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Tuning of Two Degree of Freedom PID Controller for Second Order Processes

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***Abstract*— In the classical control of PID various versions and variations of PID are exercised. For instance, series and parallel and other are according to degree of freedom for the given process. For a process two main objective of control are set point tracking and load disturbance rejection. A two Degree of Freedom controller has two separate controller units each for set-point tracking and disturbance rejection. However the use of two Degree of Freedom controllers introduces additional parameters that need to be tuned appropriately. Various paper describe tuning rule of two Degree of Freedom PID controller for the processes with time delay. This design approaches for two Degree of Freedom (2-DOF) PID controllers for second order processes without time delay that gives smooth control is presented in this paper. Two Degree of Freedom PID controller gives better result compare to PID controller for second order processes without time delay.**

***Index Terms*— 2-DOF, PID, tracking, regulatory control, servo control.**

I. INTRODUCTION

A primary objective of a control theory is to make the output of a dynamic process behave in a certain manner. Control theory is an interdisciplinary branch of engineering and mathematics that deals with the behavior of dynamical systems [1]. The desired output of a system is called the reference. In the field of control system, various control strategies and methods are implemented, devised and experienced in the process control and other control applications. In control methodology, mostly prefers controllers are proportional, Integral and derivative in parallel and/ or series combinations so far. PID controllers are used for its simplicity and better performance in majority of cases. Tuning of controllers is the main task for better performance of the system. Speaking on control techniques, there are various techniques like hierarchical control, robust control (H-infinity loop shaping, sliding mode control), adaptive control, optimal control (LQG and MPC etc.), intelligent control (fuzzy logic, Genetic algorithms etc.), adaptive control (direct and indirect), etc.

The degree of freedom of a control system is defined as the number of closed-loop transfer functions that can be adjusted independently [1]. Two-degree-of-freedom (abbreviated as 2-DOF) controller have advantages over one-degree-of-freedom (abbreviated as 1-DOF) controller. In the most of industrial process control application, the desired value of the controlled variable, or set point, normally remains constant and a good load disturbance rejection is required [2], which is usually known as regulatory control. However due to variations in process operating conditions, the control variable set point may eventually need to be changed and then a good transient response to such change is required, which is known as servo control operation. Specific criteria for command tracking include rise time and settling time. Satisfying these two operating conditions simultaneously is difficult by using one-degree-of-freedom (1-DOF) PI/PID Controller, but using two-degree-of-freedom (2-DOF) PI/PID allows tuning of controller in order to do so [3]. The extra parameter it provides is used to improve its servo control behavior. This second degree of freedom is aimed at providing additional flexibility to the control system design [4].

In [5], various 2-DOF PID controllers were proposed for industrial use and detailed analyses were made including equivalent transformations, interrelationship with previously proposed “advanced type” PID (i.e., the preceded-derivative PID and the I-PD) controllers, explanations of the effects of the 2-DOF structure, and a list of optimal parameters.

In article [1,3,6] describe various tuning methods for two-degree-of-freedom PID controller for the processes with time delay or slow processes or SOPTD system. But there are no methods to provide to tune the two-degree-of-freedom PID controller for second order processes without time delay. This paper gives simple tuning method for two-degree-of-freedom PID controller for second order processes.

II. CONTROL STRATEGIES

A. PID controller

In the field of control engineering ultimate aim of any controller is reduction of error in majority of the cases. Generally control engineer prefers PID controller for their application due to its simplicity and better performance in majority of cases. Tuning of PID controller is the main task for better performance.

The PID control scheme is named after its three correcting terms, whose sum constitutes the manipulated variable (MV). The output of the PID controller is calculated by summation

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of the proportional, integral, and derivative terms [5][6], as shown PID structure in figure 1.

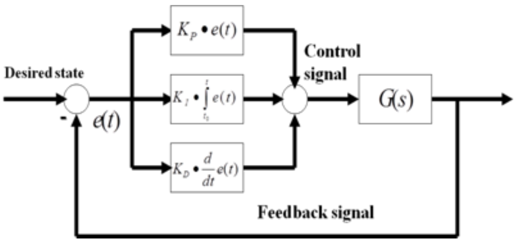

Figure-1, PID structure

Defining u(t) as the controller output, the final form of the PID algorithm is;

$$u(t) = K_P \cdot e(t) + K_I \cdot \int e(t)dt + K_D \cdot \frac{d}{dt}e(t) \qquad (1)$$

Where, K_P = Proportional Gain, K_I = Integral Gain, K_D = Derivative Gain, $e(t)$ = Error Signal, $u(t)$ = control effort.

B. 2-DOF PID controller

The design of control systems is a multi-objective problem, so a two degree-of-freedom (abbreviated as 2DOF) control system naturally has advantages over a one degree-of freedom (abbreviated as 1DOF) control system [5][7].

The process will be controlled with a two-degree-of-freedom proportional integral derivative (PID) controller whose output is expressed as in equation (1).

A general form of the 2-DOF PID controller is shown in Figure 2, where the controller consists of two compensators $G_{ff}(s)$ and $G_c(s)$, [8] which are known as

1. Set point controller transfer function also known as feed-forward compensator which is $G_{ff}(s)$ and given by

$$G_{ff}(s) = K\left(\beta + \frac{1}{sT_i}\right) \qquad (2)$$

2. Feedback transfer function (feedback compensator) which is $G_c(s)$ and given by

$$G_c(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right) \qquad (3)$$

Where, β is set point weighting factor or controller parameter $(0 \le \beta \le 1)$ and K, T_i, T_d are PID controller parameter that is Proportional Gain K_p, integral time T_i and derivative time T_d respectively. The block diagram of this controller is shown in figure 2.

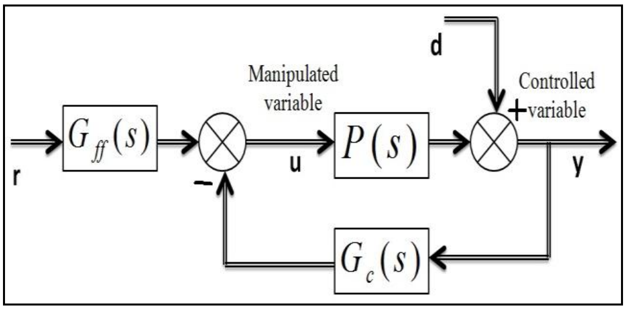

Figure-2, 2-DOF PID controller structure

Where, P(s) = Plant transfer function, u= Manipulated variable, y= controlled variable or output, r= set point.

Manipulated variable (u) for continuous controller is given as

$$u(t) = K_p\left[\beta r(t) - y(t) + \frac{1}{sT_i}(r(t) - y(t)) - sT_d y(t)\right] \quad (4)$$

For the purpose of analysis only, the controller output (4) will be rewritten as follows:

$$u(t) = K_p\left(\beta + \frac{1}{T_i s}\right)r(t) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)y(t) \quad (5)$$

$$u(t) = G_{ff}(s)r(t) - G_c(s)y(t) \qquad (6)$$

The closed-loop transfer function from the set-point to the controlled variable is given by

$$\frac{y(s)}{r(s)} = M_{yr}(s) = \frac{G_{ff}(s)P(s)}{1 + G_c(s)P(s)} \qquad (7)$$

Where, the loop sub index has been suppressed for simplicity. In addition the closed-loop transfer function from the load-disturbance to the controlled variable is given by

$$\frac{y(s)}{d(s)} = M_{yd}(s) = \frac{P(s)}{1 + G_c(s)P(s)} \qquad (8)$$

Which are related by

$$M_{yr}(s) = G_{ff}(s)M_{yr}(s) \qquad (9)$$

Many different ways of discretize the continuous controller of equation 4. However here forward difference approximation is used for integral mode and backward difference approximation is used for derivative mode [10, 11, 12]. So after discretizing, final discrete 2-DOF controller equation is

$$u(n) = \frac{T_c(z)}{R_c(z)}r(n) - \frac{S_c(z)}{R_c(z)}y(n) \qquad (10)$$

Where, $\frac{T_c(z)}{R_c(z)}$ is feed-forward compensator and $\frac{S_c(z)}{R_c(z)}$ is feedback compensator, and $R_c(z), S_c(z), T_c(z)$ are given by following polynomials.

$$
\begin{aligned}
R_c(z) &= [1 - z^{-1}] \\
T_c(z) &= [\beta + (b_i - \beta)z^{-1}] \qquad (11) \\
S_c(z) &= [(1 + b_d) + (b_i - 1 - 2b_d)z^{-1} + b_d z^{-2}]
\end{aligned}
$$

Where $b_i = (T_s / T_i)$ and $b_d = (T_d / T_s)$, T_s is sampling period.

III. CONTROLLER DESIGN

The second order processes without time delay are represented by a linear model in the form of following general transfer function:

$$P(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} \qquad (12)$$

Where, K is gain, T_1 and T_2 are time constant.

Usually the design of two-degree-of-freedom PID controller is performed in two stages. First the parameter (K_p, T_i, T_d) of the feedback controller (3) required to obtain the desired regulatory control performance. Second the set-point controller, (2) weighting parameter (β) is used to improve the servo-control performance.

In what follows a different approach is taken to obtained PID controller parameter. The complete set of PID controller parameter (K_p, T_i, T_d) are obtained using Good Gain method [9].

Second to obtained set-point controller (2) weighting parameter (β) is obtained using the equation

$$\beta = \min\left\{ \frac{T_i * T_d}{K_p}, 0.8 \right\} \qquad (13)$$

So, complete set of parameter of two-degree-of-freedom PID controller are obtained by using these two steps.

IV. EXAMPLES

Two simulation examples are provided. The first one exemplifies the application of the presented method. The system descriptions used are, however, of higher order and second order system without time delay are therefore used. The performance of the two-degree-of-freedom PID controller is compared to that one of a PID controller, therefore showing the benefits of using the 2-DOF control configuration.

A. *Example 1*

Consider that second order controlled process without time delay or fast process is given by [13]

$$P_1(s) = \frac{1}{(2s + 1)(0.5s + 1)} \qquad (14)$$

Simplifying above process

$$P_1(s) = \frac{1}{(s^2 + 2.5s + 1)} \qquad (15)$$

For above example or second order process 2-DOF PID controller is design. Controller parameter (K_p, T_i, T_d) are obtained using Good Gain method [9], and these parameter are

$$
\begin{aligned}
K_p &= 3.2, \\
T_i &= 2.55, \qquad (16) \\
T_d &= 0.6375
\end{aligned}
$$

The set point weighting factor (β) is calculated using equation (13) and value of set point weighting factor β, *is* 0.508

Second order transfer function of example 1, is modeled in MATLAB simulation file as shown in Figure 3 for, 2-DOF PID controller and PID controller.

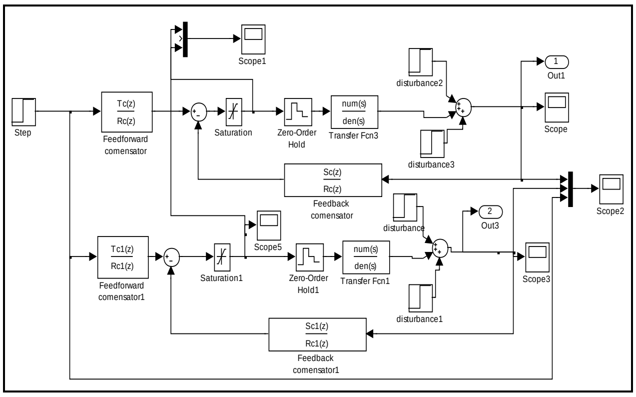

Figure 3, system 1 with PID and 2-DOF PID Controller

Using this controller setting the system response for set point tracking and disturbance rejection under load changes is shown in figure 4.

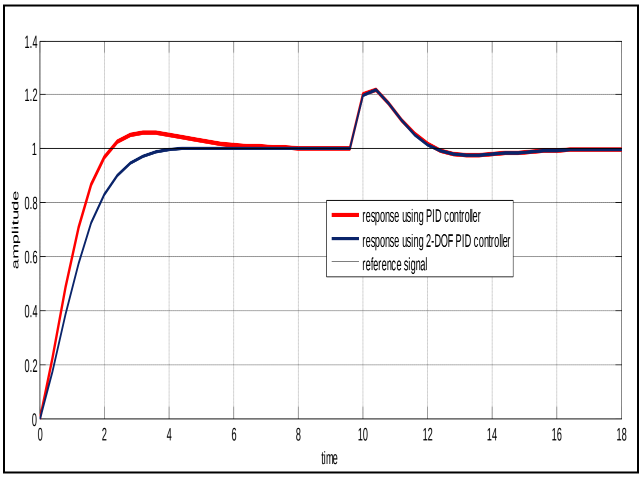

Figure 4, Simulation result for 2-DOF PID and PID Controller for tracking and disturbance rejection

The system response for set-point changes is shown in figure 5,

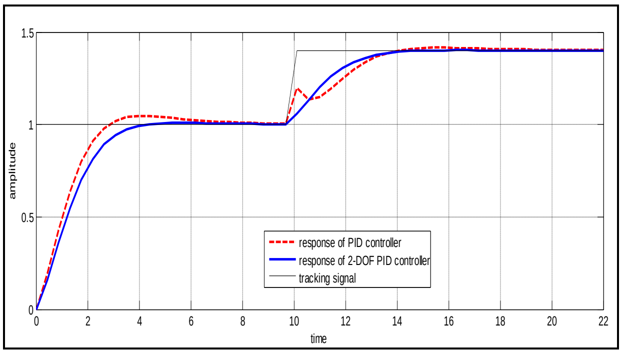

Figure 5, reference tracking under set point changes

B. *Example 2*

Consider another second order controlled process without time delay or fast process that is given by [14]

$$P_2(s) = \frac{1.4582s + 11.65}{(s^2 + 3.43s + 3.557)} \qquad (17)$$

Using Good Gain method the PID parameter are obtained for this process *$P_2(s)$*,

$K_p = 3.2,$

$T_i = 2.55,$

$T_d = 0.6375$

Similarly, the set point weighting factor (β) using (13) is 0.55. Using this controller setting the system response for set point tracking and disturbance rejection under load changes is shown in figure 6, for process $P_2(s)$

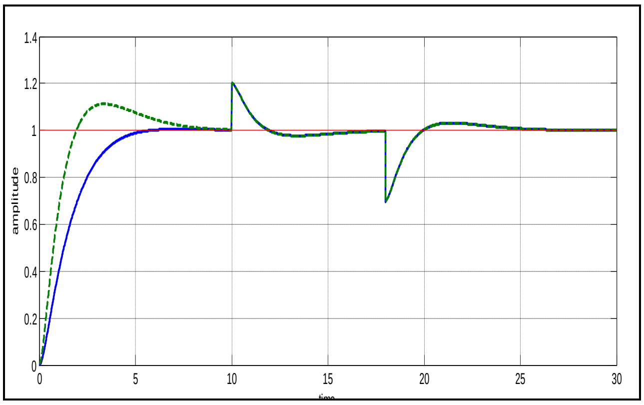

Figure 6, Simulation result for 2-DOF PID and PID Controller for tracking and disturbance rejection

The system response for set-point changes is shown in figure 7,

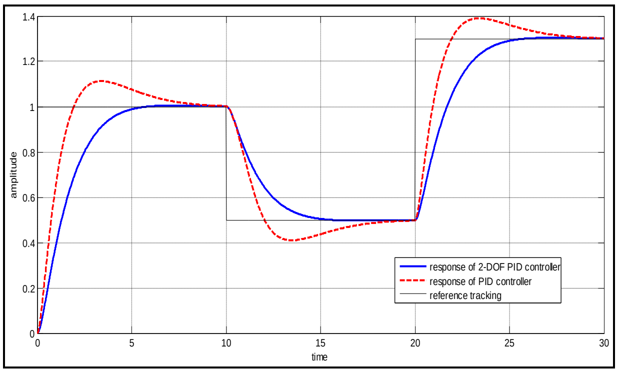

Figure 7, reference tracking under set point changes

V. CONCLUSION

Here a system actually requires a minimum number of independent controllers equal to the degree of freedom of the system or purpose of the control system. The purpose is tracking as well as disturbance rejection giving the requirement of two independent controllers; and the system is having the degree of freedom two. An effort is made to design a control proposal satisfying the two fold purpose of control. As seen from the analysis and plots the new design having a 2-DOF PID control has far reaching results. Clearly the system performs better under two-DOF PID control with a very less percentage overshoot and good load disturbance rejection with a minimum settling time, all of these compare to PID controller. So purposed tuning method is easy to understand and design of 2-DOF PID controller for Second order process of without time delay or for fast process.

REFERENCES

[1] Mituhiko Araki, Hidefumi Taguchi "Two-Degree-of-Freedom PID controller" *International Journal of control, automation and systems* 2003, pp. 401-411.

[2] F. G. shinskey, process control: as taught vs practiced, *industrial and engineering chemical research* 41 (16) (2002).

[3] V.M. Alfao, R. Vilanavo, O. Arrieta, "Simple Robust autotuning of Two-degree-of-freedom (2-DOF) PI controlers", *ISA transactios* 51(2012), pp. 30-41.

[4] M. Araki, H. Taguchi, Two-degree-of-freedom PID controllers- their functions and optimal tuning, in, *IFAC digital control: past, present and future of PID control* 2000.

[5] M. Araki, Two-degree-of-freedom PID control system: part I *systems and control* vol. 29 pp. 649-656, 1985.

[6] K.H. Ang, G. Chong and Y. Li, "PID control system analysis, design and technology," *IEEE transaction on Control System Technology, Vol.13,* No.4, 2005, pp. 559-576

[7] K. J. Astrom and Tore H, "PID controller".

[8] V.M. Alfao, R. Vilanavo, "Model-reference robust tuning of 2DOF PI controllers for 1st and 2nd order plus dead-time controlled processes", *Journal of Process control* 22(2012), pp. 359-374.

[9] F. Haugen, "The Good Gain method for simple experimental tuning of PID controllers", 2012 *Norwegian Society of Automatic Control*

[10] Megha jaiswal, Mohan phadnis "speed control of DC motor using genetic algorithm Based PID controller" 2013 *IJARCSSE* pp. 247-253.

[11] Michal Kocur, Stefan Kozak, BranislavDvorscak, "Design and Implementation of FPGA - Digital Based PID Controller", *IEEE 15th international Carpathian control conference (ICCC),* pp. 233-236, 2014.

[12] Cristina I. Muresan, SilviuFolea, georgeMois, EVA H. Dulf, "Development and implementation of an FPGA based fractional order Controller for a DC motor" *Mechatronics* 23 (2013), pp. 798-804.

[13] V.M. Alfao, R. Vilanavo, O. Arrieta, "Robust tuning of Two-degree-of-freedom (2-DOF) PI/PID based cascade control system", *Journal of Process Control* 19(2009), pp. 1658-1670.

[14] Sandeep kumar, Temperature control of CSTR using PID and PID (2-DOF) Controller, *IJARCSSE* may 2012 pp. 326-329.

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