Spatial Bias Correction Based on Gaussian Kernel Fuzzy C Means in Clustering

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Abstract--- **Clustering is the process of grouping data objects into set of disjointed classes called clusters so that objects within a class are highly similar to one another and dissimilar to the objects in other classes. K-means (KM) and Fuzzy c-means (FCM) algorithms are popular and powerful methods for cluster analysis. However, the KM and FCM algorithms have considerable trouble in a noisy environment and are inaccurate with large numbers of different sample sized clusters. The Kernel based Fuzzy C-Means (KFCM) clustering is moreover studied with associated cluster validity measures. Many numerical simulations are used to evaluate whether or not the kernelized measures are adequate for ordinary ball-shaped clusters. Finally, a new class of kernel functions is Gaussian kernel based Fuzzy C-Means (GKFCM) is proposed in this research. The proposed GKFCM algorithm becomes a generalized type of, BCFCM, and KFCM algorithms and presents with more efficiency and robustness.**

Index Terms--- **Clustering, Fuzzy C-Means, BCFCM, KFCM and GKFCM.**

I. Introduction

Clustering analysis is a scientific method utilizing unsupervised learning criteria to segment a given data set into groups of similar individuals. Clustering is a popular method used for image segmentation for its simplicity and easiness to implement. The commonly used clustering algorithms are hard c means and the fuzzy c-means algorithm. It is an important task in areas such as machine learning, pattern recognition and artificial intelligence. K means is a well known and widely used partitioned clustering algorithm that is easy to implement, very efficient and has linear time complexity. KM partitions the dataset into clusters, where one object belongs to only one cluster. The crisp membership function causes drawback of the KM algorithm. The cluster result is sensitive to the initial cluster centers and may converge to the local optima. FCM algorithms have considerable trouble in noisy environments, or when the difference of sample size in each cluster is large.

There are two approaches to mitigate the impact of the problems to improve the adaptively and robustness of FCM algorithm. On the one hand, many researchers have concentrated on extending distance functions in FCM. The FCM objective function is modified for compensating the strength in homogeneities and by allowing the pixel labeling to be influenced by the labels in its instant neighborhood. This modification of FCM is called Bias-corrected fuzzy *c*-means (BCFCM) algorithm. But this BCFCM algorithm also lacks robustness to noise and it takes more time for execution. So, kernel version of FCM is introduced to solve the drawbacks of BCFCM. This KFCM algorithm also affected by their parameters while execution. Hence, in this work Gaussian kernel-based fuzzy *c*-means algorithm (GKFCM) is proposed. The proposed GKFCM algorithm provides more efficiency and robustness than BCFCM and KFCM algorithms.

The rest of the paper provides about the related work in section 2, methodology for clustering in section 3, experimental results for proposed methodology in section 4, conclusion about this work in section 5.

II. Literature Survey

A comparative analysis over standard datasets and images has established the superiority of this algorithm over its corresponding standard algorithm. [1] introduced the kernel based rough intuitionistic fuzzy c-means algorithm and show that it is superior to all the algorithms in the sequel; i.e. both normal and the kernel based algorithms. Bias-corrected fuzzy *c*-means (BCFCM) algorithm with spatial information is especially effective in image segmentation. Since it is computationally time taking and lacks enough robustness to noise and outliers, some kernel versions of FCM with spatial constraints, such as KFCM_S1 and KFCM_S2, were proposed to solve those drawbacks of BCFCM. However, KFCM_S1 and KFCM_S2 are heavily affected by their parameters. So, [2] presented a Gaussian kernel-based fuzzy *c*-means algorithm (GKFCM) with a spatial bias correction.

[3] proposed a fuzzy kernel C-means clustering algorithm (FKCM) which is based on conventional fuzzy C-means clustering algorithm (FCM). This new FKCM algorithm integrates FCM with Mercer kernel function and deals with some issues in fuzzy clustering. [4] presented a comprehensive comparative analysis of kernel-based fuzzy clustering and fuzzy clustering. Kernel based clustering has emerged as an interesting and quite visible alternative in fuzzy clustering, however, the effectiveness of this extension vis-à-vis some generic methods of fuzzy clustering has neither been discussed in a complete manner nor the performance of clustering quantified through a convincing comparative analysis.

[5], [6], [7] introduced the kernel method is extended to the well-known fuzzy c-means (FCM) and possibilistic c-means (PCM) algorithms. It is realized by substitution of a kernel-induced distance metric for the original Euclidean distance, and the corresponding algorithms are called kernel fuzzy c-means (KFCM) and kernel possibilistic c-means (KPCM) algorithms. Next, they presented another novel for

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obtaining fuzzy segmentations of images that are subject to multiplicative intensity in homogeneities, such as magnetic resonance images. In 2004 they presented a novel algorithm for fuzzy segmentation of magnetic resonance imaging (MRI) data. The algorithm is realized by modifying the objective function in the conventional fuzzy C-means (FCM) algorithm using a kernel-induced distance metric and a spatial penalty on the membership functions.

A novel kernelized fuzzy attribute C-means clustering algorithm is proposed by [8]. Kernelized fuzzy attribute C-means clustering algorithm is a natural generalization of kernelized fuzzy C-means algorithm with stable function. [9] presented a kernel-based fuzzy clustering algorithm that exploits the spatial contextual information in image data. An entropy-based possibilistic *c*-means clustering using the kernel trick has been proposed as more robust method. Numerical examples are shown and effect of the kernel method is discussed by [10]. [11] presented that a Support vector machines (SVMs) with the Gaussian (RBF) kernel have been popular for practical use. This letter analyzes the behavior of the SVM classifier when these hyper parameters take very small or very large values. Our results help in understanding the hyper-parameter space that leads to an efficient heuristic method of searching for hyper-parameter values with small generalization errors.

Kernel methods are algorithms that, by replacing the inner product with an appropriate positive definite function, implicitly perform a nonlinear mapping of the input data into a high-dimensional feature space. In this paper, [12] presented a kernel method for clustering inspired by the classical k-means algorithm in which each cluster is iteratively refined using a one-class support vector machine. [13] realized the drawbacks of the general fuzzy c-mean algorithm and it tries to introduce an extended Gaussian version of fuzzy C-means by replacing the Euclidean distance in the original object function of FCM.

By applying kernel tricks, the kernel fuzzy c-means algorithm attempts to address this problem by mapping data with nonlinear relationships to appropriate feature spaces. Kernel width is crucial for effective kernel clustering. Unfortunately, for most applications, it is not easy to find the right width. To design and manage the uncertainty for kernel width, [14] proposed a type-2 kernelized fuzzy c-means algorithm (T2KFCM). They extended the type-1 fuzzy sets of membership to interval type-2 fuzzy sets using two widths and which creates a footprint of uncertainty for the membership.

An overview of fuzzy c-means clustering algorithms is given where [15] focused on different objective functions: they use regularized dissimilarity, entropy-based function, and function for possibilistic clustering. Classification functions for the objective functions and their properties are studied. Fuzzy c-means algorithms using kernel functions is also discussed with kernelized cluster validity measures and numerical experiments. New kernel functions derived from the classification functions are more over studied.

III. PROPOSED METHODOLOGY

The methodology discusses the KFCM and GKFCM techniques for clustering validity measurements. The procedures are mentioned for proposed GKFCM as follows.

a. Kernelized Fuzzy C-Means (KFCM)

Fuzzy C-means (FCM) clustering algorithm is the soft extension of the traditional hard C-means. This FCM clustering algorithm uses fuzzy set while the possibility is measured using membership function that each training vector belongs to a cluster. This each training vector results are assigned to multiple clusters. Thus it can overcome in some degree the drawback of dependence on initial partitioning cluster values in hard C-means. However, just like the C-means algorithm, this FCM algorithm is successful only in clustering with spherical and non-overlapping data. To rectify this problem, nonlinear version of FCM is constructed using kernel method and introduced a kernel-based fuzzy C-means clustering algorithm (KFCM).

The main aim of KFCM is to record the input data into feature space with larger dimension by nonlinear transform and performing FCM in that feature space. By this, the complex and non linear data points in input space are very simple and linearly separable in feature space after the nonlinear transform. From this, the preferred performance will be achieved. Another advantage of KFCM is that it can determine the number of clusters under some standard but in FCM which needs the desired number of clusters in advance.

b. Gaussian Kernel Of Fuzzy C-Means (GKFCM)

The Gaussian Kernel of Fuzzy c-means (GKFCM), which is also known as Alternative Fuzzy c-means (AFCM), is a derivative of FCM that replaces the Euclidean distance by Gaussian Kernel function. The objective function of GKFCM is defined by

$$J_{GKFCM} = \sum_{j=1}^{K}\sum_{i=1}^{N}\left(u_{ij}\right)^{m}\left\{1 - exp\left(-\beta\left\|x_i - c_j\right\|^2\right)\right\} \quad (1)$$

With m > 1 and the restriction $\sum_{j=1}^{K} u_{ij} = 1, i = 1, \dots, N$.

The new centers and membership grades can be updated alternatively as follows:

$$u_{ij} = \frac{1/\left(1 - exp\left(-\beta\left\|x_i - c_j\right\|^2\right)\right)^{1/(m-1)}}{\sum_{k=1}^{K} 1/\left(1 - exp\left(-\beta\left\|x_i - c_j\right\|^2\right)\right)^{1/(m-1)}} \quad (2)$$

$$c_j = \frac{\sum_{i=1}^{N}\left(u_{ij}\right)^{m} exp\left(-\beta\left\|x_i - c_j\right\|^2\right) x_i}{\sum_{i=1}^{N}\left(u_{ij}\right)^{m} exp\left(-\beta\left\|x_i - c_j\right\|^2\right)} \quad (3)$$

The main contribution of Gaussian–base kernel function is to handle outliers, as the effect of eq. (2) and (3) on the outlier will be trimmed down.

The kernel version of FCM by replacing the Euclidean distance $\left\|x_j - a_i\right\|$ with the kernel substitution as

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$$\|\emptyset(x_j) - \emptyset(a_i)\|^2 = K(x_j, x_j) + K(a_i, a_i) \pm 2K(x_j, a_i) \quad (4)$$

Where $\emptyset$a nonlinear map from the data space into the feature is space with its corresponding kernel K. the specially assumed $K(x, y) = 1$ and then proposed the kernel-type objective function J_m^K with

$$J_m^K(\mu, a) = 2\sum_{j=1}^{n}\sum_{i=1}^{c} \mu_{ij}^m (1 - K(x_j, a_i)) \quad (5)$$

Thus, by minimizing $J_{ij}^m(\mu, a)$ with necessary condition of the update equations are as follows:

$$a_i = \sum_{j=1}^{n} \mu_{ij}^m K(x_j, a_i) x_j \Big/ \sum_{j=1}^{n} \mu_{ij}^m K(x_j, a_i), \quad i = 1,2, \dots, c \quad (6)$$

$$\mu_{ij} = \left(1 - K(x_j, a_i)\right)^{-1/m-1} \Big/ \sum_{k=1}^{c} \left(1 - K(x_j, a_k)\right)^{-1/m-1}, \quad i = 1, \dots, c; \quad j = 1, \dots, n \quad (7)$$

This work point out that the necessary conditions for minimizing $J_m^K(\mu, a)$ are update above (6) and (7) equations only when the kernel function K is chosen to be the Gaussian function with $K(x_j, a_i) = \exp(-\|x_j - a_i\|^2/\sigma^2)$.

$$JS_m^K(\mu, a) = \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m \left(1 - K(x_j, a_i)\right) + \alpha \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m (1 - K(\bar{x}_j, a_i)) \quad (8)$$

When $K(x, y) = \exp(-\|x - y\|/\sigma^2)$ is taken. Thus, the necessary conditions for the minimizing $JS_m^k(\mu, a)$.

For tuning the spatial bias correction term, parameter α is used to control the effect of the neighbors. In fact, the parameter α heavily affects the clustering results of BCFCM, KFCM. Spontaneously, each spatial bias correction can be adjusted individualy for each cluster i so that performance will be better.

That is, the overall parameter α is better replaced with η_i that is correlated to each cluster i. In this sense, will consider the following modified objective function JS_m^G with

$$JS_m^k(\mu, \alpha) = \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m \left(1 - K(x_j, a_i)\right) + \sum_{i=1}^{c}\sum_{j=1}^{n} \eta_i \mu_{ij}^m \left(1 - K(x_j, a_i)\right) \quad (10)$$

Where K(x,y)= $exp((-\|x - y\|/\sigma^2)$.

However, there are parameters σ^2 and η_i in the proposed objective function JS_m^k. Since σ is presented as a dispersion, this work use the sample variance to estimate σ^2 with

$$\sigma^2 = \sum_{j=1}^{n} \|x_j - \bar{x}\|^2/n \quad with\ \bar{x} = \sum_{j=1}^{n} x_j/n \quad (11)$$

a. GKFCM Procedure

Input:

$X = \{x_1, \dots, x_n\}, x_i \in R^s$, the data set

$c, 2 \le c \le n$, the number of clusters

$\in > 0$, the stopping criterion of algorithm

$a^{(0)} = (a_1^{(0)}, \dots, a_c^{(0)})$, the initials of cluster centers

Output: $a = \{a_1, a_2, \dots, a_c\}$, the initials of cluster centers

Algorithm:

Step 1: let s=1 and estimate σ^2 using (11)

Step 2: Compute η^2 using

$$\eta_i = \frac{min_{i' \ne i}(1 - K(a_i, a_i))}{max_k(1 - K(a_k, \bar{x}))}, \quad i = 1, \dots, c$$

Step 3: Compute $\mu^{(s)}$ with $a^{(s-1)}$ and $\eta_i^{(S)}$

$$\mu_{ij} = \frac{((1 - K(x_j, a_i)) + \eta_i(1 - K(\bar{x}_j, a_i))^{-1/m-1}}{\sum_{k=1}^{c}((1 - K(x_j, a_k)) + \eta_i(1 - K(\bar{x}_j, a_k))^{-1/m-1}}, \quad i = 1, \dots, c: j = 1, \dots, n$$

Step 4: Update $a^{(s)}$ with $a^{(s-1)}$,$\mu^{(s)}$ and $\eta_i^{(S)}$ using

$$a_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m (K(x_j, a_i)x_j + \eta_i K(\bar{x}_j, a_i)\bar{x}_j)}{\sum_{j=1}^{n} \mu_{ij}^m K(x_j, a_i) + \eta_i K(\bar{x}_j, a_i)}, \quad i = 1, \dots, C$$

Step 5: IF $\|a^{(s)} - a^{(s-1)}\| < \varepsilon$, STOP and OUTPUT. ELSE s = s + 1 and return to step 2.

IV. Experimental Results

This section describes the proposed methodology efficiency for clustering application. Here comparison of the proposed and existing methods is shown. All algorithms are implemented under the same initial values and all with the stopping condition. The experiments are all performed on MATLAB.

The below table 1 shows the better performance of proposed method with improved accuracy and less execution time.

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Table 1: Calculation of Accuracy and Execution Time for Clustering Techniques

Techniques	Accuracy (%)	Execution Time (Seconds)
BCFCM	69	39
KFCM	76	30
GKFCM	87	22

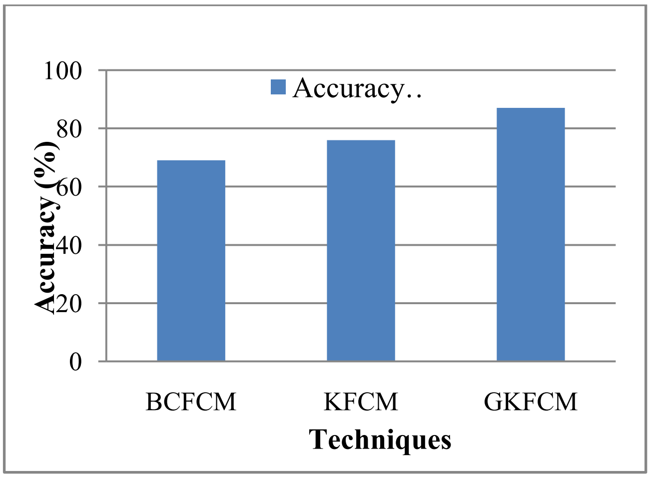

Figure 1: Accuracy

In figure 1 describes the accuracy for clustering techniques such as BCFCM, KFCM, and proposed GKFCM approaches. The proposed method has high accuracy when compared to preceding BCFCM and KFCM. And, in figure 2 describes the execution time for the clustering techniques. The proposed GKFCM has less execution time when compared to existing BCFCM and KFCM.

Figure 2: Execution Time

V. CONCLUSION

In this research, summarize that the bias-corrected FCM (BCFCM) is proposed by minimizing the FCM objective function. Since it is computationally time taking and lacks enough robustness to noise and outliers. However, the parameters of BCFCM and KFCM heavily affect the final clustering results. Thus, this work proposed a Gaussian kernel-based FCM (GKFCM) algorithm with a spatial bias correction. The proposed GKFCM algorithm is a generalized version of BCFCM, KFCM. Moreover, GKFCM can automatically learn the parameters by the prototype-driven learning scheme. The experimental results prove that the GKFCM has high accuracy with less execution time when compared to preceding clustering techniques.

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