ElzakiTransform: A Solution of Differential Equations

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***Abstract*-Elzaki transform is a new integral transform was applied tosolve variestype of differential equations such as linear ordinary differential equations with constant coefficients, linear system of partial differential equations with constant coefficients, nonlinear ordinary differential equations.**

***Index Terms*—Elzaki transform, ordinary differential equations, partial differential equations.**

I. INTRODUCTION

Elzaki transform is a new integral transform was introduced by Tarig ELzaki in 2010[1].In Elzaki transform solve differential equations with variable coefficients which were not solved by Sumudu transform and Laplace transform, that means Sumudu and Laplace transforms failed to solvethese types of differential equations [4].Elzaki transform is modified transform of Sumudu and Laplace transforms .Elzaki transform can be used to solve ordinary differential equations [8]. Elzaki transform and differential transform methods to solve nonlinear problems [5].Elzaki transform applied to solution of system of partial differential equations [2].

II. ELZAKI TRANSFORM

ELzaki transform is a new integral transform, it is defined by

$$A=\left\{ f(t):\exists M,k_1 and\, k_2>0:\left|f(t)\right|<Me^{|t|k_j}, if\, t\in(-1)^j\times\left[0,\infty\right)\right\} \quad (1)$$

In a set A , M is constant must be finite , $k_1 and k_2$ be infinite , v be any variable is used to the variable t in the argument of the function f , in the ELzaki transform .The ELzaki transform is defined by

$$E\left[f(t)\right]=v\int_0^\infty f(t)e^{-t/v}\,dt=T(v),\qquad v\in(-k_1,k_2) \quad (2)$$

III. PROPERTIES

Following properties very useful in study of differential equations

If ELzaki transform of the function f (t) given by $E\left[f(t)\right]=t(v)$, then:

- $E\left[t\,f(t)\right]=v^2\frac{d}{dv}\left[\frac{T(v)}{v}-vf(0)\right]-v\left[\frac{T(v)}{v}-vf(0)\right]$
- $E\left[t^2\,f'(t)\right]=v^4\frac{d^2}{dv^2}\left[\frac{T(v)}{v}-v\,f(0)\right]$
- $E\left[t\,f''(t)\right]=v^2\frac{d}{dv}\left[\frac{T(v)}{v^2}-f(0)-v\,f'(0)\right]-v\left[\frac{T(v)}{v^2}-f(0)-v\,f'(0)\right]$
- $E\left[f'(t)\right]=\frac{T(v)}{v}-v\,f(0)$
- $E\left[f''(t)\right]=\left[\frac{T(v)}{v^2}-f(0)-v\,f'(0)\right]$
- $E\left[\frac{\partial f}{\partial t}(x,t)\right]=\frac{T(x,v)}{v}-v\,f(x,0)$
- $E\left[\frac{\partial^2 f}{\partial t^2}(x,t)\right]=\frac{1}{v^2}T(x,v)-f(x,0)-v\frac{\partial f}{\partial t}(x,v)$

Now we apply the given properties to solve some differential equations:

IV. SECOND ORDER ORDINARY DIFFERENTIAL EQUATION [8]

Example 1:

Consider second order differential equation:

$$3y''+3y'=0 \qquad y(0)=2,\ y'(0)=1$$

Solution:

Taking ELzaki transform to given equations,

$$3\frac{E(v)}{v^2}-2-v+3E(v)=0 \quad (3)$$

After solving to get,

$$E(y)=\frac{2v^2}{3(1+v^2)}+\frac{v^3}{3(1+v^2)} \quad (4)$$

By using inverse ELzaki transform to get,

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$$y(x)=\frac{2\sin x}{3}+\frac{\cos x}{3}$$

V. ORDINARY DIFFERENTIAL EQUATION WITH VARIABLE COEFFICIENTS [4]

Example 2:

Solve the differential equation:

$$2y''+ty'-2y=0 \qquad y(0)=0, y'(0)=1$$

Solution:

Taking ELzaki transform to given equations,

By using initial condition we get,

$$E'(y)+\left(\frac{2}{v^3}-\frac{4}{v}\right)E(y)=1 \qquad (5)$$

Solution in the form

$$E(y)=v^3+C\,v^4\,e^{\frac{1}{2v^2}} \qquad C=0\text{, then} \qquad (6)$$

$$E(y)=v^3,$$

By using inverse ELzaki transform we get,

y = t

VI. NONLINEAR DIFFERENTIAL EQUATION [5]

Example 3:

Consider the nonlinear first order differential equation.

$$y'=y^2 \quad ,y(0)=1$$

Solution:

Appling ELzaki transform on both sides to get,

$$\frac{Y(u)}{u}-uy(0)=\mathrm{E(y)}^2 \qquad (7)$$

$$Y(u)=u^2+uE(y)^2 \qquad (8)$$

The standard ELzaki transform methods defines the solution y (t) by the series

$$y=\sum_{n=0}^{\infty}y(n) \qquad (9)$$

Taking inverse ELzaki transform on both sides,

$$y(t)=1+E^{-1}\left[u\,E(y)^2\right](10)$$

Substituting Eq (9) into Eq(10) we get,

$$y(n+1)=E^{-1}\left\{u\,E(A_n)\right\} \quad , \quad n\geq 0 \qquad (11)$$

Where, $A_n=\sum_{r=0}^{n}y(r)\,y(n-r)$, *and* $A_0=1$ (12)

For n=0, we have: $y(1)=t$

For n=1, we have: $A_1=2t \quad and \quad y(2)=t^2$

For n=2, wehave: $A_2=3t^2 \quad and \quad y(3)=t^3$

In series form solution is given by,

$$y(t)=y(0)+y(1)+y(2)+y(3)+..........$$

$$y(t)=1+t+t^2+t^3+............$$

$$y(t)=\frac{1}{1-t}$$

VII. PARTIAL DIFFERENTIAL EQUATIONS [5]

Example 4:

Consider the following first order initial value problem

$$\begin{cases}\frac{\partial U}{\partial x}(x,t)+\frac{\partial V}{\partial t}(x,t) = \mathrm{x}\\ 2\frac{\partial U}{\partial t}(x,t)-3\frac{\partial V}{\partial x}(x,t)=t\end{cases} \qquad (13)$$

With the initial conditions

$$\mathrm{U}(x,0)=x \qquad ,\mathrm{V}(x,0)=0 \qquad (14)$$

Solution:

Taking ELzaki transform to given equations (13)

$$\begin{cases}\frac{\partial^2\overline{U}}{\partial x^2}+\frac{\overline{V}}{v}-vV(x,0)=xv^2\\ 2\frac{\overline{U}}{v}-2vU(x,0)-3\frac{\partial\overline{V}}{\partial x}=v^3\end{cases} \qquad (15)$$

Equations (15) becomes,

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$$\begin{cases} 3v^2 \dfrac{\partial^2 \overline{U}}{\partial x^2} + 3v \dfrac{\partial \overline{V}}{\partial x} = v^4 \\ 2\overline{U} - 3v \dfrac{\partial \overline{V}}{\partial x} = v^4 + 2xv^2 \end{cases} \quad (16)$$

Then, $3v^2 \dfrac{\partial^2 \overline{U}}{\partial x^2} + 2\overline{U} = 2v^4 + 2xv^2$

$$\overline{U} = \frac{2v^4}{3v^2 D^2 + 2} + \frac{2xv^2}{3v^2 D^2 + 2}$$

$$\overline{U} = v^4 + v^2 x$$

Taking inverse ELzaki transform,

$$U = t^2 + x$$

$$\frac{\partial V}{\partial t}(x,t) = 3x - \frac{\partial U}{\partial x} = 3x - 1$$

$$V(x,t) = (3x-1)\,\mathrm{t} + \mathrm{f(x)}$$

Thus, $V = (3x-1)t$

VIII. APPENDIX

ELzaki transform of some function

$f(t)$	$E[f(t)] = T(\mathrm{v})$
1	v^2
t	v^3
t^n	$n!\, v^{n+2}$
$\sin at$	$\dfrac{av^3}{1+a^2v^2}$
$\cos at$	$\dfrac{v^2}{1+a^2v^2}$

IX. CONCLUSIONS

In this paper, we have introduced new modified transform form Sumudo and Laplace namely ELzaki transform for solving varies types of ordinary differential equations and partial differential equations.

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