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Kinematic, Dynamic Modeling and Simulation of Biped Robot

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***Abstract*—The idea of having a biped robot walking like a human is found to interesting in many aspects. Biped is a multi-jointed mechanism that performs a human's motions. It seems more difficult to analyze the behavioral character of walking robot due to the complexity of mathematical description involved. This work is focused on developing a methodology for deriving mathematical modeling of a biped robot. The work is aimed to build the lower side, the locomotion part of a biped robot. The model used consists of 5-links which are connected through revolute joints. The identical legs have hip joint, knee joints and ankle joint. The Kinematical model is obtained using D - H Technique. It couples a design considerations and simplicity of design to provide inverse kinematics analysis of 11 degree-of-freedom (DOF) biped robot. Lagrangian formulation is applied to obtain dynamic model of robot. MAPLE software is used for mathematical modeling. The trajectory planning is done in Matlab for kinematic analysis and robot's motions. By applying it, the user specific walking parameters, joint trajectories of the robot are computed. The parameters for these motions are found in simulation, under a criterion of stability of walking. Simulations are done in Matlab software to test the behavior of the humanoid. The results show that the proposed motions give an efficient and stable walking of the robot. This method presents a simple and efficient procedure for finding the joint solution of bipeds.**

***Index Terms*—Degree of freedom (DOF),Denavit–Hartenberg(D-H)parameters, Jacobian, Lagrangian.**

I. INTRODUCTION

From ancient times, man has tried to create the mechanism that resembles the human body. Bipedal locomotion involves a large number of degrees of freedom. In the last decades we have seen a rapid growth in use of Humanoid Robotics, which leads to an autonomous research field. Humanoid robots are used in all situations of human's everyday life, cooperating with us. They will work in services, in homes and hospitals, and they are even expected to get involved in sports. Another important fact about today's and especially tomorrow's humanoid robots resembles human-like in their shape and behavior. The research on humanoid biped robot includes various areas such as mechanical design, mathematical modeling. Besides this, there are many problems that involve kinematics analysis, dynamic analysis. All this makes the study of bipedal robot a complex subject .The design for range of motion of each joint is same as standard human so that a humanoid robot performs human tasks.

In this paper, we proposed a models by viewing the kinematic chain of a leg of a biped in forward order. This paper is organized as follows: An outline of the mechanical design of the developed biped robot is given in Section 2. The kinematics for the proposed humanoid robot is obtained using

theDenavit-Hartenberg convention in Section 3. The jacobian matrix which is prerequisite for dynamics is described in Section 4. The discussion about the dynamic model in the Sagittal and Frontal planes using Lagrange equations is in Section 5. Section 6 presents validation of kinematic and dynamic model by simulation .Finally, Section 7 presents some important conclusions for biped robot.

II. MECHANICAL DESIGN

The design of biped is based on human body in terms of ratios, body proportions, and range of motion. This paper propose to have sufficient DOF to imitate human motion. The model used consists of 5-links which are connected through revolute joints, 2-links for each leg and 1-link for torso. It is considered as a robot with waist or torso, linking two legs which are linked together through hip joints to emulate a human's activities. The identical legs have hip joint between torso and thigh, knee joints between the thigh and shank, ankle joint between shank and foot, and a rigid body forms the torso. The joint structure of the biped has eleven degrees of freedom, 5 DOF for each leg and 1 DOF for waist or torso. DOF for waist is shared between legs. The Hip joint has 2-DOF, which allows it motion in the sagittal and the lateral plane. The range of motion in the sagittal plane is between +70º to -50º and +50º and -60º in the lateral plane. The Knee joint has 1-DOF, which allows it motion in the sagittal and the lateral plane. The mobility for the knee joint is +140º in the sagittal plane. The ankle joint has 2-DOF, which allows it motion in the sagittal and the lateral plane. The range of motion in the sagittal plane is between +70º to -50º and +50º and -60º in the lateral plane.

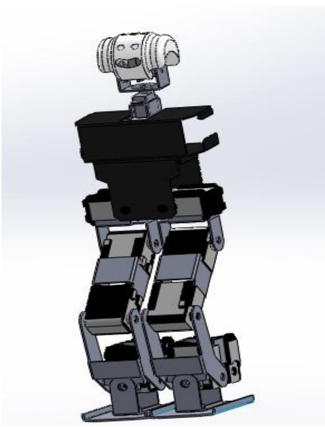

Fig.1: CAD model of biped

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Servos are mounted on the biped robot serves as actuators for the system. One servo is attached to torso. On each leg, two servos are attached to the hip, one servo is attached to the knee and two servos are attached to the ankle. The mechanical design of the bipedal robot is modular, making it easy to change and replace parts. The frameworks of biped will be fabricated from acrylic in order to obtain light weight, and a wide range of motion.

III. KINEMATIC MODEL

Kinematic model depending upon above planned movements, can be formulated. Kinematic analysis is based on the basic equation of the geometric model that aids in determining the position and orientation of a foot with a reference to torso for known values of the joint variables of kinematic chain that compose the robot. Denavit-hartenberg formulation is used to model biped. Each part is considered as a link represented by a line along its joint axis and common normal to next joint axis. Coordinate system is attached to each link illustrating relative position amongst various links. A 4×4 transformation matrix relating i+1 frame to i frame is given by,

$$^{i-1}H_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i-1} & \sin\theta_i \sin\alpha_{i-1} & a_{i-1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i-1} & -\cos\theta \sin\alpha_{i-1} & a_{i-1}\sin\theta_i \\ 0 & \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots (1)$$

Where,

θ_i = Rotation angle is angle between X_{i-1} and X_i measured about Z_i.

α_{i-1} = Twist angle is angle between lines along joints i-1 and i measured about common perpendicular X_{i-1}.

A_{i-1} = link length is the distance between the lines along joints i-1 and i along common perpendicular.

D_i = link offset is distance along Z_i from line parallel to X_{i-1} to the line parallel to X_i and are called as Denavit-hartenberg (D-H) parameters.

Equation 1 is homogeneous transformation matrix indicating position and orientation of each joint. An origin(X_0, Z_0) is established at the torso and each joint has a coordinate frames are attached following D-H definition. For the biped robot with all revolute joints, we have formulated $\theta_i, \alpha_{i-1}, a_{i-1}, d_i$.Table 1 and Table 2 lists D-H parameter used to solve transformation matrix. Transformation matrix of each joint can be obtained by substituting D-H parameters into Equation 1.

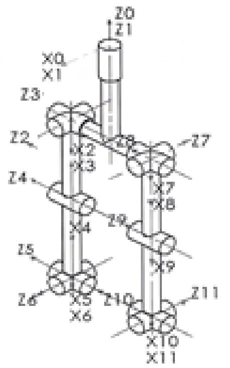

Fig. 2: Frame assignment

Table1: Denavit-Hatenberg parameters for left leg

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	Θ_1
2	90	l_2	0	Θ_2+90
3	90	0	0	Θ_3
4	0	l_4	0	Θ_4
5	0	l_5	0	Θ_5
6	-90	0	0	Θ_6

Table2: Denavit-Hatenberg parameters for right leg

i	α_{i-1}	a_{i-1}	d_i	Θ_i
1	0	0	0	Θ_1
7	-90	l_7	0	Θ_7+90
8	-90	0	0	Θ_8
9	0	l_9	0	Θ_9
10	0	l_{10}	0	Θ_{10}
11	90	0	0	Θ_{11}

The continuous homogeneous transformation from $^0H_1$ to $^5H_6$ transform ankle coordinate to base torso coordinate, shown in equation 2.Pose of ankle with respect to torso is given by,

$$^0H_6 = {}^0H_1 \cdot {}^1H_2 \cdot {}^2H_3 \cdot {}^3H_4 \cdot {}^4H_5 \cdot {}^5H_6 \ldots\ldots (2)$$

$$P = {}^0H_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & Px \\ r_{21} & r_{22} & r_{23} & Py \\ r_{31} & r_{32} & r_{33} & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots (3)$$

Equation 3 provides solution of forward kinematics with matrix P being result. The translation vector$\{Px, Py, Pz\}$ gives position of foot and orientation matrix$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$shows direction of foot.

IV. JACOBIAN MATRIX

In this we are interested to derive the velocity relationships that relate the linear and angular velocities of the end-effector to the joint velocities. We will find the angular velocity of the end-effector frame which gives the rate of rotation of the frame and the linear velocity of the origin. Then we relate these velocities to the joint velocities.Jacobian matrix forms the basic elements in building a dynamic model of biped walking. On the basis of motion i.e rectilinear or rotary, the

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jacobian matrixes are divided as linear or revolutary. In this design, all joints are revolute, so general form of matrix can be written as,

$$J_i = \begin{bmatrix} J\nu_i \\ J\omega_i \end{bmatrix} = \begin{bmatrix} \frac{Z_{i-1} \times (On - O_{i-1})}{Z_{i-1}} \end{bmatrix} \text{........ (4)}$$

Masses are considered as two concentrated material points such as thigh, shin or leg. We can define the dynamic system as figure 3.

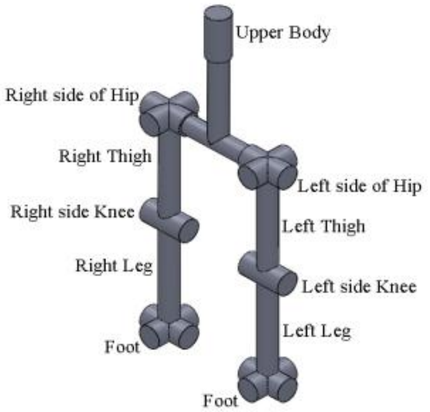

Fig 3: Masses and joints for thigh and shin or legs.

Each mass needs a jocobian matrix J_4 and J_6 means jacobian of thigh and shin. By separating this jacobian matrix into rectilinear and rotary elements, jacobian matrix can be written as shown below. Linear and rotary components of jacobian of thigh are given by,

$$J\nu_4 = [Z_0 \times (O_4c - O_0) \quad Z_1 \times (O_4c - O_1) \quad Z_2 \times (O_4c - O_2) \quad Z_3 \times (O_4c - O_3) \quad 0 \quad 0] \text{........ (5)}$$

$J\nu_4$ =[[- l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)),- l_4 s(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), 0, 0],[l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)),- l_4 s(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), 0, 0],[0, 0, 0, - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1))² - l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2))², 0, 0]]

$$J\omega_4 = [Z_0 \quad Z_1 \quad Z_2 \quad Z_3 \ \ 0 \ \ 0\ ] \text{........ (6)}$$

$J\omega_4$ =[[0, 0, 0, - c(θ_1) s(θ_2) - c(θ_2) s(θ_1), 0, 0],[0, 0, 0, c(θ_1) c(θ_2) - s(θ_1) s(θ_2), 0, 0][1, 1, 1,0, 0, 0]]

Similarly, Linear and rotary components of jacobian of shin can be calculated as,

$$J\nu_6 = [Z \times (O_6c - O_0) \quad Z_1 \times (O_6c - O_1) \quad Z_2 \times (O_{64}c - O_2) \quad Z_3 \times (O_6c - O_3) \quad Z_4 \times (O_6c - O_4) \quad 0]$$

$$J\omega_6 = [Z_0 \quad Z_1 \quad Z_2 \quad Z_3 \ \ Z_4 \ \ 0\ ]$$

Where,

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} {}^0H_1(1,3) \\ {}^0H_1(2,3) \\ {}^0H_1(3,3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} {}^0H_2(1,3) \\ {}^0H_2(2,3) \\ {}^0H_2(3,3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} {}^0H_3(1,3) \\ {}^0H_3(2,3) \\ {}^0H_3(3,3) \end{bmatrix} = \begin{bmatrix} -c(\theta_1)s(\theta_2) - c(\theta_2)s(\theta_1) \\ c(\theta_1)c(\theta_2) - s(\theta_1)s(\theta_2) \\ 0 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} {}^0H_4(1,3) \\ {}^0H_4(2,3) \\ {}^0H_4(3,3) \end{bmatrix} = \begin{bmatrix} -c(\theta_1)s(\theta_2) - c(\theta_2)s(\theta_1) \\ c(\theta_1)c(\theta_2) - s(\theta_1)s(\theta_2) \\ 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} {}^0H_1(1,4) \\ {}^0H_1(2,4) \\ {}^0H_1(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} {}^0H_2(1,4) \\ {}^0H_2(2,4) \\ {}^0H_2(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} {}^0H_3(1,4) \\ {}^0H_3(2,4) \\ {}^0H_3(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_4 = O_4c = \begin{bmatrix} {}^0H_4(1,4) \\ {}^0H_4(2,4) \\ {}^0H_4(3,4) \end{bmatrix} = \begin{bmatrix} l_4\, c(\theta_3)(c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) \\ l_4\, c(\theta_3)\, (c(\theta_1)s(\theta_2) + c(\theta_2)s(\theta_1)) \\ -l_4\, s(\theta_3) \end{bmatrix}$$

$$O_6c = \begin{bmatrix} {}^0H_6(1,4) \\ {}^0H_6(2,4) \\ {}^0H_6(3,4) \end{bmatrix}$$

=[{l5 (c(θ_3) c(θ_4) (c(θ_1) c(θ_2)-s(θ_1) s(θ_2)) s(θ_3) s(θ_4) (c(θ_1) c(θ_2)s(θ_1) s(θ_2)))+ l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2))},{l5 (c(θ_3) c(θ_4) (c(θ_1) s(θ_2)+c(θ_2) s(θ_1))s(θ_3) s(θ_4) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1))) + l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1))},{- l5 (c(θ_3) s(θ_4) + c(θ_4) s(θ_3)) - l_4 s(θ_3)}]

V. LAGRANGE FORMULATION

The aim to solve dynamics is to obtain equation of motion of system. Because of multiple degree of freedom system, it is difficult to obtain equation of motion. In this paper, principles of Lagrangian dynamics is used for determining the gait locomotion equations for obtaining the torque in each joint of the biped. By representing variables of system as generalized coordinate, we can write equation of motion for an n-DOF system using Euler-Lagrange Equation as,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \text{........ (7)}$$

L=K-P........ (8)

Where L is Lagrangian, K is kinetic energy and P is potential energy.

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The kinetic energy of a rigid body is sum of two terms.

$K=\frac{1}{2}mv^Tv+\frac{1}{2}\omega^TI\omega$…….. (9)

The inertia tensor is required to be transferred into global coordinate, so equation 6 should be multiplied by rotational transfer matrix R.

$K=\frac{1}{2}mv^Tv+\frac{1}{2}\omega^TRIR^T\omega$ …….. (10)

Total kinetic energy is sum of each links.

$K=\sum_{i=1}^{n}\left\{\frac{1}{2}m_iv_i^Tv_i+\frac{1}{2}\omega_i^TR_iI_iR_i^T\omega_i\right\}$ …….. (11)

By using the Jacobian matrix, the kinetic energy can be written as the function of the joint variables like Equation 12.

$K=\frac{1}{2}\dot{q}^T[\sum_{i=1}^{n}\{m_i\,J\nu_i(q)^TJ\nu_i(q)+J\omega_i(q)^TR_i(q)I_iR_i(q)^TJ\omega_i(q)\}]\dot{q}$ …….. (12)

$K=\frac{1}{2}\dot{q}^T\{m_4\,J\nu_4(q)^TJ\nu_4(q)+m_6J\nu_6(q)^TJ\nu_6(q)+J\omega_4(q)^TR_4(q)I_4R_4(q)^TJ\omega_4(q)+J\omega_6(q)^TR_6(q)I_6R_6(q)^TJ\omega_6(q)\}\dot{q}$…….. (13)

Inertia matrix D(q) can be given by equation 11

$D(q)=m_4\,J\nu_4(q)^TJ\nu_4(q)+m_6J\nu_6(q)^TJ\nu_6(q)+J\omega_4(q)^TR_4(q)I_4R_4(q)^TJ\omega_4(q)+J\omega_6(q)^TR_6(q)I_6R_6(q)^TJ\omega_6(q)\}$ …….. (14)

Kinetic energy can be written as,

$K=\frac{1}{2}\dot{q}^TD(q)\dot{q}=\frac{1}{2}\sum_{j=1}^{n}d_{ij}(q))\dot{q}_i\dot{q}_j$ …….. (15)

Where D(q) is 6×6 symmetric matrix. Equation 16shows its elements.

$$D(q)=\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16}\\ * & d_{22} & d_{23} & d_{24} & d_{25} & d_{26}\\ * & * & d_{33} & d_{34} & d_{35} & d_{36}\\ * & * & * & d_{44} & d_{45} & d_{46}\\ * & * & * & * & d_{55} & d_{56}\\ * & * & * & * & * & d_{66}\end{bmatrix} \text{…….. (16)}$$

Potential Energy of leg is,

$P=m\,g\,h=\sum_{i=1}^{n}m_i\,gh_{ci}$ …….. (17)

Lagrangian L is the function of the joint variables given by Equation 18.

$L=K-P=\frac{1}{2}\dot{q}^TD(q)\dot{q}-P(q)=\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}d_{ij}(q))\dot{q}_i\dot{q}_j-\sum_{i=1}^{n}m_i\,gh_{ci}(q)$…….. (18)

The partial derivatives of the Lagrangian with respect to the velocity is,

$$\frac{\partial L}{\partial\dot{q}k}=\frac{\partial}{\partial\dot{q}k}\left(\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}d\,ij\,(q))\dot{q}i\,\dot{q}j\right)-\frac{\partial}{\partial\dot{q}k}P(q)=\sum_{j=1}^{n}dkj(q)\dot{q}j \quad \text{…….. (19)}$$

Differential of equation 19 is,

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}k}=\sum_{j=1}^{n}dkj\,\ddot{q}j+\sum_{j=1}^{n}\frac{d}{dt}dkj\,\dot{q}j=\sum_{j=1}^{n}dkj\,\ddot{q}j+\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial dkj}{\partial qi}\,\dot{q}i\,\dot{q}j$$

…….. (20)

The partial derivatives of the Lagrangian with respect to the position is

$\frac{\partial L}{\partial qk}=\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial ij}{\partial qk}\dot{q}i\,\dot{q}j-\frac{\partial P}{\partial k}$…….. (21)

Euler-Lagrangian equation can be obtained by substraction of equation 21 from equation 20.

$\sum_{j=1}^{n}dkj\,\ddot{q}j+\sum_{j=1}^{n}\sum_{i=1}^{n}\left\{\frac{\partial dkj}{\partial qi}-\frac{1}{2}\frac{\partial dij}{\partial qk}\right\}\dot{q}i\,\dot{q}j+\frac{\partial P}{\partial qk}=\tau_k$ …….. (22)

If we define the Christoffel symbols C_{ijk} and gravity force $g_k(q)$as in Equation and

$C_{ijk}=\frac{1}{2}\left(\frac{\partial dkj}{\partial qi}+\frac{\partial dki}{\partial qj}-\frac{\partial dij}{\partial qk}\right)$…….. (23)

$g_k(q)=\frac{\partial P(q)}{\partial qk}$…….. (24)

Equations of motion are given by equation 25,

$\sum_{j=1}^{6}dkj(q)\,\ddot{q}j+\sum_{j=1}^{6}\sum_{i=1}^{6}Cijk(q)\,\dot{q}i\,\dot{q}j+gk(q)=\tau_k$ …….. (25)

The second term of equation 25,$\sum_{j=1}^{6}\sum_{i=1}^{6}Cijk(q)\,\dot{q}i\,\dot{q}j$has two meanings. When i= j, term indicates centrifugal force. When i≠ j, term indicates Coriolis Effect. Since the product of inertia is much smaller than moment of inertia, Corioliseffect can be disregarded. So the equations of motion can written as equation 26.

$\tau_1=d_{11}\ddot{\theta}_1+d_{12}\ddot{\theta}_2+d_{13}\ddot{\theta}_3+d_{14}\ddot{\theta}_4+d_{15}\ddot{\theta}_5+d_{16}\ddot{\theta}_6+C_{111}\dot{\theta}_1^2+C_{221}\dot{\theta}_2^2+C_{331}\dot{\theta}_3^2+C_{441}\dot{\theta}_4^2+C_{551}\theta_5^2+C_{661}\theta_6^2+g_1.$

$\tau_2=d_{21}\ddot{\theta}_1+d_{22}\ddot{\theta}_2+d_{23}\ddot{\theta}_3+d_{24}\ddot{\theta}_4+d_{25}\ddot{\theta}_5+d_{26}\ddot{\theta}_6+C_{112}\dot{\theta}_1^2+C_{222}\dot{\theta}_2^2+C_{332}\dot{\theta}_3^2+C_{442}\dot{\theta}_4^2+C_{552}\theta_5^2+C_{662}\theta_6^2+g_2.$

$\tau_3=d_{31}\ddot{\theta}_1+d_{32}\ddot{\theta}_2+d_{33}\ddot{\theta}_3+d_{34}\ddot{\theta}_4+d_{35}\ddot{\theta}_5+d_{36}\ddot{\theta}_6+C_{113}\dot{\theta}_1^2+C_{223}\dot{\theta}_2^2+C_{333}\dot{\theta}_3^2+C_{443}\dot{\theta}_4^2+C_{553}\theta_5^2+C_{663}\theta_6^2+g_3.$

$\tau_4=d_{41}\ddot{\theta}_1+d_{42}\ddot{\theta}_2+d_{43}\ddot{\theta}_3+d_{44}\ddot{\theta}_4+d_{45}\ddot{\theta}_5+d_{46}\ddot{\theta}_6+C_{114}\dot{\theta}_1^2+C_{224}\dot{\theta}_2^2+C_{334}\dot{\theta}_3^2+C_{444}\dot{\theta}_4^2+C_{554}\theta_5^2+C_{664}\theta_6^2+g_4.$

$\tau_5=d_{51}\ddot{\theta}_1+d_{52}\ddot{\theta}_2+d_{53}\ddot{\theta}_3+d_{54}\ddot{\theta}_4+d_{55}\ddot{\theta}_5+d_{56}\ddot{\theta}_6+C_{115}\dot{\theta}_1^2+C_{225}\dot{\theta}_2^2+C_{335}\dot{\theta}_3^2+C_{445}\dot{\theta}_4^2+C_{555}\theta_5^2+C_{665}\theta_6^2+g_5.$

$\tau_6=d_{61}\ddot{\theta}_1+d_{62}\ddot{\theta}_2+d_{63}\ddot{\theta}_3+d_{64}\ddot{\theta}_4+d_{65}\ddot{\theta}_5+d_{66}\ddot{\theta}_6+C_{116}\dot{\theta}_1^2+C_{226}\dot{\theta}_2^2+C_{336}\dot{\theta}_3^2+C_{446}\dot{\theta}_4^2+C_{556}\theta_5^2+C_{666}\theta_6^2+g_6.$

…….. (26)

Element of inertia matrix is,

$d_{11}=(I_{xx4}\,(c(\theta_3)\,(1-l_4^2\,l_5^2\,((l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))+l_4\,c(\theta_3)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))^2/2+(l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2)))+l_4\,c(\theta_3)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2)))^2/2-l_4^2/2-l_5^2/2+(l_5\,(c(\theta_3)\,s(\theta_4)+c(\theta_4)\,s(\theta_3))+l_4\,s(\theta_3))^2/2)^2)^{1/2}+l_4\,l_5\,s(\theta_3)\,((l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))+l_4\,c(\theta_3)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))^2/2+(l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2)))+l_4\,c(\theta_3)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2)))^2/2-l_4^2/2-l_5^2/2+(l_5\,(c(\theta_3)\,s(\theta_4)+c(\theta_4)\,s(\theta_3))+l_4\,s(\theta_3))^2/2))-Iyx4\,(s(\theta_3)\,(1-l_4^2\,l_5^2\,((l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))+l_4\,c(\theta_3)\,(c(\theta_1)\,s(\theta_2)+c(\theta_2)\,s(\theta_1)))^2/2+(l_5\,(c(\theta_3)\,c(\theta_4)\,(c(\theta_1)\,c(\theta_2)-s(\theta_1)\,s(\theta_2))-s(\theta_3)\,s(\theta_4)\,(c(\theta_1)\,c(\theta_2)$

ISSN: 2278 – 7798

$- s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))).$

Similarly all other $d_{12},d_{13},d_{14},d_{15},d_{16},d_{21},d_{22},d_{23},d_{24},d_{25},d_{26},d_{31},d_{32},d_{33},d_{34},d_{35},d_{36},d_{41},d_{42},d_{43},d_{44},d_{45},d_{46},d_{51},d_{52}, d_{53},d_{54}, d_{55},d_{56}, d_{61}, d_{62},d_{63}, d_{64}, d_{65},d_{66}$ can be found out.

Each element of the Christoffel symbols is,

$$C_{113}=\frac{1}{2}\left\{\frac{\partial d31}{\partial q1}+\frac{\partial d31}{\partial q1}-\frac{\partial d11}{\partial q3}\right\}=\frac{\partial d13}{\partial q1}-\frac{1}{2}\frac{\partial d11}{\partial q3}$$

$= ((I_{xx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2) + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) + c(\theta_4) s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) - (l_5 (c(\theta_3) c(\theta_4) - s(\theta_3) s(\theta_4)) + l_4 c(\theta_3)) (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3)) + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) + c(\theta_4) s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))) - (l_4^2 l_5^2 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) + c(\theta_4) s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) - (l_5 (c(\theta_3) c(\theta_4) - s(\theta_3) s(\theta_4)) + l_4 c(\theta_3)) (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3)) + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) + c(\theta_4) s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))/(1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))$

Equation 17 is about potential energy which is function of joint angles and is given by,

$P = m_4 g (h_0-l_4c c(\theta_2) c(\theta_3)+m_6 g (h_0- c(\theta_2)(l_4 c(\theta_3)+l_{6c} c(\theta_3+ \theta_4)$

$P = (981 m_4 (h_0-l_4c c(\theta_2) c(\theta_3)))/100 + (981 m_6 (h_0- c(\theta_2) (l_{6c} c(\theta_3+ \theta_4) + l_4 c(\theta_3))))/100$

Element of gravity force is given by,

$g_2=\frac{\partial P}{\partial q2} =(981 m_6 s(\theta_2) (l_{6c} c(\theta_3 + \theta_4) + l_4 c(\theta_3)))/100 + (981 l_{4c}m_4 c(\theta_3) s(\theta_2))/100$

Similarly each element of gravity force g_1, g_3, g_4, g_5, g_6 can be calculated.

Torque required for yaw motion of torso is given by,

$\tau_1= \ddot{\theta}_1 ((I_{xx4} (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4s(\theta_3))^2/2)) – I_{yx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4s(\theta_3))^2/2))).$

Torque required for rolling motion at hip joint is given by,

$\tau_2= \ddot{\theta}_1 ((I_{xx4} (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) – I_{yx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^2 - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))).$

Torque required for pitching motion at hip joint is given by,

ISSN: 2278 – 7798

$$\tau_3 = \ddot{\theta}_1((I_{xx4}\,(c(\theta_3)(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} + l_4\,l_5\,s(\theta_3)((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)) - Iyx_4\,(s(\theta_3)(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} - l_4\,l_5\,c(\theta_3)((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2})\,\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)) - s(\theta_5)(s(\theta_3)(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} - l_4\,l_5\,c(\theta_3)((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)))).$$

Torque required for pitching motion at knee joint is given by,

$$\tau_4 = \ddot{\theta}_4((c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))(s(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))\,(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} - l_4\,l_5\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} - l_4\,l_5\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2))) + c(\theta_2)\,s(\theta_1))(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} + l_4\,l_5\,s(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)) - (c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} + l_4\,l_5\,s(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2))).$$

Torque required for pitching motion at ankle joint is given by,

$$\tau_5 = \ddot{\theta}_4\,((c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2}) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2})(c(\theta_5)(c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))(1 - l_4^2\, l_5^2((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3)) + l_4\,s(\theta_3))^2/2)^2)\tfrac{1}{2} + l_4\,l_5\,s(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))((l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1))) + l_4\,c(\theta_3)(c(\theta_1)\,s(\theta_2) + c(\theta_2)\,s(\theta_1)))^2/2 + (l_5\,(c(\theta_3)\,c(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)) - s(\theta_3)\,s(\theta_4)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2))) + l_4\,c(\theta_3)(c(\theta_1)\,c(\theta_2) - s(\theta_1)\,s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\,(c(\theta_3)\,s(\theta_4) + c(\theta_4)\,s(\theta_3))$$

ISSN: 2278 – 7798

$$+ l_4\ s(\theta_3))^2/2)) - s(\theta_5)(s(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)$$
$$s(\theta_1))(1 - l_4^2\ l_5^2((l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)$$
$$s(\theta_1)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1))) + l_4$$
$$c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)))^2/2 + (l_5\ (c(\theta_3)$$
$$c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ c(\theta_2) -$$
$$s(\theta_1)\ s(\theta_2))) + l_4\ c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)))^2/2 - l_4^2$$
$$- l_5^2/2 + (l_5\ (c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3)) + l_4$$
$$s(\theta_3))^2/2)^2)\frac{1}{2} - l_4\ l_5\ c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1))((l_5$$
$$(c(\theta_3)\ c(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)) - s(\theta_3)$$
$$s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1))) + l_4\ c(\theta_3)(c(\theta_1)\ s(\theta_2) +$$
$$c(\theta_2)\ s(\theta_1)))^2/2 + (l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)$$
$$s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4$$
$$c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5$$
$$(c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3)) + l_4\ s(\theta_3))^2/2))) -$$
$$(c(\theta_5)(c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))(1 - l_4^2\ l_5^2((l_5$$
$$(c(\theta_3)\ c(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)) - s(\theta_3)$$
$$s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1))) + l_4\ c(\theta_3)(c(\theta_1)\ s(\theta_2) +$$
$$c(\theta_2)\ s(\theta_1)))^2/2 + (l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)$$
$$s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4$$
$$c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5$$
$$(c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3)) + l_4\ s(\theta_3))^2/2)^2)\frac{1}{2} + l_4\ l_5$$
$$s(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))((l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)$$
$$s(\theta_2) + c(\theta_2)\ s(\theta_1)).$$

Torque required for rolling motion at ankle joint is given by,

$$\tau_6=\theta_1\ ((1 - l_4^2\ l_5^2((l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)$$
$$s(\theta_1)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1))) + l_4$$
$$c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)))^2/2 + (l_5\ (c(\theta_3)$$
$$c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ c(\theta_2) -$$
$$s(\theta_1)\ s(\theta_2))) + l_4\ c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)))^2/2 -$$
$$l_4^2/2 - l_5^2/2 + (l_5\ (c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3)) + l_4$$
$$s(\theta_3))^2/2)^2)\frac{1}{2}) + (l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)$$
$$s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4$$
$$c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5$$
$$(c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3)) + l_4\ s(\theta_3))^2/2)^2)\frac{1}{2}) + ((I_{yz6}pi$$
$$c((\ l_4\ c(\theta_4)\ c(\theta_5)\ s(\theta_6) - l_4\ s(\theta_4)\ s(\theta_5)\ s(\theta_6) + l_5\ c(\theta_4)^2$$
$$c(\theta_5)\ s(\theta_6) + l_5\ c(\theta_5)\ s(\theta_4)^2\ s(\theta_6))/(c(\theta_4)^2\ c(\theta_5)^2\ c(\theta_6)^2$$
$$+ c(\theta_4)^2\ c(\theta_5)^2\ s(\theta_6)^2 + c(\theta_4)^2\ c(\theta_6)^2\ s(\theta_5)^2 + c(\theta_5)^2$$
$$c(\theta_6)^2\ s(\theta_4)^2 + c(\theta_4)^2\ s(\theta_5)^2\ s(\theta_6)^2 + c(\theta_5)^2\ s(\theta_4)^2\ s(\theta_6)^2$$
$$+ c(\theta_6)^2\ s(\theta_4)^2\ s(\theta_5)^2 + s(\theta_4)^2\ s(\theta_5)^2\ s(\theta_6)^2))\ c((pi\ s((\ l_4$$
$$c(\theta_4)\ c(\theta_5)\ s(\theta_6) - l_4\ s(\theta_4)\ s(\theta_5)\ s(\theta_6) + l_5\ c(\theta_4)^2\ c(\theta_5)$$
$$s(\theta_6) + l_5\ c(\theta_5)\ s(\theta_4)^2\ s(\theta_6))/(c(\theta_4)^2\ c(\theta_5)^2\ c(\theta_6)^2 +$$
$$c(\theta_4)^2\ c(\theta_5)^2\ s(\theta_6)^2 + c(\theta_4)^2\ c(\theta_6)^2\ s(\theta_5)^2 + c(\theta_5)^2c(\theta_6)^2$$
$$s(\theta_4)^2 + c(\theta_4)^2\ s(\theta_5)^2\ s(\theta_6)^2 + c(\theta_5)^2\ s(\theta_4)^2\ s(\theta_6)^2 +$$
$$c(\theta_6)^2\ s(\theta_4)^2\ s(\theta_5)^2 + s(\theta_4)^2s(\theta_5)^2$$
$$s(\theta_6)^2)))/2)(c(\theta_5)(c(\theta_3)(1 - l_4^2\ l_5^2((l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)$$
$$s(\theta_2) + c(\theta_2)\ s(\theta_1)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)$$
$$s(\theta_1))) + l_4\ c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)))^2/2 + (l_5$$
$$(c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)$$
$$c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4\ c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)$$
$$s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\ (c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3))$$
$$+ l_4\ s(\theta_3))^2/2)^2)\frac{1}{2} + l_4\ l_5\ s(\theta_3)((l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)$$
$$s(\theta_2) + c(\theta_2)\ s(\theta_1)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)$$
$$s(\theta_1))) + l_4\ c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)))^2/2 + (l_5$$
$$(c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)$$
$$c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4\ c(\theta_3)(c(\theta_1)\ c(\theta_2) - s(\theta_1)$$
$$s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\ (c(\theta_3)\ s(\theta_4) + c(\theta_4)\ s(\theta_3))$$
$$+ l_4\ s(\theta_3))^2/2)) - s(\theta_5)(s(\theta_3)(1 - l_4^2\ l_5^2((l_5\ (c(\theta_3)$$
$$c(\theta_4)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)) - s(\theta_3)\ s(\theta_4)(c(\theta_1)\ s(\theta_2)$$
$$+ c(\theta_2)\ s(\theta_1))) + l_4\ c(\theta_3)(c(\theta_1)\ s(\theta_2) + c(\theta_2)\ s(\theta_1)))^2/2$$
$$+ (l_5\ (c(\theta_3)\ c(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2)) - s(\theta_3)$$
$$s(\theta_4)(c(\theta_1)\ c(\theta_2) - s(\theta_1)\ s(\theta_2))) + l_4\ c(\theta_3)(c(\theta_1)\ c(\theta_2) -$$
$$s(\theta_1)\ s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\ (c(\theta_3)\ s(\theta_4) + c(\theta_4)$$
$$s(\theta_3)) + l_4\ s(\theta_3))^2/2)^2)\frac{1}{2} - l_4\ l_5\ c(\theta_3)).$$

VI. SIMULATION

A simulation is the imitation of the operation of real-world process or system over time. It is a tool to evaluate the performance of a system, existing or proposed, under different configurations of interest and over long periods of real time. The behavior of a system that evolves over time is studied by developing a simulation model. Simulation consists of Building a computer model that describes the behavior of a system and Experimenting with this model to reach conclusions that support decisions It Use the mathematical model to determine the response of the system in different situations. Objective in simulation is for given external torque/forces obtain motion of robot. Simulation can be used as an Analysis tool for predicating the effect of changes as well as Design tool to predicate the performance of new system. It is better to do simulation before Implementation.

In order to establish the kinematic and the dynamic model of the humanoid robot, both the mechanical properties of all its components, such as mass and inertia, as well as its servos dynamic behavior, must be known to a certain degree of accuracy. These dynamic properties will be used in Matlab in order to get an accurate simulation for the real humanoid robot aiming at a good control. Dynamic model of the biped is obtained based on the physical properties of their components. Typically, by knowing the mass, center of mass and the inertia tensor of each element of the robot it is possible to get mathematical model. The centroid of each mass was then found by using the SolidWorks software package, after the detailed elements of all the pieces involved were drawn in AutoCAD Inventor software. Finally, the inertia tensor of each element was determined through the SolidWorks software.

We have done the simulation of the above-mentioned motions of the biped using MATLAB as the programming tool. This work presents trajectory planning of the joint angles of a five link biped for stable walking. The locomotion is constrained to a plane perpendicular to the ground that separates the biped into left and right halves. Their model describes five links, which consists of the two legs and the torso driven by independent servos.

A. Trajectories for Walking

The biped walking cycle is broken into various phases depending on how the feet are in contact with the ground. There are three phases in the biped walking i.e the starting phase from the vertical position and two steady walking steps. The walking is started with right leg. In the first step, the right leg is moved forward and placed on the ground with support of the left leg. The first steady walking step involves lifting the left leg with single leg support of the right leg until the left leg is planted on the ground again. The second steady walking step is similar to the first steady walking step, but this step has single leg support of the left leg until the right leg is lifted and planted on the ground again. The repetitions of walking steps result in a continued locomotion in the sagittal plane. At the end of each step the biped has two legged support. This walking can be defined as one starting step and a repetition of steady walking steps with left and right feet alternately switched for supports. The results are

ISSN: 2278 – 7798

provided in the form of graphs with joint angles against time giving the trajectory.

B. Simulation and Results for Biped Walking

The biped model discussed in this work is simulated using MATLAB, due to the simplicity of its use and good quality graphics. A program 'walking_slope, walking_ground' created for simulation and various other functions double_stance, heelstrike_ds, heelstrike_ss, single_stance that support the main program are created. The lengths and the radii of the links of the humanoid robot in meters are obtained by measuring a human subject. Initial joint angles are defined for vertical position where legs of the humanoid hands are aligned to each other.

C.Simulation of biped walking on sloped ground

The trajectory of the joint angles obtained from the work is converted into a set of equations defining the relationship between the joint angles and time for various links of the humanoid during a walk cycle. In this function, the equations are given for various links at different time periods. For each link the times and the joint angles are collected as arrays in steps of time which is same as the time increment for every iteration of the simulation. Plotting the coordinates of the joints obtained by this function would result in stick figure. To obtain better graphics, the links of the humanoid robot can be represented as cylinder. The parameters that will be used are listed asM = 1000, m = 1,l = 0, L = 1,w = 0, c = 1,r = 0, d = 0,g = 1, gam = 0.009.

Foot has a radius of zero, so we have to set two of the angles to be constant. We have set q_2 and q_4 at pi. Swing leg is allowed to pass for near vertical stance leg angles. A step be viewed as a stride function takes a vector of the angles and angles rates at a particular point in the motion .It returns the angles and rates at the next occurrence of that point. The result of the stride function are found by integrating the equations of motion over one step.

If for a given set of initial conditions if stride function returns the same conditions then one gait cycle exists. If the stride function returns the initial conditions after two steps then two gait cycle exists. The zeroes of g (θ_0) is defined as, g (θ_0) = f (θ_0) - θ_0.

The gait cycles are periodic walking solutions .The initial conditions for which a gait cycle exists are called fixed points. We have used MATLAB to find fixed points and create periodic walking simulations. Stability of the cycles is checked by finding the eigenvalues of the Jacobian of the stride function. J is constructed by evaluating the stride function with small perturbations to the components of the particular fixed point that is being evaluated. Small perturbations, θ, to the state vector, θ_0, at the start of a step will either grow or decay to the step by $\theta^{k+1} = J\,\theta^k$.

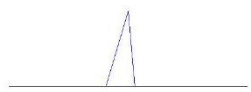
If the eigenvalues of the Jacobian are within the unit circle, then sufficiently small perturbations will decay to zero and the system will return to the gait cycle. If any eigenvalues are outside the unit circle, perturbations along the corresponding eigenvector will grow and drive the gait cycle unstable. If any eigenvalues lie on the unit circle, the cycle is neutrally stable for small perturbations along the corresponding eigenvector and these perturbations will remain constant. We have used MATLAB to find the Jacobian and its eigenvalues.

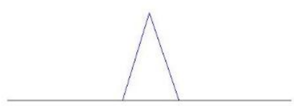
As q_2 and q_4 are locked, we eliminate the angular momentum balance equations about points A_1 and A_2. M_{11}, M_{13}, M_{31}, M_{33}, RHS_1, and RHS_3 are required to find the necessary angles. A collision detection function takes in the current time, positions, and parameters and determines whether a collision has occurred. This function is used with the integrator.

The main driver are needed to be set up. Initially, the initial conditions vector and parameters are setup. The initial conditions vector only needs q_1, u_1,q_3,$u_3$since q_2 and q_4 are constant. We have used the fsolve function to calculate the fixed point, 0 = f (θ_0) - θ_0. fsolve will iterate upon the provided initial conditions until it finds the fixed points within the tolerance provided. The fixed point function calls the onestep function which integrates the equations of motion and uses the heelstrike equations over the number of steps specified. The fixed point function calls onestep for one step. We have found out the stability of the found fixed points.

We have defined a function partialder which calculate the Jacobian of the stride function. The Jacobian is estimated using central difference method of approximating derivatives. Central difference is accurate to the perturbation size squared as opposed to the perturbation size for forward difference but requires a little less than twice the number of evaluations. We will use perturbations of size 1×10^{-5}.onestep function is created which will integrate the equations of motion over the specified number of steps. The function takes in the initial conditions, the parameters and the number of steps.

We have set up the function for calculating the fixed points. If no number of steps is specified, it is assume that we are trying to find the fixed points and we only want to return the final state of the robot. The motion can be integrated and an event will be set in the options. This event will call collision function to detect for collisions while the equations of motion are being integrated. When a collision is detected, the integrator is stopped and the heelstrike equations will be called using the final conditions from the integration of the single stance equations. The resulting state vector and time is then set to the initial conditions for integration of the next step. At the end of the steps, if flag is one, all of the positions and times are returned. If not, only the last positions are returned. Lastly, we animate the robot over the steps and create plots of the stance leg and swing leg angles. The simulation results are as listed below.





ISSN: 2278 – 7798

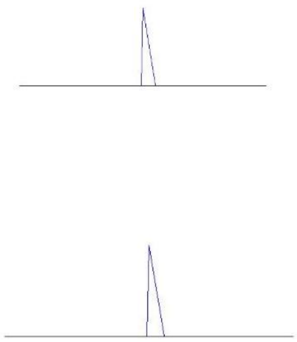


Fig 4: Walking pattern of biped on sloped ground

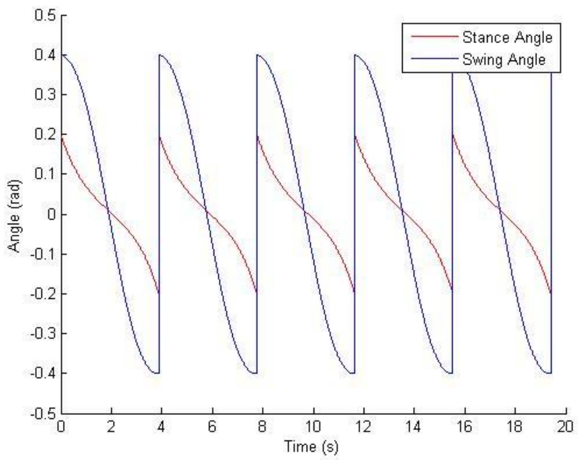


Fig 5: Stance and Swing angle plot against time for sloped ground

D.Simulation of biped walking on flat surface

The final step in creating walking robot simulations is adding control. The simple biped can walk only when the ground is sloped. We have designed a biped that also walk on flat ground. This includes simple biped with motor at each joint for actuation. The parameters that are used are as M = 1, m = 0.5, I = 0.02, l = 1, w = 0, c = 0.5, r = 0.2, d = 0.00, g = 1, gam = 0.00.Then add a global variable for the hip state and define the two possible states. The initial state will be Hipswing.

It is required to set up a controller function that will take in the state variables, time, and parameters. The controller return the torque at that instance. Then define the proportional and derivative gains as well as the reference angle for the hip swing. State the robot is determined by controller. After leg reaches the reference angle, the robot transform to free swing. If Hipstate is Hipswing, the controller calculates the torque that will be applied based on the control law, $T_h = -P(q_3-q_{3REF})-Du_1$. If Hipstate is Hipfree, the controller applies zero torque.

Finally we have the controller act in the single stance function. We set to be equal to the result of the controller function. The simulation results are as listed below.

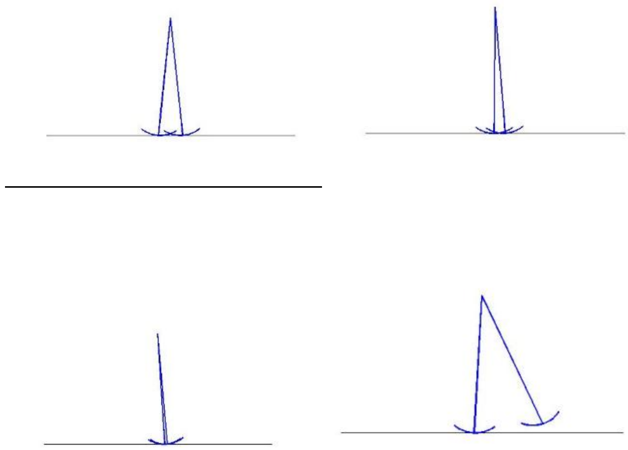


Fig 6:Walking pattern of biped on flat ground

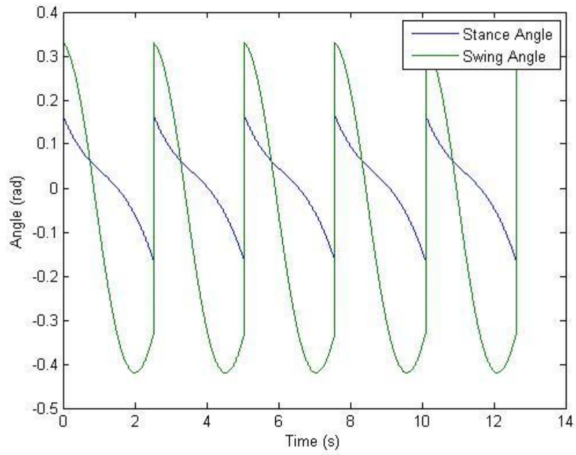


Fig 7: Stand and Swing angle plot against time for flat ground

VII. CONCLUSION

Biped which had been previously modelled have less number of DOF, which restrict the motion of biped in working environment. Also the mathematical model which had been used, take into account either kinematic or dynamic approach. Both approaches were not considered. This paper presented an easy way to visualize movement of a 5-link, 11–DOF biped considering kinematic and dynamic aspects. This is the first step for deriving dynamic model of biped. Lagrangian formulation is applied to obtain dynamic model of robot. There are two sets of the humanoid 3D CAD drawing. One set is detailed includes resembling the reality pieces for mechanical analysis. Second is a less detailed one with precise real measurements is used in animation. The process of modeling the multi-body structure of a biped robot for generating a stable walking motion is a very complex .It deal with the formulation and solution of highly complex dynamics equations of a very large size. Complexity of mathematical models is due to geometry of biped. This work can be seen responsible for the creation of the strong foundations for future developments in humanoid robots. During this project, the following topics have been successfully achieved:

i. The external physical parameter identification of the

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humanoid structure, such as the mass, inertia tensor and centers of gravity of its main parts.

ii. The dynamic analysis and identification of the behavior of the servo-actuators.

iii. The development of a simulator for the biped using Matlab as animation.

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