in Table I, Simulation studies determine the efficiency, reliability and accuracy of two controllers, FSF and LQR. The controllers not only meet the design requirements but also are robust to the parameter variations. The LQR controller is more robust and reliable than the FSF controller in successfully swinging the pendulum to the upright position. As the data indicates, the LQR controller is also faster than the FSF controller. Overall, it is seen that the LQR controller is more convenient to swing up the pendulum to its upright mode and maintain stability on the unstable equilibrium point. From the experimental results, however, that both controllers (FSF and LQR) can be effective in maintaining the rotary inverted pendulum stable. We were successfully able to design controllers that performed the desired tasks of stabilizing the Rotary Inverted Pendulum System. During this process, we learnt how a physical system can be analyzed as a simplistic model and how the digital control techniques, which we learnt during the course, could be used to achieve the desired outputs. We also learnt that the physical system is not always a perfect match to the simplistic model we suppose it to be. We have to make some adjustments in the designs so as to accommodate factors like friction and backlash. We could use PD controller to improve systems' margin. The approximation to continuous 2nd order system gives reasonably good results for dominant closed loop pole location. The tradeoff between settling time of pendulum and motor swing angle is very important to avoid saturation in voltage.

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