Hybrid Particle Swarm Algorithm with Levy Mutation

Jyoti Garg[1], Ashutosh Gupta[2]
[1]Department of Electronics and Communication Engineering, ABES Engineering College Ghaziabad-201009, INDIA
[2]Central Electronics Limited, Ghaziabad, INDIA

Abstract*—The standard Particle Swarm Optimization (PSO) studies its own previous best solution and the group's previous best to optimize problems. One problem exists in PSO is its tendency of trapping into local optimum. To avoid this problem, in this paper a new PSO (LHPSO)is presented by combining a Lévy mutation based on Lévy Distribution on the best particle so that the mutated best particle could lead all the rest of particles to the better positions. The idea to introduce Lévy Mutation operator is to increase the probability of a particle escaping from a local optimum. As it is known the variance of Lévy distribution is infinite, so that Lévy mutation could make a particle to have a long jump. LHPSO has been compared with Standard PSO on a set of benchmark functions. The results show that proposed idea is an effective and efficient idea.

***Keywords*- PSO, Lévy Probability Distribution, Mutation, Exploration, Exploitation.**

I. INTRODUCTION

The Particle Swarm Optimization (PSO) was firstly introduced by Kennedy and Eberhart in 1995 [1]. It is a simple evolutionary algorithm which differs from other evolutionary algorithms in which it is motivated from the simulation of social behavior. PSO has shown good performance in finding better solutions to optimize problems and turned out to be another powerful tool besides other evolutionary algorithms such as genetic algorithms [2].Clerc and Kennedy [11] introduced a constriction factor based on dynamic systems theory in order to guarantee the stability of the swarm, which may be necessary to ensure convergence of the PSO algorithm and also some guidelines for choosing the constriction and weighting coefficients. In a similar way, van den Bergh [4] has also presented some results in this direction. However, a deterministic approach might notbe straightforward generalized for the analysis of trajectory of stochastic particles in multi-dimensional search space. Particle Swarm Optimization is a evolutionary algorithm to find optimal regions of search spaces through the interaction ofindividuals in a population of particles [3]. Each particle is assigned a random velocity and repeatedly moved through search space. Every particle is attracted towards a location of best fitness achieved so far by particle itself and by location of best fitness achieved so far across whole population [1]. Theanalysis on simple PSO is presented in [5]. One problem found in the standard PSO is that it could easily fall into local optima in many optimization problems. Recently some research has been done in this direction [6]. One reason for PSO to converge to local optima is that particles in PSO can quickly converge to the best position once the best position has no change in a local optimum. When all particles become similar, there is little hope to find a better position to replace the previous best position found so far. Another reason is the lack of population diversity in PSO algorithms is understood to be a factor in their convergence on local optima [3]. Therefore, the addition of a mutation operator to PSO should enhance its global search capacity and thus improve its performance. There are severalinstances in PSO where mutation is introduced in the swarm.Some mutation operators that have been applied to mutate the position vector in PSO include Gaussian Mutation [8], Cauchy [9], Chaos mutation [10] etc.

This paper presents a new PSO algorithm based on Lévy mutation with the help of Lévy distribution on the best particle so that the mutated best particle could lead all the rest of particles to the global positions. As it is known the variance of Lévy distribution is infinite, so that it could make a particle to have a long jump. Results have been tested on standard benchmark functions**.** The rest of the paper is organized as follows: Section II describes the original Particle Swarm Optimization algorithm, Section III describes Lévy distribution, In Section IV the proposed LHSPO algorithm is discussed; section V deals with the results and discussion. Finally, this paper concludes with section VI.

.

II. PARTICLE SWARM OPTIMIZATION

The Particle Swarm Optimization is motivated from the social behavior of bird flocking. Particles try to get their best position in search space by their best position as well as following their neighbor's best position. For a D-dimensionalsearch space, the position of the i_{th} particle is represented as $X_i=(x_{i1},x_{i2},x_{i3},\ldots,x_{iD})$. Each Particle maintains a memory of its previous best position

ISSN: 2278 – 7798

$P_i=(P_{i1},P_{i2},P_{i3},\ldots,P_{iD})$.The Best one among all the particles in the population is represented as global best $P_g=(P_{g1},P_{g2},P_{g3},\ldots,P_{gD})$.The velocity of each particle is represented $V_i=(v_{i1},v_{i2},v_{i3},\ldots,v_{iD})$.During each iterationeach particle isaccelerated toward its previous best position and the neighbor'sglobal best position. At each iteration velocity of the particle isupdated according to equation (1) that depends on the currentvelocity, and global position. The next position of the particleis updated according to equation (2) using new velocity andprevious position in the search space. This process is iterated anumber of times until best global position is achieved.

$$v_{iD} = wv_{iD} + c_1 r_1 (P_{iD} - x_{iD}) + c_2 r_2 (P_{gD} - x_{iD}) \tag{1}$$

$$x_{iD} = x_{iD} + v_{iD} \tag{2}$$

In Equation(1) w is inertia weight, $c_1$is cognitiveacceleration constant and $c_2$is social acceleration constants. $x_{iD}$is the position vector of ith particle at that particular iteration and$P_{iD}$is personal best of particle i. $P_{gD}$is the global best of all particles at that particular iteration. The second part tells us that any particle that is distant from its cognitive best will accelerate toward it more strongly than were it nearby. Third part represents any particle which is distant from the social best will accelerate toward it more strongly than a particle nearby. The constantsr_1,$r_2$are the uniformly generated random numbers in the range of [0, 1]. Equation (2) position of the particle is updated according to the velocity as stated in equation (1).

I. LEVY DISTRIBUTION

For a Levy Process Probability density function is defined as

$$L_{\alpha,\gamma}(z) = \frac{1}{\pi}\int_0^\infty \exp(-\gamma q^\alpha)\cos(qz)\,dq \quad , z \in R \tag{3}$$

Where α and γ both are distribution parameters.γDefines the scaling of process and α defines the index ofdistribution and controls the scale properties of stochastic process {z}[7]. Above defined pdf is symmetric w.r.t.z=0. γ is taken 1 for simplicity. If the series expansion of equation (1) is considered, the approximation of levy stableprocess is taken as [7].

$$L_{\alpha,1}(z) \approx \frac{C_{LS}(\alpha)}{z^{(1+\alpha)}} \tag{4}$$

As it is known that analytical form of equation (4) is notknown for general αthen a random number in Levydistribution is generated in this way [7]. First step is twonormal stochastic variables p,q having standard deviation σ_p,$\sigma_q$are taken as shown in equation(5)

$$v = \frac{p}{|q|^\alpha} \tag{5}$$

By using probability theory, probability distribution ofstochastic processcan be found out. Probability density has
an important property the asymptotic approximation that is usedfor large arguments [7]. As in this distribution σ_p,σ_q Standard deviation can't be chosen independently for an arbitrary valueofα. So for convenience $\sigma_q=1$ is assumed and accordingly $\sigma_p$is calculated. If n Number of different variables are having similarprobability distribution are taken according to eqauation (6).

$$z_n = \frac{1}{n^{\frac{1}{\alpha}}}\sum_{j=1}^{n} v_k \tag{6}$$

In equation (6) value of $n^{1/\alpha}$ should be considered as it is characterizing variable z_n, γ. But the convergence of equation (6)is very slow. To improve this, non linear transformationfunction is used as shown in equation (7).

$$W = \{[K(\alpha)-1]\exp(\frac{-v}{C(\alpha)})+1\}v \tag{7}$$

If Proper value of $K(\alpha)$,$C(\alpha)$ is used, equation (7)shows a faster convergence. As $K(\alpha)$,$C(\alpha)$ dependson α and their values can be calculated in[7]. For efficientalgorithm linear transformation can be used as shown inequation (8).

$$Z_s = \gamma^{\frac{1}{\alpha}} Z \tag{8}$$

II. IV.PROPOSED IDEA (LEVY MUTATION WITH PSO)

Lévy Mutation Particle Swarm Optimization is simple and modified version of Particle Swarm Optimization algorithm, as it uses Levy distribution to mutate the best particle's position. Sometimes particles don't explore them by assuming that their neighbor particle is global best particle and start to converge around that neighbor. To avoid this premature convergence, levy mutation is applied on position of best particle so that particles don't trap in local optima. In the proposed algorithm, initial position and velocity is calculated according to equation (1) and (2).After that, levy mutation operator is used as

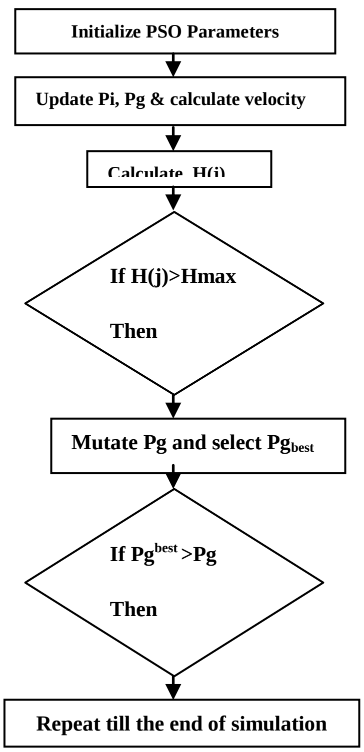
ISSN: 2278 – 7798

$$H(j) = \frac{\sum_{j=1}^{popsize} v[j][i]}{popsize} \qquad (9)$$

Where $v[j][i]$ is the ith velocity vector of jth particle in thepopulation, popsize is the population size. $H(j)$ is a weightvector within $[-H_{max}, H_{min}]$ and H_{max} is set to 1.Then thisoperator with Levy Distribution is applied on best particle asshown in equation(10).

$$P_g^{best} = P_g + H(j) * L_j(\alpha) \qquad (10)$$

Where $L_j(\alpha)$represents Lévy Distribution. Computational Steps of LHPSO are shown in Flow graph.



V. RESULT AND DISCUSSION

For comparison between PSO and LHPSO, four Standardbenchmark functions have been used as shown in Table I.Function f_1,f_2, $f_3$are unimodal functions while f_4 is multimodal function. InExperiments dimension of functions are taken 30. Parameters $c_1=c_2=1.496$and $w=$ 0.7298 isconsidered. Population size is taken 20 and generations aretaken 100. Comparisons between PSO and LHPSO are shownin Table II.

Fineness of function f_1,f_2 has been shown in Figure 2,3.

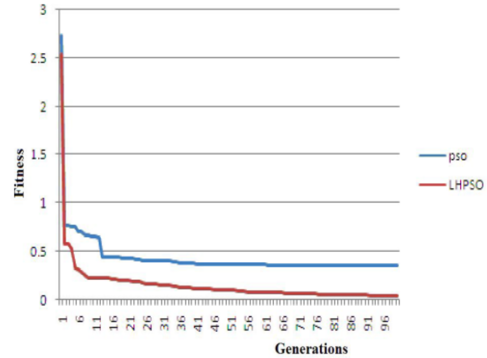


Fig 2. Fitness of function $f_1$in PSO and LHPSO

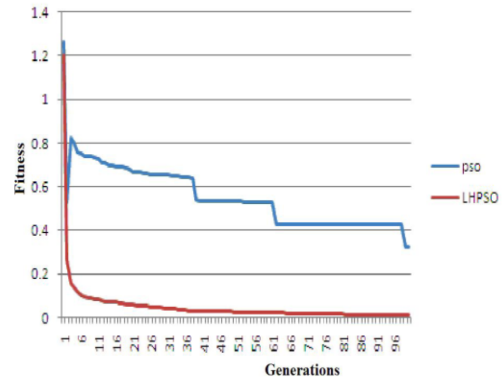


Fig 3. Fitness of function $f2$ in PSO and LHPSO

Figure 2,3 shows that LHPSO gives better results thanStandardPSO.

TABLE I. BENCMARK TEST FUNCTIONS USED FOR LHPSO

Function	Dimension	Search Space	fmin
$f_1(x) = \sum_{i=1}^{n} x_i^2$	**30**	**[-5.12,5.12]**	**0**
$f_2(x) = \sum_{i=1}^{n} (\sum_{j=1}^{j} x_j)^2$	**30**	**[-65536,65536]**	**0**
$f_3(x) = \sum_{i=1}^{\frac{n}{2}} (100(x_{2i} - x_{2i-1}^2$	**30**	**[-30,30]**	**0**
$f_4(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i$	**30**	**[-5.12,5.12]**	**0**

ISSN: 2278 – 7798

TABLE II. BENCMARK TEST FUNCTIONS USED FOR LHPSO

Test Function	LHPSO		PSO	
	Mean Best	Standard Deviation	Mean Best	Standard Deviation
F1(x)	1.65e-8	7.954e-8	1.79e-7	3.51e-7
F2(x)	0.165	0.1701	0.398	0.3082
F3(x)	0.959	1.4376	1.8016	2.8389
F4(x)	15.04	8.16	37.0721	9.7295

VI. CONCLUSION

In this paper Hybrid PSO is implemented with LévyMutation based on Levy Distribution. Due to this PSO will notget stuck into local optima. This also shows a trade of betweenExploitation and Exploration. To show LHPSO gives betterresults than PSO standard benchmark functions have beenused. Also in terms of fitness value, LHPSO is more efficientalgorithm than Standard PSO.

VII.REFERENCES

[1] J. Kennedy and R. Eberhart, Particle Swarm Optimization, IEEE International Conference on Neural Networks, Perth, Australia. 1995.

[2] R. Eberhart and Y. Shi, Comparison between Genetic Algorithms andParticle Swarm Optimization, The 7th Annual Conference on Evolutionary Programming, San Diego, USA. 1998.

[3] A. Ratnaweera, S. K. Halgamuge, and H. C. Watson, Self-organizinghierarchical particle swarm optimizer with time-varyingaccelerationcoefficients, IEEE Transactions on Evolutionary Computation, vol. 8,no.3, pp. 240–255, 2004.

[4] F. van den Bergh, A. P. Engelbrecht, Cooperative Learning in NeuralNetworks using Particle Swarm Optimization, South African ComputerJournal, 84-90, Nov. 2000.

[5] X, Xie, W, Zhang, Z, Yang, Hybrid Particle Swarm Optimizer withMass Extinction, International Conf. on Communication,Circuits andSystems (ICCCAS), Chengdu, China.1170-1174, 2002.

[6] M. Lovbjerg, T. Krink, Extending Particle Swarm Optimisers with Self-Organized Criticality, Proceedings of Fourth Congress on EvolutionaryComputation, vol. 2, 1588-1593, 2002.

[7] R. Mantegna, Fast, accurate algorithm for numerical simulation of L´evystable stochastic process, Physics Review E, vol.49, pp.4677-4683,1994

[8] Wei,C., He, Z., Zheng, Y.and Pi, W., “Swarm Directions EmbeddedinPast Evolutionary Programming”,In Proc. of IEEE Congress onEvolutionary Computation, Vol. 2, 2002, pp. 1278- 1283

[9] Stacey, A., Jancic, M. and Grundy, I., “Particle Swarm Optimizationwith Mutation”, In Proc. of the IEEE Congress on EvolutionaryComputation, 2003, pp.1425 – 1430

[10] Dong, D., Jie, J., Zeng, J. and Wang, M.,“Chaos Mutation-BasedParticle Swarm Optimize"r for Dynamic Environment”, In Proc. of the3rd Int. Conf. on Intelligent System and Knowledge Engineering.

[11] M.Clerc, and J.Kennedy,‘‘The particle swarm-explosion,stability,andconvergence in a multidimensional complex space,”IEEE Trans. onEvolutionary Computation, vol.6,no.1,pp.58-73,2002

ISSN: 2278 – 7798