A graph containing some subgraphs with Spectral radius conditions

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Abstract Here, in this paper, we are using the upper bound for the spectral radius for a graph obtained by Cao, to present adequate conditions based on the spectral radius for a graph containing some sub graphs

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I. Introduction

We consider only finite undirected graphs without loops or multiple edges. Notation and terminology not defined here follow those in [2]. For a graph G = (V; E), we use n and e to denote its order IVI and size IEI, respectively. The largest and smallest degrees of a graph G are denoted by Δ(G) and δ(G), respectively. The eigenvalues of a graph are defined as the eigenvalues of its adjacency matrix. The largest eigenvalue of a graph G, denoted ρ(G), is called the spectral radius of G. If no confusion arises, we may drop G for those invariants. We use C_k to denote a cycle of length k. We also call C_3 as a triangle. The circumference of a graph is defined as the length of the longest cycle in the graph.

Cao [3] obtained the following upper bound for the spectral radius of a graph.

Lemma 1.1. [3] Let G be a graph of order n and size e with minimum degree δ≥1 and maximum degree Δ. Then

$$\rho(G)=\sqrt{2e-\delta(n-1)+(\delta-1)\Delta}$$

with equality if and only if G is regular, a star plus copies of K_2, or a complete graph plus a regular graph with smaller degree of vertices.

2 Main Results

Using Lemma 1.1, Li [4] obtained sufficient conditions which are based on the spectral radius for some Hamiltonian properties of graphs. In this note, we use some of the ideas in [4] to obtain spectral conditions for a connected graph to contain some subgraphs.

Lemma 2.1. Let G be a connected graph of order *n* and size *e*. Assume $k \geq 2$ is an integer. If

$$\rho > \sqrt{\left(1-\frac{1}{k}\right)n^2-\delta(n-1)+(\delta-1)\Delta}$$,then G contains k_{k+1}

Proof: Let G be a connected graph satisfying the conditions in Lemma 2.1. Turan´ [5

Graph G doesn't contain k_{k+1} then $e \leq \left(1-\frac{1}{k}\right)\frac{n^2}{2}$.

Assume that G does not contain K_{k+1}. Then, by Lemma 1.1, we have that

$$\rho \leq \sqrt{2e-\delta(n-1)+(\delta-1)\Delta} \leq \sqrt{\left(1-\frac{1}{k}\right)n^2-\delta(n-1)+(\delta-1)\Delta}$$

This is a contradiction. This completes the proof.

Let k=2 in Lemma 2.1. Then we have following result.

Result 2.2. Let G be a connected graph of order *n* and size e. If $\rho > \sqrt{\frac{n^2}{2}-\delta(n-1)+(\delta-1)\Delta}$

Then G contains a triangle.

Let H = $K_{r;\,r}$, where $r \geq 2$. Then, for any $\varepsilon > 0$

$$r \equiv \rho(H) > r - c \equiv \sqrt{\frac{(n(H))^2}{2}-\delta(H)(n(H)-1)+(\delta(H$$

And H does not contain a triangle. Thus Result 2.2 is best possible.

Lemma 2.3. Let G be a connected graph of order n and size e. Assume G is not bipartite.

If $$\rho > \sqrt{\frac{(n-1)^2}{2}+2-\delta(n-1)+(\delta-1)\Delta}$$

Then G contains a triangle.

$$\rho > \sqrt{\frac{(n-1)^2}{2}+2-\delta(n-1)+(\delta-1)\Delta}$$

Then G contains a triangle.

Proof: Let G be a connected graph satisfying the conditions in Lemma 2.

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Page 111 in [2], we have that if a non-bipartite graph G is without a triangle then
Assume that the non-bipartite graph G does not contain a triangle. Then, by Lemma 1.1, we have

$$\rho \le \sqrt{2e-\delta(n-1)+(\delta-1)\Delta} \le \sqrt{\left(1-\frac{1}{k}\right)n^2-\delta(n-1)+(\delta-1)\Delta}$$

Which is a contradiction. This completes the proof.

Lemma 2.4. Let G be a connected graph of order n and size e. If

$$\rho > \sqrt{\frac{n}{2}\left(1+\sqrt{4n-3}\right)-\delta(n-1)+(\delta-1)\Delta},$$ contains C_4

Proof: Let G be a connected graph satisfying the given Conditions. Reiman [5] proved that if a ______

Graph G does not contain C_4, then

$e \le \frac{n}{4}\left(1+\sqrt{4n-3}\right)$. Assume that G does not contain

C_4,Then by **Lemma 1.1, we have that**

$$\rho \le \sqrt{2e-\delta(n-1)+\delta(n-1)+(\delta-1)\Delta} \le \sqrt{\frac{n}{2}\left(1+\sqrt{4n-3}\right)-\delta(n-1)+(\delta-1)\Delta}$$

This is a contradiction. This completes the proof.

Lemma 2.5. Let G be a connected graph of order n and size e. If

$$\rho > \sqrt{n\sqrt{(r-1)n}+\frac{n}{2}-\delta(n-1)+(\delta-1)\Delta}$$

Then G contains $K_{2,r}(r \ge 2)$

Proof: Let G be a connected graph satisfying the given conditions. By Exercise 7:3:4(b) on Page 111 in [2], we have that if a graph G does not contain $K_{2,r}(r \ge 2)$ then $e \le \frac{n\sqrt{(r-1)n}}{2}+\frac{n}{4}$.

Assume that G does not contain $K_{2;r}$. Then, by Lemma 1.1, we have

$$\rho \le \sqrt{2e-\delta(n-1)+\delta(n-1)+(\delta-1)\Delta} \le \sqrt{n\sqrt{(r-1)n}+\frac{n}{2}-\delta(n-1)+(\delta-1)\Delta}$$

This is a contradiction. This completes the proof.

Lemma 2.6. Let G be a connected graph of order n and size e. assume c satisfies $3 \le c \le n$. If

$$\rho \ge \sqrt{(n-1)(c-1-\delta)+(\delta-1)\Delta+2}$$ Then the circumference of G ≥ c.

Proof: Let G be a connected graph satisfying the given conditions. By Lemma 4:9 on Page 137 in [1], we have that if the circumference of a graph G < c then
Assume that the circumference of G is less than c. Then, by Lemma 1.1, we have

$$\rho \le \sqrt{2e-\delta(n-1)+(\delta-1)\Delta} \le \sqrt{(n-1)(c-1-\delta)+(\delta-1)\Delta}$$

Which is a contradiction. This completes the proof.

Result 2.8. Let G be a connected graph of order n and size e. Assume G is Hamiltonian. If

$$\rho > \sqrt{\frac{n^2}{2}-\delta(n-1)+(\delta-1)\Delta}$$

Then G contains C_r for each r with $3 \le r \le n$

Let H = $K_{r;r}$, where $r \ge 2$. Then H is Hamiltonian and, for any $\varepsilon > 0$,

$$r=\rho(H) > r-\varepsilon = \sqrt{\frac{(n(H))^2}{2}-\delta(H)(n(H)-1)+(\delta(H)-1)\Delta(H)-\varepsilon},$$

and H does not contain C_s when s is odd such that 3 s n(H).
Thus Result 2.8 is possible.

REFERENCES

[1] Extremal Graph Theory by **B. Bollobas**, Academic Press Inc., London, 1978.

[2] Graph Theory with Applications by **J. A. Bondy and U. S. R. Murty,** Macmillan, London and Elsevier, New York, 1976.

[3] Bounds on eigenvalues and chromatic numbers by **D. Cao**, Linear Algebra Appl., 270(1998), 1 - 15.

[4] Spectral radius and some Hamiltonian properties of graphs by **R. Li**, Manuscript, Sept. 2014.

[5] An extremal problem in graph theory by **P. Turan**, Math. Fiz. Lapok, 48(1941), 436 - 454.

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