Contact Stress Analysis for 'Gear' to Optimize Mass using CAE Techniques

Abstract: Gears have a wide variety of applications. Their applications vary from watches to very large mechanical units like the lifting devices and automobiles. Gears generally fail when the working stress exceeds the maximum permissible stress. Contact stress analysis between two spur gear teeth was considered in different contact positions representing a pair of mating gears during rotation. These stresses are proportional to the amount of power transmitted while the design could offer favorable or adverse conditions for generation of the same. This dissertation work would identify the magnitude of the stresses for a given configuration of a gear transmitting power while trying to find ways for reducing weight of the gear. The philosophy for driving this work is the lightness of the gear for a given purpose while keeping intact its functionality. The process constraints for manufacturing the gear also need to be considered while recommending alternative/s. Ease of incorporating the new feature for weight reduction over the existing process of manufacturing and the magnitude of volume of mass (or weight) reduced could be considered as the key parameters for assessment for this work.

Keywords: Contact stress, FEA, Hertzian theory, Spur Gear

I INTRODUCTION

Gears are the most common means of transmitting power in the modern mechanical engineering world. Gearing is one of the most effective methods for transmitting power and rotary motion from the source to its application with or without change of speed or direction. Gears will prevail as a critical machine element for transmitting power in future machines due to their high degree of reliability and compactness. The rapid development of heavy industries such as vehicle, shipbuilding and aircraft industries require advanced application of gear technology. Spur gear is a cylindrical shaped gear in which the teeth are parallel to the axis. It is easy to manufacture and it is mostly used in transmitting power from one shaft to another shaft up to certain distance & it is also used to vary the speed & Torque. e.g. Watches, gearbox etc. The cost of replacement of spur gear is very high and also the system down time is one of the effect in which these gears are part of system. Failure of gear causes breakdown of system which runs with help of gear, e.g. automobile vehicle.

When gear is subjected to load, high stresses developed at the root of the teeth, Due to these high Stresses, possibility of fatigue failure at the root of teeth of spur gear increases. There is higher chance of fatigue failure at these locations. So to avoid fatigue failure of the gear, the stresses should be minimized at maximum stress. Spur gear is a cylindrical shaped gear in which the teeth are parallel to the axis. It is easy to manufacture and it is mostly used in transmitting power from one shaft to another shaft up to certain distance & it is also used to vary the speed & Torque. e.g. Watches, gearbox etc. The cost of replacement of spur gear is very high and also the system down time is one of the effect in which these gears are part of system. Failure of gear causes breakdown of system which runs with help of gear, e.g. automobile vehicle.
gears for stresses. Gears have been analysed for different points of contact on the tooth profile and the corresponding points of contact on the pinion.

II FINITE METHOD ANALYSIS

In FEA modeling element quality greatly effects the accuracy of analysis results. Many modern finite element analyser solvers have routines to compensate for some measure of poor quality element but it is not a good practice to rely on these compensations. The FEA modeler must take into consideration element quality and thereby judge whether the analysis results are meaningful. The ideal four node (quad) plate element is a planner square; two types of error can result from translating a single node. If one of the nodes is translated in the plane remaining nodes, interior angles change & edge lengths vary between sides introducing skew and aspect ratio into the element if one of the nodes is translated out of the plans of others result is warpage. With first order tria elements warpage is not possible but aspect ratio & skew remain valid. It measures element quality. The element checks in Hyper mesh test their properties and provide feedback as to quality of element.

A STRESS CONTOUR OF GEAR & PINION

STRESS IN GEAR

MAXIMUM STRESS OBSERVED IN GEAR IS 255.36 N/MM²

STRESS IN PINION

MAX STRESS FOUND IN PINION IS 240.46 N/MM²

CONTACT PRESSURE AT PINION

MAX STRESS FOUND IN CONTACT STRESS AT PINION IS 65.46 N/MM²
III  HERTZ CONTACT STRESS (CONTACT STRESS ANALYSIS)

The transfer of power between gears takes place at the contact between the acting teeth. The stresses at the contact point are computed by means of the theory of Hertz. The theory provides mathematical expressions of stresses and deformations of curved bodies in contact. Fig. 4 shows a model applied to the gear-two parallel cylinders in contact.

According to the theory, the deformed distance \(a\) equals,

\[
a = 2 \sqrt{\frac{W(1-v_1^2) + (1-v_2^2)}{\frac{E_1}{r_1} + \frac{E_2}{r_2}}}\]

where,

- \(W\) = load acting at tooth in N
- \(v_1\) = Poisson's ratio of pinion
- \(v_2\) = Poisson's ratio of Gear
- \(E_1\) = Elasticity of pinion in N/MM\(^2\)
- \(E_2\) = Elasticity of Gear in N/MM\(^2\)
- \(F\) = Face Width of Pinion in MM
- \(r_1\) = Pitch radius of Pinion in MM
- \(r_2\) = Pitch radius of Gear in MM

\[
\sigma = \sqrt{\frac{w(1-v_1^2) + (1-v_2^2)}{\frac{E_1}{r_1} + \frac{E_2}{r_2}}}\]

Where \(W\) is the load, \(E_1\) and \(E_2\) are the Modulus of Elasticity of pinion and gear respectively, \(v_1\) and \(v_2\) are the Poisson’s ratios of pinion and gear respectively and \(F\) is the face width of pinion. Same equation can be apply for teeth, assuming for \(R_1\) and \(R_2\) the respective radii of the involute curve at the contact point, as shown in Fig. 4. Let us assume that the contact stress Analysis in Pinion. Where \(r_0\) and \(P_0\) are the pitch radii of the pinion and gear and \(\phi\) is the pressure angle. The stress correlations derived heretofore and Eq. are based on a number of simplifying assumptions, such as pure bending of short beam and elliptic distribution of stresses at tooth contact. A question therefore arises concerning their accuracy. contact takes place at point 1, and then the respective radii are equal to:

\[
R_1 = r_0 P_1 \sin \phi
\]

\[
R_2 = r_0 P_2 \sin \phi
\]

The elastic compression of two-dimensional bodies in contact cannot be calculated solely from the contact stresses given by the Hertz theory. Some account must be taken for the shape and size of the bodies themselves and the way in which they are supported. In most practical circumstances such calculations are difficult to perform, which have resulted in a variety of approximate formulae for calculating the elastic compression of bodies in line contact such as gear teeth and roller bearings in line contact. The pitting problems, design needs and safety requirements make far in depth and complicated study of this contact. The current project aims to arriving at these very solutions.

\[
\sigma = 243.75 \text{N/MM}^2
\]
IV DESIGN SPACE FOR TOPOLOGY OPTIMIZATION

A Design Variable
we remove the material from Gear of tooth because to create design space. Optimization will be perform over the design variable. We define the separately property & material.

B CONSTRAINT
Stress limit should not go beyond the Endurance limit while performing topology optimization then force is provided tip of the tooth which will generate equal and stresses in contact analysis is 65 N/MM$^2$. Then we created objective is minimization of mass.

Fig.6 Design Space for Topology Optimization
1. To minimize the volume fraction of gear
2. Constraint :- stress should not be more than of yield stress i.e. 355 N/MM2
3. Endurance limit 0.4 times ultimate tensile strength = Ultimate tensile strength of material is 700 N/MM2
   = 0.4*700
   = 280 N/MM2
Blue colour shows where topology optimization is perform

Fig.7 Element Density plot of Gear
Geometry of gear and pinion after topology optimization.

Fig. 8 Geometry of gear & Pinion after topology
we have created eight holes of 5mm radius uniformly along radial circumference of 32 mm from the center of the gear.

C Results After Topology Optimization
Stress contour of gear tooth after topology optimization
Stress in gear after modification are 272 N/mm²
Stress contour of pinion after topology optimization
Stress in pinion after modifications done in gear are 240.46 N/mm²
VI Results and Discussion

<table>
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<tr>
<th>Sr. No.</th>
<th>Part</th>
<th>Analysis Result</th>
<th>Analytical Result</th>
<th>After Topology</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Pinion</td>
<td>240.46 N/MM²</td>
<td>243.75 N/MM²</td>
<td>240.46 N/MM²</td>
</tr>
<tr>
<td>2</td>
<td>Gear</td>
<td>255.36 N/MM²</td>
<td>272 N/MM²</td>
<td>272 N/MM²</td>
</tr>
</tbody>
</table>

- Contact stress of pinion & Gear before Topology 65.46 N/mm²
- Contact stress of pinion & Gear after Topology 65 N/mm²

From above result we can conclude that contact stresses of Gear & Pinion is somewhat similar. Before topology weight of Gear is 749gm and After Topology weight of gear is 638gm. It means that Ease of incorporating the new feature for weight reduction over the existing process of manufacturing and the magnitude of volume of mass (or weight) reduced. It will be considered as the key parameters for assessment for this work.

CONCLUSION

1. This dissertation work would identify the magnitude of the stresses for a given configuration of a gear transmitting power before optimization contact stress is 65.16 N/MM² and after optimization contact stress is 65 N/MM². It is observed that very slight reduction in stress.

2. In this study, the best result is obtained by introducing holes on Gear before that weight of gear is 770gm after topology weight is 638gm.

3. Stress relieving feature having a shape of circular hole is used in the path of stress flow. when compared to elliptical hole it will be easy for manufacturing.

REFERENCES


