Markovian Inventory Policy with Application to Petrol Stations

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***Abstract*— Optimizing replenishment decisions under demand uncertainty are major challenges confronted by most petrol stations. In this paper, a new mathematical model is developed to optimize inventory replenishment policies of kerosene product at selected petrol stations under Markovian demand. In the given model, the distribution of demand over successive periods is dependent on a Markov chain that represents possible states of demand for kerosene product. The objective is to determine in each period of the planning horizon an optimal inventory replenishment policy so that the total inventory costs are minimized for a given state of demand. Using weekly equal intervals, the decisions of when to replenish additional units to stock are made using dynamic programming over a finite period planning horizon. A numerical example demonstrates the existence of an optimal state dependent inventory replenishment policy as well as the corresponding total inventory costs of kerosene on three instances of real-life test problem**

*Key*words: Demand, Inventory policy, Markovian, Petrol stations

1. INTRODUCTION

Market turbulence forces petrol stations to ensure success in planning optimal replenishment decisions of inventory items under demand uncertainty. In practice, optimizing economic order quantity poses a great challenge for inventory management when the demand for items follows a stochastic trend. In order to cope with current turbulent market trends of petroleum products; optimal guidelines for replenishment policies at petrol stations are paramount. This improves both customer retention and goodwill in business transactions. To achieve this goal, two major problems are usually encountered: (i) Determining the most desirable period during which to replenish additional fuel and (ii) determining the optimal inventory replenishment policy given a periodic review inventory system when demand is uncertain.

Oded B and Larson C [1] examined deliveries in an inventory/routing problem using stochastic dynamic programming. In this paper, the amount of needed product at each customer is a known random process, typically a Wiener process. The objective here is to adjust dynamically the amount of product provided on scene to each customer so as to minimize total expected costs.

Cornillier F,Boctor F,Laporte G and Renand J[2] developed an exact algorithm for the petrol station replenishment problem. The algorithm decomposes the problem into a truck loading and routing problem. The authors determine quantities to deliver within a given interval of allocating products to tank truck compartments and of designing delivery routes to stations. In related work by Cornillier F,Boctor F,Laporte G and Renand J([3] , a heuristic for the multi-period petrol station replenishment problem was developed. In his article, the objective is to maximize the total profit equal to the revenue minus the sum of routing costs and of regular and overtime costs. Procedures are provided for the route construction, truck loading and route packing enabling anticipation or the postponement of deliveries. Cornillier F,Boctor F,Laporte G and and Renand J[4] extended the problem by analyzing the petrol station replenishment problem with time windows. In this article, the aim is to optimize the delivery of several petroleum products to a set of petrol stations using limited heterogeneous fleet of trucks by assigning products to truck compartments, delivery routes and schedules.

In related work by Popovic D,Bjelic N and Radivojevic G[5], a simulation approach is presented to analyze the deterministic inventory routing problem solution of the stochastic fuel delivery problem. The method is extended to stochastic problems with planning periods of different lengths by analyzing different process performances. Solutions based on deterministic consumption can be applied to stochastic case by balancing emergency deliveries and safety stocks. Triki C[6] in addition suggested solution methods for the petrol station replenishment problem over T-day planning horizon. Four heuristic methods are described for its solution, tested and compared on two instances of real-life test problem. Computational results show encouraging improvements with respect to human planning solution. The literature cited provide profound insights that are crucial in analyzing the petrol station replenishment problem within the context of transportation and logistics framework. However, a new dynamic approach is sought for petrol stations that relate demand uncertainty with customers and inventory positions as a strategy to optimize economic order quantity and inventory costs in a multi-stage decision setting.

In this paper, an inventory model is considered whose goal is to optimize economic order quantity and total inventory costs associated with holding inventory. At the beginning of each period, a major decision has to be made, namely whether to replenish additional kerosene or not to replenish and keep kerosene at the current inventory position in order to sustain demand. The paper is organized as follows. After describing the mathematical model in §2, consideration is given to the process of estimating model parameters. The model is solved in §3 and applied to a special case study in §4.Some final remarks lastly follow in §5.

2. MODEL FORMULATION

2.1 Notation and Assumptions

i,j	=	States of demand
F	=	Favorable state

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U = Unfavorable state
P = Petrol station
n,N = Stages
D^Z = Demand matrix
Z = Replenishment policy
N^Z = Customer matrix
I^Z = Inventory matrix
D^Z_{ij} = Quantity demanded
N^Z_{ij} = Observed customers
I^Z_{ij} = Quantity in inventory
Q^Z = Demand transition matrix
Q^Z_{ij} = Demand transition probability
C^Z = Inventory cost matrix
C^Z_{ij} = Total Inventory costs
e^Z_i = Expected future inventory costs
a^Z_i = Accumulated inventory costs
c_r = Unit replenishment costs
c_g = Unit storage costs
c_s = Unit shortage costs
i,j ε{ F,U} P ε {1,2,3} Z ε {0,1}
n= 1,2,3……………….N

We consider a designated number of petrol stations whose demand for kerosene during each time period over a fixed planning horizon is classified as either *Favorable*(denoted by state F) or *Unfavorable*(denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining the optimal course of action, namely to replenish additional stock (a decision denoted by Z=1) or not to replenish additional stock (a decision denoted by Z=0) during each time period over the planning horizon, where Z is a binary decision variable. Optimality is defined such that the lowest expected total inventory costs are accumulated at the end of N consecutive periods spanning the planning horizon under consideration. In this case, three petrol stations (P=3) are considered over a two-period (N=2) planning horizon.

2.2 Finite Dynamic Programming Problem Formulation

Recalling that demand can either be in state F or in state U, the problem of finding an optimal replenishment policy may be expressed as a finite period dynamic programming model. Let $C_n(i,P)$ denote the optimal expected total inventory costs accumulated at petrol station P during periods *n,n+1,……..,N* given the state of the system at the beginning of period *n* is i ε { F,U}. The recursive equation relation C_c and C_{n+1} is

$$C_n(i,P) = min_Z[Q^Z_{iF}(P)C^Z_{iF}(P)+C_{n+1}(F,P), Q^Z_{iU}(P)C^Z_{iU}(P)+C_{n+1}(U,P)] \quad (1)$$

i ε { F,U} n= 1,2,3……………….N , P ε { 1,2,3}
together with the conditions

$$C_{N+1}(F,P) = C_{N+1}(U,P) = 0$$

The recursive relationship may be justified by noting that the cumulative total inventory costs $C^Z_{ij}(P) + C_{N+1}(j)$ resulting from reaching state j ε { F,U} at the start of period n+1 from state i ε { F,U} at the start of period *n* occurs with probability $Q^Z_{ij}(P)$. Hence, the dynamic programming recursive equations

$$C_n(i,P) = min_Z[e^Z_i(P) + Q^Z_{iF}(P)C_{n+1}(F,P) + Q^Z_{iU}(P)C_{n+1}(U,P)] \quad (2)$$

$$C_N(i,P) = min_Z[e^Z_i(P)] \quad (3)$$

result where (3) represents the Markov chain stable state.

2.2.1 Computing $Q^Z(P)$ and $C^Z(P)$

The demand transition probability from state i ε { F,U} to state j ε { F,U} given replenishment policy Z ε { 0,1} may be taken as the number of customers observed at petrol station P with demand initially in state i and later with demand changing to state j ,divided by the sum of customers over all states..

$$Q^Z_{ij}(P) = N^Z_{ij}(P)/[N^Z_{iF}(P) + N^Z_{iU}(P)] \quad (4)$$

i,j ε { F,U} P ε {1,2,3} Z ε {0,1}
n= 1,2,3……………….N

When demand outweighs on-hand inventory, the inventory cost matrix $C^Z(P)$ may be computed by means of the relation

$$C^Z(P) = (c_r + c_g + c_s)[D^Z(P) - I^Z(P)]$$

Therefore

$$C^Z_{ij}(P) = \begin{cases} (c_r + c_g + c_s)[D^Z_{ij}(P) - I^Z_{ij}(P] & if \ D^Z_{ij}(P) > i^Z_{ij}(P) \\ c_g[I^Z_{ij}(P) - D^Z_{ij}(P] & if \ D^Z_{ij}(P) \leq I^Z_{ij}(P) \end{cases} \quad (5)$$

for all i,j ε { F,U} P ε {1,2,3} Z ε {0,1}
n= 1,2,3……………….N

A justification for expression (5) is that $D^Z_{ij}(P) – I^Z_{ij}(P)$ units must be stocked in order to meet the excess demand. Otherwise additional units to stock are cancelled when demand is less than or equal to on-hand inventory.
The following conditions must however, hold:

1. Z=1 when $c_r > 0$ and Z=0 when $c_r =0$,
2. $c_s > 0$ when shortages are allowed, and $c_s = 0$ when shortages are not allowed.

3. OPTIMIZATION

The optimal replenishment policy and inventory costs are found in this section for each period separately.

3.1 Optimization during period 1

When demand is favorable (ie. in state F), the optimal replenishment policy and associated inventory costs during period 1 are

$$Z = \begin{cases} 1 & if \ e^1_F(P) < e^0_F(P) \\ 0 & if \ e^1_F(P) \geq e^0_F(P) \end{cases}$$

and

$$C_1(F,P) = \begin{cases} e^1_F(P) & if \ Z = 1 \\ e^0_F(P) & if \ Z = 0 \end{cases}$$

respectively.

Similarly, when demand is Unfavorable (ie. in state U), the optimal replenishment policy and associated inventory costs during period 1 are

$$Z = \begin{cases} 1 & if \ e^1_U(P) < e^0_U(P) \\ 0 & if \ e^1_U(P) \geq e^0_U(P) \end{cases}$$

and

$$C_1(U,P) = \begin{cases} e^1_U(P) & if \ Z = 1 \\ e^0_U(P) & if \ Z = 0 \end{cases}$$

3.2 Optimization during period 2

Using the dynamic programming recursive equation (1) and recalling that $a^Z_i(P)$ denotes the already accumulated total inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a^Z_i(P,n) = e^Z_i(P)+Q^Z_{iF}(P)min(e^1_F(P), e^0_F(P))+Q^Z_{iU}(P)min(e^1_U(P), e^0_U(P))$$

$$= e^Z_i(P) + Q^Z_{iF}(P)C_2(F,P) + Q^Z_{iU}(P)C_2(U,P)$$

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Hence, using results of $a^Z_i(P,n)$ above, when demand is favorable(i.e in state F),the optimal replenishment policy during period 2 is

$$Z = \begin{cases} 1 & if \;\; a_F^1(P,2) < a_F^0(P,2) \\ 0 & if \;\; a_F^1(P,2) \geq a_F^0(P,2) \end{cases}$$

while the associated total inventory costs are then

$$C_2(F,P) = \begin{cases} a_F^1(P,2) & if \;\; Z = 1 \\ a_F^0(P,2) & if \;\; Z = 0 \end{cases}$$

Similarly, when demand is Unfavorable (ie. in state U), the optimal replenishment policy during period 2 is

$$Z = \begin{cases} 1 & if \;\; a_U^1(P,2) < a_U^0(P,2) \\ 0 & if \;\; a_U^1(P,2) \geq a_U^0(P,2) \end{cases}$$

In this case, the associated total inventory costs are

$$C_2(U,P) = \begin{cases} a_U^1(P,2) & if \;\; Z = 1 \\ a_U^0(P,2) & if \;\; Z = 0 \end{cases}$$

respectively

4. CASE STUDY

In order to demonstrate the use of the model in §2-3, real case applications from three petrol stations: *Fuelex, Petro City* and *Hassel Petroleum* in Uganda are presented in this section. Kerosene is among the petroleum products sold and the demand for kerosene fluctuates every week. The petrol stations want to avoid excess inventory when demand is Unfavorable(state U) or running out of stock when demand is Favorable(state F) and hence seek decision support in terms of an optimal replenishment policy and the associated inventory costs of kerosene over a two-week period is required at Fuelex, Petrol City and Hassel Petroleum.

4.1 Data Collection

The unit costs(in UGX), number of customers, demand and inventory levels (in thousand liters) of kerosene at the three petrol stations are presented in Tables 1 and 2 below:

Table I
Unit costs of Kerosene product

Petrol Station (P)	Unit Replenishment Costs in UGX (c_r)	Unit storage Costs in UGX (c_g)	Unit shortage Costs in UGX (c_s)
Fuelex	4500	1200	300
Petrol City	4800	900	300
Hassel Petroleum	5100	600	300

Table II
Customers, demand and inventory levels of kerosene given state-transitions and replenishment policies over twelve weeks

State Transition (i,j)	**Petrol Station (P)**	**Replenishment Policy (Z)**	**Customers $N^Z_{ij}(P)$**	**Demand $D^Z_{ij}(P)$**	**Inventory $I^Z_{ij}(P)$**
FF	Fuelex (1)	1	15	1.56	0.95
FU		1	10	1.15	0.93
UF		1	8	1.07	0.93
UU		1	2	0.11	0.94
FF		0	10	1.22	0.44
FU		0	8	0.78	0.45
UF		0	5	0.78	0.47
UU		0	4	0.15	0.46
FF	Petrol City (2)	1	20	0.93	1.45
FU		1	14	0.60	0.45
UF		1	10	0.59	0.79
UU		1	12	0.11	0.80
FF		0	12	0.72	0.81
FU		0	7	0.77	0.79
UF		0	4	0.75	0.80
UU		0	2	0.11	0.79
FF	Hassel Petroleum (3)	1	18	0.82	0.68
FU		1	10	0.93	0.69
UF		1	8	0.84	0.72
UU		1	10	0.09	0.69
FF		0	10	0.51	1.05
FU		0	7	0.70	1.00
UF		0	4	0.72	1.00
UU		0	2	0.10	0.25

Hence, past data revealed the following demand pattern and inventory levels of kerosene when demand was favorable (F) or Unfavorable (U).If additional kerosene is ordered during week 1, the customer matrix, the demand matrix and inventory matrix are given by

$$D^1(1) = \begin{bmatrix} 1.56 & 1.15 \\ 1.07 & 0.11 \end{bmatrix} \qquad N^1(1) = \begin{bmatrix} 15 & 10 \\ 8 & 2 \end{bmatrix}$$

$$I^1(1) = \begin{bmatrix} 0.95 & 0.93 \\ 0.93 & 0.94 \end{bmatrix} \qquad N^1(2) = \begin{bmatrix} 20 & 14 \\ 10 & 12 \end{bmatrix}$$

$$D^1(2) = \begin{bmatrix} 0.93 & 0.60 \\ 0.59 & 0.11 \end{bmatrix} \qquad N^1(3) = \begin{bmatrix} 18 & 10 \\ 8 & 10 \end{bmatrix}$$

$$I^1(2) = \begin{bmatrix} 1.45 & 0.45 \\ 0.79 & 0.80 \end{bmatrix} \qquad I^1(3) = \begin{bmatrix} 0.68 & 0.69 \\ 0.72 & 0.69 \end{bmatrix}$$

$$D^1(3) = \begin{bmatrix} 0.82 & 0.93 \\ 0.84 & 0.09 \end{bmatrix}$$

while if additional kerosene is not ordered during week 1, the matrices are

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$$N^0(1) = \begin{bmatrix} 10 & 8 \\ 5 & 4 \end{bmatrix} \qquad D^0(1) = \begin{bmatrix} 1.22 & 0.78 \\ 0.78 & 0.15 \end{bmatrix}$$

$$I^0(1) = \begin{bmatrix} 0.44 & 0.45 \\ 0.47 & 0.46 \end{bmatrix} \qquad D^0(2) = \begin{bmatrix} 0.72 & 0.77 \\ 0.75 & 0.11 \end{bmatrix}$$

$$N^0(2) = \begin{bmatrix} 12 & 7 \\ 4 & 2 \end{bmatrix} \qquad I^0(2) = \begin{bmatrix} 0.81 & 0.79 \\ 0.80 & 0.89 \end{bmatrix}$$

$$N^0(3) = \begin{bmatrix} 10 & 7 \\ 4 & 2 \end{bmatrix} \qquad D^0(3) = \begin{bmatrix} 0.51 & 0.70 \\ 0.72 & 0.10 \end{bmatrix}$$

$$I^0(3) = \begin{bmatrix} 1.05 & 1.00 \\ 1.00 & 0.25 \end{bmatrix}$$

4.2 Computation of Model Parameters

Using (4) and (5), the state-transition matrices and inventory cost matrices at each respective petrol station are

$$Q^1(1) = \begin{bmatrix} 0.600 & 0.400 \\ 0.800 & 0.200 \end{bmatrix} \quad Q^1(2) = \begin{bmatrix} 0.588 & 0.412 \\ 0.455 & 0.545 \end{bmatrix}$$

$$Q^1(3) = \begin{bmatrix} 0.643 & 0.357 \\ 0.455 & 0.545 \end{bmatrix} \quad C^1(1) = \begin{bmatrix} 3.66 & 1.32 \\ 0.84 & 0.25 \end{bmatrix}$$

$$C^1(2) = \begin{bmatrix} 0.156 & 0.255 \\ 0.060 & 0.207 \end{bmatrix} \quad C^1(3) = \begin{bmatrix} 0.896 & 1.920 \\ 0.768 & 0.180 \end{bmatrix}$$

for the case when additional kerosene was replenished (Z=1) while these matrices are given by

$$Q^0(1) = \begin{bmatrix} 0.556 & 0.444 \\ 0.556 & 0.444 \end{bmatrix} \qquad Q^0(2) = \begin{bmatrix} 0.632 & 0.368 \\ 0.667 & 0.333 \end{bmatrix}$$

$$Q^0(3) = \begin{bmatrix} 0.588 & 0.412 \\ 0.667 & 0.333 \end{bmatrix} \qquad C^0(1) = \begin{bmatrix} 4.74 & 1.98 \\ 1.86 & 0.09 \end{bmatrix}$$

$$C^0(2) = \begin{bmatrix} 0.27 & 0.006 \\ 0.015 & 0.204 \end{bmatrix} \qquad C^0(3) = \begin{bmatrix} 0.162 & 0.090 \\ 0.084 & 0.300 \end{bmatrix}$$

for the case when additional kerosene is *not* replenished(Z=0) during week 1. When additional kerosene is replenished (Z=1), the state-transition matrices and inventory cost matrices above yield the following future expected inventory costs (in million UGX).

$$e_F^1(1) = (0.600)(3.66) + (0.400)(1.320) = 2.724$$
$$e_U^1(1) = (0.800)(0.840) + (0.200)(0.25) = 0.722$$
$$e_F^1(2) = (0.588)(0.156) + (0.412)(0.255) = 0.197$$
$$e_U^1(2) = (0.455)(0.06) + (0.545)(0.207) = 0.140$$
$$e_F^1(3) = (0.643)(0.896) + (0.357)(1.920) = 1.262$$
$$e_U^1(3) = (0.444)(0.768) + (0.556)(0.180) = 0.449$$

However, when additional kerosene is *not* replenished (Z=0), the state-transition matrices and inventory cost matrices above yield the following future expected inventory costs (in million UGX).

$$e_F^0(1) = (0.556)(4.74) + (0.444)(1.98) = 3.515$$
$$e_U^0(1) = (0.556)(1.86) + (0.444)(0.09) = 1.074$$
$$e_F^0(2) = (0.632)(0.27) + (0.368)(0.006) = 0.173$$
$$e_U^0(2) = (0.667)(0.015) + (0.333)(0.209) = 0.080$$
$$e_F^0(3) = (0.588)(0.162) + (0.412)(0.09) = 0.132$$
$$e_U^0(3) = (0.667)(0.084) + (0.333)(0.300) = 0.156$$

When additional kerosene was replenished (Z=1), the accumulated inventory costs (in million UGX) for weeks 1 and 2 can be computed as follows:

$$a_F^1(1,2) = 2.724 + (0.600)(2.724) + (0.400)(0.722) = 4.647$$
$$a_U^1(1,2) = 0.722 + (0.800)(2.724) + (0.200)(0.722) = 3.046$$
$$a_F^1(2,2) = 0.197 + (0.588)(0.173) + (0.412)(0.080) = 0.332$$

$$a_U^1(2,2) = 0.140 + (0.455(0.173 + (0.545)(0.080) = 0.262$$
$$a_F^1(3,2) = 1.262 + (0.647)(0.132) + (0.353)(0.156) = 1.402$$
$$a_U^1(3,2) = 0.449 + (0.444)(0.132) + (0.556)(0.156) = 0.594$$

Similarly, the accumulated inventory costs (in million UGX) for weeks 1 and 2 are computed below when additional kerosene is *not* replenished (Z=0).

$$a_F^0(1,2) = 3.515 + (0.556)(2.724) + (0.444)(0.722) = 5.350$$
$$a_U^0(1,2) = 1.074 + (0.556)(2.724) + (0.444)(0.722) = 2.909$$
$$a_F^0(2,2) = 0.173 + (0.632)(0.173) + (0.368)(0.078) = 0.311$$
$$a_U^0(2,2) = 0.080 + (0.667)(0.173) + (0.333)(0.078) = 0.221$$
$$a_F^0(3,2) = 0.132 + (0.588(0.132) + (0.412)(0.180) = 0.283$$
$$a_U^0(3,2) = 0.156 + (0.667)(0.132) + (0.333)(0.156) = 0.296$$

4.3 The Optimal Replenishment Policy and Costs

Week 1

Fuelex : Since 2.724 < 3.515,it follows that Z=1 is an optimal replenishment policy for week 1 with associated total inventory costs of 2.724 million UGX when demand is Favorable. Since 0.722 < 1.074, it follows that Z=1 is an optimal replenishment policy for week 1 with associated total inventory costs of 0.722 million UGX when demand is Unfavorable.

Petrol City: Since 0.173 < 0.197, it follows that Z=0 is an optimal replenishment policy for week 1 with associated total inventory costs of 0.173 million UGX when demand is Favorable. Since 0.080< 0.140, it follows that Z=0 is an optimal replenishment policy for week 1 with associated total inventory costs of 0.080 million UGX when demand is Unfavorable.

Hassel Petroleum: Since 0.132<1.257, it follows that Z=0 is an optimal replenishment policy for week 1 with associated total inventory costs of 0.132 million UGX when demand is Favorable. Since 0.156 < 0.449, it follows that Z=0 is an optimal replenishment policy for week 1 with associated total inventory costs of 0.156 million UGX when demand is Unfavorable.

Week 2

Fuelex : Since 4.647 5.350,it follows that Z=1 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 4.647million UGX when demand is Favorable. Since 2.909 < 3.046, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 2.909 million UGX when demand is Unfavorable.

Petrol City: Since 0.311 < 0.332, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 0.311 million UGX when demand is Favorable. Since 0.221 < 0.262, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 0.221 million UGX when demand is Unfavorable.

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Hassel Petroleum: Since 0.283 < 1.402, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 0.283 million UGX when demand is Favorable. Since 0.296 < 0.594, it follows that Z=0 is an optimal replenishment policy for week 2 with associated accumulated inventory costs of 0.296 million UGX when demand is Unfavorable.

5. CONCLUSION

The paper provides some interesting results as a cost minimization strategy for the petrol station replenishment problem using Markov decision processes. It would however be worthwhile to extend the research and examine the petrol station replenishment problem under non stationary demand conditions. In the same spirit, the model developed raises a number of salient issues to consider: Lead time of kerosene during the replenishment cycle, vehicle routings for deliveries and customer response to abrupt changes in price of kerosene at each individual petrol station. Finally, special interest is sought in further extending the model by considering inventory replenishment policies in the context of Continuous Time Markov Chains (CTMC).

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