"An EPQ Model of Deteriorating Items using Three Parameter Weibull Distribution with Constant Production Rate and Time Varying Holding Cost"

KIRTAN PARMAR, U. B. GOTHI

***Abstract*— In this paper, we have analyzed a deterministic inventory model where deterioration rate follows three parameter Weibull distribution with constant demand and production rate. In the model considered here, shortages are not allowed and holding cost is time-dependent. The model is solved analytically to obtain the optimal solution of the problem. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.**

***Keywords*— Deteriorating items, Inventory, Three parameter Weibull distribution, Time-varying holding cost.**

I. INTRODUCTION

Inventory system is one of the main streams of the Operation Research which is essential in business enterprises and industries. Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. It needs scientific way of exercising inventory model.

The pioneering work of Harris [7] in inventory models is being treated by mathematical techniques. He developed the simplest inventory model, the Economic Order Quantity (EOQ) model which was later popularized by Wilson [18].

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Relaxation of some assumptions in the formulation of the EOQ model led to the development of other inventory models that effectively tackle several other inventory problems occurring in day-to-day life.

An optimal replenishment policy is dependent on ordering cost, inventory carrying cost and shortage cost. An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Fortunately, the rate of deterioration is too small for items like steel, toys, glassware, hardware, etc. There is little requirement for considering deterioration in the determination of economic lot size. Deterioration is defined as change, damage, decay, spoilage, obsolescence and loss of marginal value of a commodity that results in decreasing usefulness from the original one.

Inventory of deteriorating items first studied by Whitin [17], he considered the deterioration of fashion goods at the end of prescribed storage period. Covert and Philip [5] considered the assumption of constant deterioration rate to represent the distribution of time to deterioration by using a two parameter Weibull distribution. Further, Philip [12] generalised this model by assuming three parameter Weibull distribution. These investigations were followed by works by several researchers like Shah and Jaiswal [13]. It has been empirically observed that the failure and life expectancy of items can be expressed in terms of Weibull distribution. This empirical observation has prompted researchers to represent the time of deterioration of a product by a Weibull distribution. The model Ghare and Schrader [6] was extended by Covert and Philip [5] who obtained an EOQ model with a variable rate of

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deterioration by assuming a two parameter Weibull distribution.

Aggarwal and Bahari-Hasani [1], studied a model assuming that items are deteriorating at a constant rate, in which, the production rate is known but can vary from one period to another period over a finite planning period. Pakkala and Achary [11] developed a production inventory model of deteriorating items with two storage facilities and a constant demand rate. Jong – Wuu Wu at el. [8] developed inventory model for time varying demand with shortages under Weibull deterioration rate. Begam, Sahoo, Sahu and Mishra [2] developed and instantaneous replenishment policy for Weibull deteriorating items with price dependent demand and Chaitanya. K. Tripathy and U. Mishra [3] used Weibull distribution for deterioration in their research for price dependent demand but holding cost is time varying.

Instead of determining Economic Order Quantity some researchers focus their work to control the production to optimize total cost in inventory control. In this regard, the work has been concentrated to obtain the Economic Production Quantity (EPQ) in inventory management. Mishra [10] firstly introduced this Economic Production Quantity model for deteriorating item. Sugapriya C. and Jeyaraman K. [14] considered the EPQ model for non-instantaneous deterioration. Choi and Hwang [4] worked in their research for optimization of production planning for deteriorating items to minimize the total cost function over a finite planning period.

Recently, Kirtan Parmar and U. B. Gothi[9] developed a deterministic inventory model for deteriorating items with time to deterioration having Exponential distribution and with time-dependent quadratic demand. In this model, shortages are not allowed and holding cost is time-dependent. Also, U. B. Gothi and Kirtan Parmar [16] have extended above deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration and shortages are allowed and partially backlogged.

Sunil Kawale and PravinBansode [15] have developed an EPQ model in which they assume that deterioration rate is a random variable following Weibull distribution with three parameters. But in the paper we find that the p.d.f. is taken as $f(t)=\alpha\beta\left(\mu-\gamma\right)^{\beta-1}e^{-\alpha\left(\mu-\gamma\right)^{\beta}}$ which is constant function andsodeterioration rate is also constant (fixed). Also, on verification almost all the results derived are not true.

In this paper, we have analyzed an EPQ model of Sunil Kanwale and PravinBansode [15] of deteriorating items with the deterioration rate $\theta=\alpha\beta(t-\gamma)^{\beta-1}$ which is a function of time t, with constant production rate and time varying IHC without shortages.

II. NOTATIONS

The mathematical modelis developed using the following notations:

01. $Q(t)$: The instantaneousstateof the inventory level at any time t ($0 \leq t \leq T$).
02. IM : Maximum inventory levelof the product.
03. p : Production rate per unit time.
04. d : Actual demand of the product perunit time; ($d < p$).
05. A : Ordering cost per order.
06. C_h : Inventory holdingcost per unit per unit time.
07. k : Production cost per unit.
08. r : Price discount per unit cost.
09. t_1 : Production period (decision variable).
10. T : Length of one cycle (decision variable)
11. $[t_1, T]$: Time interval during which there is no production of the product.
12. $TC(t_1, T)$: Total cost per unit time.

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III. ASSUMPTIONS

1. The demand rate for the product is known and finite.
2. Once aunit of the product isproduced, it is available to meet the demand.
3. Once the production is terminated the product starts
being deterioratedandthe price discount is considered.
4. Holding cost is linear function of time and it is $C_h = h+rt$ (h, r > 0)
5. An infinite planning horizon is assumed.
6. Shortages are not allowed to occur.
7. Repair orreplacement of the deteriorated items does not take place during a given cycle.
8. Total inventory cost is a real, continuous function which is convex to the origin.

IV. MATHEMATICAL MODEL & ANALYSIS

The distribution of the time to deteriorate is random variable following the three parameter Weibull distribution. The probability density function for three parameter Weibull distribution is given by

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1} e^{-\alpha(t-\gamma)^{\beta}}$$

(where $t \geq \gamma$, $0 < \alpha < 1$ and $\beta, \gamma > 0$)

The instantaneous rate of deterioration θ(t) of the non-deteriorated inventory at time t, can be obtained from $\theta(t) = \frac{f(t)}{1-F(t)}$, where $F(t) = 1-e^{-\alpha(t-\gamma)^{\beta-1}}$ is the cumulative distribution function for the three parameter Weibull distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t) = \alpha\beta(t-\gamma)^{\beta-1}$. The probability density function represents the distribution of the time to deteriorate which may have a decreasing, constant or increasing rate of deterioration. The three parameter Weibull distribution is suitable for items with any initial value of the rate of deterioration (Begum at el. [2]).

Initially, inventory is zero. At time t = 0, the production starts and simultaneously supply also starts. The production stops at $t = t_1$ where the maximum inventory level S is reached. In the interval $[0, t_1]$ the inventory built up at a rate (p – d) and there is no deterioration in this interval. The production stops at time t_1 and then deterioration starts and supply is also with discounted rate. The inventory is finitely decreasing until inventory reaches zero in the time interval $[t_1, T]$. As soon as inventory level is zero the next production run starts.

The graphical representation is shown in the Figure 1.

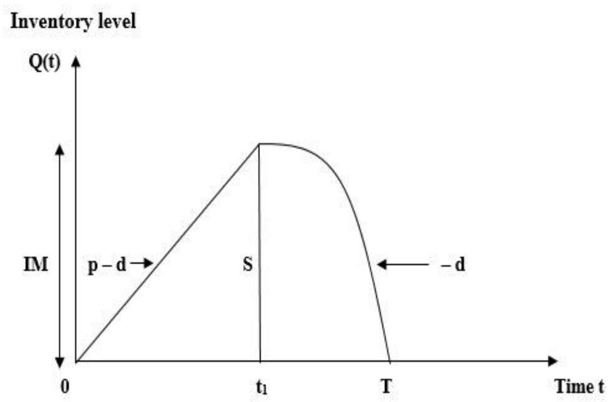

Figure 1: Graphical representation of the inventory system

The inventory level of the product at time 't' over period [0, T] can be described by the following differential equations.

$$\frac{dQ(t)}{dt} = p-d \qquad (0 \leq t \leq t_1) \quad (1)$$

$$\frac{dQ(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} Q(t) = -d \qquad (t_1 \leq t \leq T) \quad (2)$$

The boundary conditions are

$$Q(0) = 0, \qquad Q(t_1) = S, \qquad Q(T)=0 \qquad (3)$$

Solution of equation (1) is

$$\Rightarrow Q(t) = (p-d)t \qquad (0 \leq t \leq t_1) \quad (4)$$

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Solution of equation (2) is given by

$$e^{\alpha(t-\gamma)^{\beta}} Q(t) = -d\left(\int e^{\alpha(t-\gamma)^{\beta}}\, dt\right)$$

$$= -d\int\left[1+\alpha\left(t-\gamma\right)^{\beta}\right]dt$$

(neglecting higher powers of α)

With the boundary condition Q(T) = 0, the solution is

$$e^{\alpha(t-\gamma)^{\beta}} Q(t) = d\left(T-t\right)+\frac{\alpha d}{\beta+1}\left[\left(T-\gamma\right)^{\beta+1}-\left(t-\gamma\right)^{\beta+1}\right]$$

$$\Rightarrow Q(t) = \left\{\begin{aligned} & d\left(T-t\right)\left[1-\alpha\left(t-\gamma\right)^{\beta}\right] \\ & +\frac{\alpha d}{\beta+1}\left[\left(T-\gamma\right)^{\beta+1}-\left(t-\gamma\right)^{\beta+1}\right]\end{aligned}\right\} \quad (t_1 \le t \le T) \qquad (5)$$

The total cost comprises of the following costs

1) The production cost during the period [0, T]

$$PC = p\cdot k\cdot t_1 \qquad (6)$$

2) The ordering cost during the period [0, T]

$$OC = A \qquad (7)$$

3) The inventory holding cost during the period [0,T]

$$IHC = \int_0^{t_1}(h+rt)Q(t)dt + \int_{t_1}^{T}(h+rt)Q(t)dt$$

$$= \int_0^{t_1}(h+rt)(p-d)t\ dt$$

$$+\int_{t_1}^{T}\left\{(h+rt)\left[\begin{aligned} & d\left[T-t\right]\left[1-\alpha\left(t-\gamma\right)^{\beta}\right] \\ & +\frac{\alpha d}{\beta+1}\left[\left(T-\gamma\right)^{\beta+1}-\left(t-\gamma\right)^{\beta+1}\right]\end{aligned}\right]\right\}dt$$

$$\Rightarrow IHC = (p-d)\left(\frac{ht_1^2}{2}+\frac{rt_1^3}{3}\right)$$

$$+d\left\{\begin{aligned}
& h\left[T+\frac{\alpha}{\beta+1}(T-\gamma)^{\beta+1}\right][T-t_1]-\frac{r}{3}\left(T^3-t_1^3\right) \\
& +\frac{1}{2}\left[r\left(T+\frac{\alpha}{\beta+1}(T-\gamma)^{\beta+1}\right)-h\right]\left(T^2-t_1^2\right) \\
& +\left[-h\alpha T+\frac{(h-\beta T)\alpha\gamma}{(\beta+1)(\beta+2)}+\frac{2\alpha r\gamma^2}{(\beta+1)(\beta+2)(\beta+3)}\right]\left[\begin{aligned}&(T-\gamma)^{\beta+1}\\ &-(t_1-\gamma)^{\beta+1}\end{aligned}\right] \\
& +\left[\frac{\alpha(h-rT)+\frac{2\alpha r\gamma}{(\beta+3)}}{(\beta+2)}\right]\left[T(T-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+1}\right] \\
& +\left[\frac{\alpha r}{(\beta+3)}\right]\left[T^2(T-\gamma)^{\beta+1}-t_1^2(t_1-\gamma)^{\beta+1}\right] \\
& -\left[\frac{h\alpha+\frac{\alpha r\gamma}{(\beta+3)}}{(\beta+1)(\beta+2)}\right]\left[(T-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right] \\
& -\left[\frac{\alpha r}{(\beta+1)(\beta+3)}\right]\left[T(T-\gamma)^{\beta+2}-t_1(t_1-\gamma)^{\beta+2}\right]
\end{aligned}\right\} \qquad (8)$$

4) The deterioration cost during the period [0, T]

$$DC = C_d k\left[Q(t_1)-\int_{t_1}^{T} d\cdot dt\right]$$

$$\Rightarrow DC = C_d\cdot\alpha\cdot d\cdot k\left\{\begin{aligned} & \left(t_1-T\right)\left(t_1-\gamma\right)^{\beta} \\ & +\frac{1}{\beta+1}\left[\left(T-\gamma\right)^{\beta+1}-\left(t_1-\gamma\right)^{\beta+1}\right]\end{aligned}\right\} \qquad (9)$$

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5) Price discount isofferedas a fraction of production cost for the units in the period [0, T] and it is

$$PD = k \cdot r \int_{t_1}^{T} d \cdot dt$$

$$\Rightarrow PD = k \cdot r \cdot d \cdot [T - t_1] \tag{10}$$

Hence the total cost per unit time is given by

$$TC(t_1, T) = \frac{1}{T}(PC + OC + DC + PD + IHC)$$

$$= \frac{1}{T}\left\{ \begin{array}{l} \left\{ \begin{array}{l} p \cdot k \cdot t_1 + A \\ + C_d \cdot \alpha \cdot d \cdot k \left\{ \begin{array}{l} (t_1 - T)(t_1 - \gamma)^{\beta} \\ + \frac{1}{\beta+1}\left[(T - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \right] \end{array} \right\} \\ + k \cdot r \cdot d \cdot [T - t_1] \\ + (p - d)\left(\frac{h t_1^2}{2} + \frac{r t_1^3}{3} \right) \end{array} \right\} \\ + d \left\{ \begin{array}{l} h\left[T + \frac{\alpha}{\beta+1}(T - \gamma)^{\beta+1} \right][T - t_1] - \frac{r}{3}\left(T^3 - t_1^3 \right) \\ + \frac{1}{2}\left[r\left(T + \frac{\alpha}{\beta+1}(T - \gamma)^{\beta+1} \right) - h \right]\left(T^2 - t_1^2 \right) \\ + \left[\begin{array}{l} -h\, \alpha\, T + \frac{(h - \beta T)\, \alpha\, \gamma}{(\beta+1)(\beta+2)} \\ + \frac{2\, \alpha\, r\, \gamma^2}{(\beta+1)(\beta+2)(\beta+3)} \end{array} \right]\left[(T - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1} \right] \\ + \left[\frac{\alpha(h - rT) + \frac{2\alpha r \gamma}{(\beta+3)}}{(\beta+2)} \right]\left[T(T - \gamma)^{\beta+1} - t_1 (t_1 - \gamma)^{\beta+1} \right] \\ + \left[\frac{\alpha\, r}{(\beta+3)} \right]\left[T^2 (T - \gamma)^{\beta+1} - t_1^2 (t_1 - \gamma)^{\beta+1} \right] \\ - \left[\frac{h\, \alpha + \frac{\alpha\, r\, \gamma}{(\beta+3)}}{(\beta+1)(\beta+2)} \right]\left[(T - \gamma)^{\beta+2} - (t_1 - \gamma)^{\beta+2} \right] \\ - \left[\frac{\alpha\, r}{(\beta+1)(\beta+3)} \right]\left[T(T - \gamma)^{\beta+2} - t_1 (t_1 - \gamma)^{\beta+2} \right] \end{array} \right\} \end{array} \right\} \tag{11}$$

Our objective is to determine optimum value of t_1 and T so that TC (t_1, T) is minimum. The values of t_1 and T, for which the total cost TC (t_1, T) is minimum, are the solutions of equations $\frac{\partial TC(t_1, T)}{\partial t_1} = 0$ and $\frac{\partial TC(t_1, T)}{\partial T} = 0$ satisfying the condition

$$\left\{ \left(\frac{\partial^2 TC(t_1, T)}{\partial t_1^2} \right)\left(\frac{\partial^2 TC(t_1, T)}{\partial T^2} \right) - \left(\frac{\partial^2 TC(t_1, T)}{\partial t_1 \partial T} \right)^2 \right\} > 0$$

The optimal solution of the equations (11) can be obtained by using appropriate software.

V. NUMERICAL EXAMPLE

The above model is illustrated by the following numerical illustration. We consider the following parametric values for A = 500, p = 40, d = 25, h = 1, r = 13, k = 5, $\alpha = 0.05$, $\beta = 0.5$, $\gamma = 3$, $C_d = 1$ and r = 1 (with appropriate units).

We obtain the optimal value of t_1 = 3.000003240 units, T = 3.708939862 units,PC = 600.000648 units, IHC = 2156.115672 units, DC = 2.479179448 units, PD = 88.61707775 units and optimal total cost per unit time $TC(t_1,T)$ = 902.4715153 units.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes or errors in its input parameter values. In this section, we study the sensitivity of the total cost

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per time unit TC(t_1,T) with respect to the changes in the values of the parametersα, β, γ.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other parameters the same. The results are presented in **Table – I.**

Table – I :Partial Sensitivity Analysis

Para-meter	Values	t_1	T	TC(t_1,T)
α	0.03	3.00000125	3.72968	898.25
	0.04	3.00000215	3.71931	900.38
	0.05	3.00000324	3.70894	902.47
	0.06	3.00000449	3.69856	904.53
	0.07	3.00000589	3.68819	906.55
β	0.30	3.00005770	3.70523	905.62
	0.35	3.00003450	3.70606	904.73
	0.40	3.00001831	3.70698	903.91
	0.45	3.00000843	3.70794	903.16
	0.50	3.00000022	3.70894	902.47
γ	2.8	2.80000425	3.49680	840.36
	2.9	2.90000370	3.60247	870.69
	3.0	3.00000324	3.70894	902.47
	3.1	3.10000286	3.81613	935.70
	3.2	3.20000255	3.92396	970.35

VII. GRAPHICAL REPRESENTATION

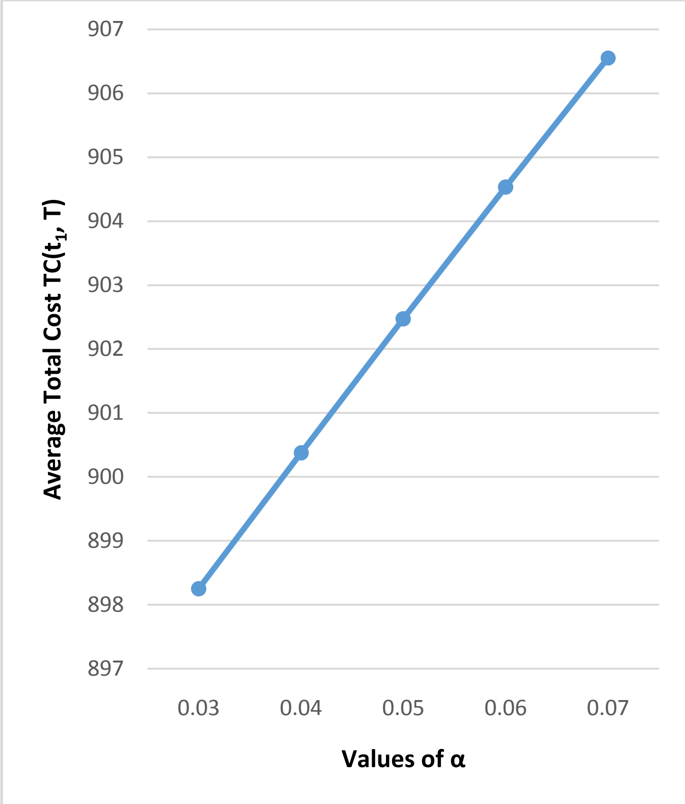

Figure – 2 : Comparison of TC for parameter α

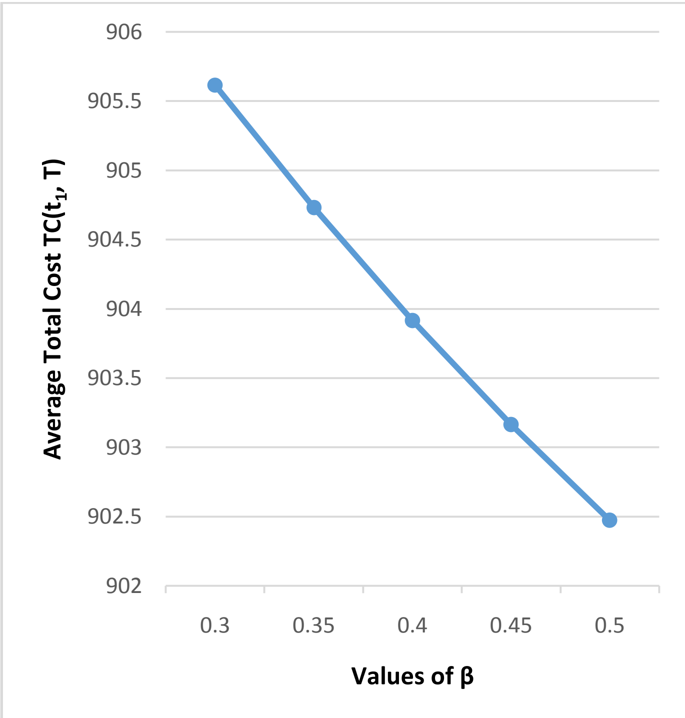

Figure – 3 : Comparison of TC for parameter β

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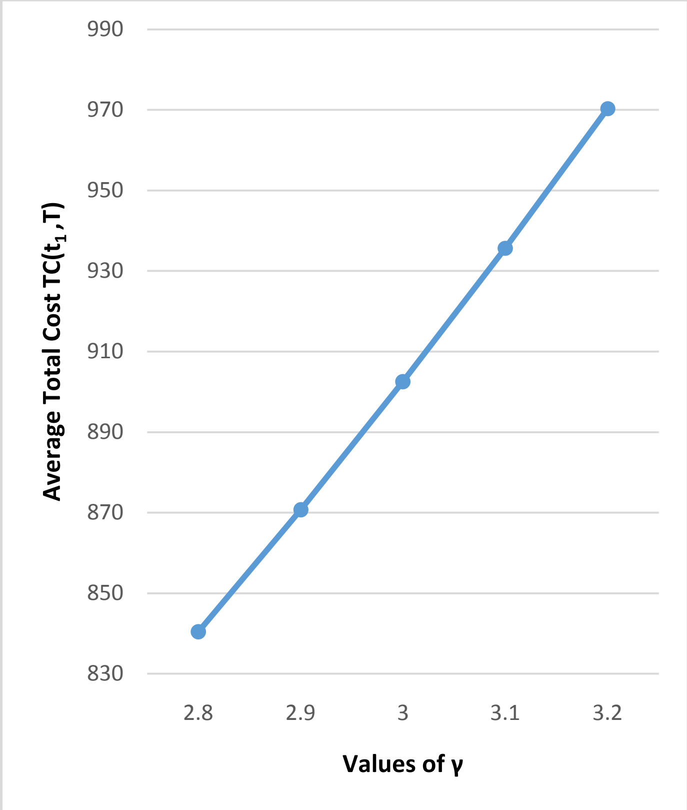

Figure – 4 : Comparison of TC for parameter γ

VIII. CONCLUSIONS

- It is observed from **Figure – 2** that when the values of scale parameter alpha (α) increase simultaneously the average total cost (TC) also increases.
- It is observed from **Figure – 3** that when the values of shape parameter beta (β) increase then the average total cost (TC) has reverse effect.
- It is observed from **Figure – 4** that when the values of location parameter gamma (γ) increase the average total cost (TC) also increases.

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