Magnetohydrodynamics Rivlin-Ericksen Flow Through Porous Medium In Slip Flow Regime

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Abstract— **A study of Rivlin-Ericksen flow through porous medium in slip flow regime subjected to transverse magnetic field is considered. Approximate solutions of primary velocity and secondary velocity distributions are obtained using perturbation technique. Velocity profiles are studied with the help of plotted graphs. Skin frictions of both the cases are discussed by using tabulated values.**

Index Terms— **Magnetohydrodynamics, Porous medium Perturbation technique and Rivlin-Ericksen.**

I. INTRODUCTION

The study of problems in slip flow regime has attracted the researchers due to its application in the high speed flight problems. The problem of high speed flight in the upper atmosphere besides the kinetic approach has been studied through a continuous approach along with consideration of the velocity slip at a rigid boundary. Panda and Goudas [7] have studied the Rayleigh problem in MHD for a porous wall in a slip flow regime. Jain and Teneja [3] have solved the problem of magnetopolar flow through a porous medium in slip flow regime. Khandelwal et al. [4] have studied effect of couple stresses on the flow through a porous medium of variable permeability in slip flow regime. Singh and Gupta [9] have discussed the MHD-free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Khandelwal and Jain [5] have investigated the unsteady MHD flow of a stratified fluid through porous medium over a moving plate in slip flow regime.

Das and Panda [2] have reported the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow regime. Varshney et al. [10] have discussed the effect of heat transfer on the flow through a porous medium of variable permeability in slip flow regime with couple stress. Das et al. [1] have studied the magnetohydrodynamic unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with heat source. Mishra et al. [6] examines the problem of mass transfer on MHD unsteady free convective flow of a polar fluid through a porous medium of variable permeability bounded by an infinite horizontal porous plate in slip flow regime. Sahoo [8] has presented the unsteady flow of an electrically conducting and incompressible viscoelastic liquid of the Walter model with simultaneous heat and mass transfer near an oscillating porous plate in slip flow regime under the influence of a transverse magnetic field of uniform strength.

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II. FORMULATION OF PROBLEM

Consider the steady flow of Rivlin memory flow past an infinite porous flat plate embedded in a porous medium in a rotating system. The fluid and the plate rotate in unison with uniform angular velocity $\boldsymbol{\Omega}_\infty$ about the z- axis taken normal to the plate upward. Since the plate occupying the plane at z=0 is of infinite extent, all physical quantities depend on z only in the steady state. The pressure gradient $\frac{\partial \mathbf{p}}{\partial \mathbf{x}}$, far away from the plate is balanced by the coriolis force $\mathbf{2\Omega U_\infty}$. All the physical variables are function of z only. If (u, v, w) be the components of the velocity field then the governing equations of Rivlin-Ericksen flow in rotating system through porous medium are given by

$$w = -w_0 \quad (1)$$

$$-w_0 Du = \vartheta D^2 u + 2\Omega v - \frac{\vartheta}{K}(u - u_0) - \frac{\sigma \mu_e^2 H_0^2}{\rho}(u - u_0) + \beta w_0 D^3 u \quad (2)$$

$$-w_0 Dv = \vartheta D^2 v - 2\Omega (u - u_0) - \frac{\vartheta v}{K} - \frac{\sigma \mu_e^2 H_0^2 v}{\rho} + \beta w_0 D^3 v \quad (3)$$

where u, v, w are the velocity components, $D = \frac{d}{dz}$, σ is the electrical conductivity of the fluid, ϑ is kinematic coefficient of viscosity, Ω is angular velocity, K is permeability of porous medium, μ_e is magnetic permeability, H_0 is intensity of uniform magnetic field along the z-axis, ρ is the density of fluid and β is the kinematic visco-elasticity. It is supposed that the magnetic Reynolds number R_m is very small so those induced

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magnetic fields is neglected and above equations are valid in absence of Hall current.

Combining Eq. (2) and Eq. (3) we get

$$\vartheta D^2F + w_0DF - \left(\frac{\vartheta}{K} + \frac{\sigma\beta_0^2}{\rho}\right)F - 2i\Omega F + \beta w_0 D^3F = 0 \quad (4)$$

where β_0 is uniform magnetic filed

$$F = \frac{u + iv - U_\infty}{U_\infty} \quad (5)$$

The boundary conditions at the plate corresponding to slip flow are given by

$$u = LDu, \quad v = LDv \quad \text{at } z = 0$$

$$u \longrightarrow U_\infty, \qquad v \longrightarrow 0 \quad \text{as } z \longrightarrow \infty \quad (6)$$

Hence $L = \left(\frac{2}{f} - 1\right) l$ (7)

Where f and *l* are the fraction of molecules reflected diffusely from the plate and the molecular mean free path of the fluid respectively.

III. METHOD OF SOLUTION

Introducing the following non-dimensional quantities:

$$z' = \frac{zu_0}{\vartheta} \text{ (Non-dimensional z-coordinate)},$$

$$S_0 = \frac{w_0}{U_\infty} \text{(Suction parameter)},$$

$$\Omega' = \frac{2\Omega\vartheta}{U_\infty^2} \text{ (Rotation parameter)},$$

$$K' = \frac{KU_\infty^2}{\vartheta^2} \text{ (Permeability parameter)},$$

$$M^2 = \frac{\sigma\mu_e^2 H_0^2 \vartheta}{\rho U_\infty^2} \text{ (Magnetic parameter)},$$

$$R = \frac{LU_\infty}{\vartheta} \text{ (Rare faction parameter)},$$

$$R_m = \frac{\beta w_0 U_\infty}{\vartheta^2} \text{ (Magnetic Reynolds number)},$$

$$Q^2 = M^2 + \frac{1}{K} \quad (8)$$

Using Eq. (8) in Eq. (4), Eq. (6) & Eq. (7), suppressing primes, give rise to

$$D^2F + S_0DF - (Q^2 + i\Omega)F + R_m D^3F = 0 \quad (9)$$

The corresponding boundary conditions become

$$F = RDF - 1 \quad \text{at } z = 0$$

$$F \rightarrow \infty \qquad \text{as } z \rightarrow \infty \quad (10)$$

A perturbation technique is used with R_m as the small perturbation parameter is given by

$$F = F_0 + R_m F_1 + 0(R_m^2) \quad (11)$$

Where F, F_0, F_1 are functions of z.

Making use of Eq. (11) into Eq. (9), yields

Zeroth order equation

$$D^2F_0 + S_0DF_0 - (Q^2 + i\Omega)F_0 = 0 \quad (12)$$

First order equation is

$$D^2F_1 + S_0DF_1 - (Q^2 + i\Omega)F_1 - D^3F_1 = 0 \quad (13)$$

Corresponding boundary conditions Eq. (10) become

$$F_0 = RDF_0 - 1, \qquad F_1 = RDF_1 \qquad \text{at } z = 0$$

$$F_0 \rightarrow 0 \quad \text{and} \quad F_1 \rightarrow 0 \text{ as } z \rightarrow \infty \quad (14)$$

The solutions of Eq. (12) and Eq. (13) satisfying the boundary condition Eq. (14) are given by

$$F_0 = \frac{e^{-\alpha^* z}}{A_1}[R\beta^* \sin\beta^* z - (1 + R\alpha^*)\cos\beta^* z] + i\frac{e^{-\alpha^* z}}{A_1}[R\beta^* \cos\beta^* z + (1 + R\alpha^*)\sin\beta^* z] \quad (15)$$

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$$F_1 = \frac{e^{-\alpha^* z}}{A_1}[(A_5 z + A_7)\cos\beta^* z - (A_6 z + A_8)\sin\beta^* z] - i\frac{e^{-\alpha^* z}}{A_1}[(A_6 z + A_8)\cos\beta^* z + (A_5 z + A_7)\sin\beta^* z] \quad (16)$$

The primary and secondary velocity profiles are given by

$$u = \frac{u}{U_\infty} = 1 + \frac{e^{-\alpha^* z}}{A_1}[R\beta^* \sin\beta^* z - (1 + R\alpha^*)\cos\beta^* z] - R_m \frac{e^{-\alpha^* z}}{A_1}[(A_5 z + A_7)\cos\beta^* z - (A_6 z + A_8)\sin\beta^* z] \quad (17)$$

$$v = \frac{v}{U_\infty} = \frac{e^{-\alpha^* z}}{A_1}[R\beta^* \cos\beta^* z + (1 + R\alpha^*)\sin\beta^* z] + R_m \frac{e^{-\alpha^* z}}{A_1}[(A_6 z + A_8)\cos\beta^* z + (A_5 z + A_7)\sin\beta^* z] \quad (18)$$

Where

$$\alpha^* = \frac{S_0}{2} + \frac{1}{2\sqrt{2}}\sqrt{\left(S_0^2 + 4Q^2 + \sqrt{(S_0^2 + 4Q^2)^2 + 16\Omega^2}\right)} \quad (19)$$

$$\beta^* = \frac{1}{2\sqrt{2}}\sqrt{\left(\sqrt{(S_0^2 + 4Q^2)^2 + 16\Omega^2}\right) - (S_0^2 + 4Q^2)} \quad (20)$$

For blowing S<0, taking S = $-S_1$ where $S_1>0$, gives

$$\alpha_1 = \frac{-S_1}{2} + \frac{1}{2\sqrt{2}}\sqrt{\left({S_1}^2 + 4Q^2 + \sqrt{\left({S_1}^2 + 4Q^2\right)^2 + 16\Omega^2}\right)} \quad (21)$$

$$\beta_1 = \frac{1}{2\sqrt{2}}\sqrt{\left(\sqrt{\left({S_1}^2 + 4Q^2\right)^2 + 16\Omega^2}\right) - \left({S_1}^2 + 4Q^2\right)} \quad (22)$$

Non-dimensional skin frictions at the plate for primary and secondary flow are

$$\tau_1 = \frac{1}{A_1}\left[\alpha + R\left(\alpha^2 + {\beta^*}^2\right) + R_m\left(A_5 - \alpha A_7 - {\beta^*}^2 A_8\right)\right] \quad (23)$$

$$\tau_2 = \frac{1}{A_1}\left[{\beta^*}^2(1 - R\alpha) + R_m\left(\alpha A_8 + {\beta^*}^2 A_7 - A_6\right)\right] \quad (24)$$

IV. DISCUSSION AND CONCLUSIONS

It is noted that M (Magnetic parameter), R (Rare-fraction parameter) and Ω (Rotation parameter) are influencing primary velocity profiles at all points near the plate in figure. 1. The effect of S_0 (Suction parameter) is to suppress the primary velocity profiles at all points. It is also noted that Primary velocity is more in visco-elastic case than viscous case.

S_0 (Suction parameter), K (Permeability parameter) enhance the secondary velocity profiles, whereas M (Magnetic parameter) and R (Rare-faction parameter) reduce the secondary velocity profiles at all points. In case of Ω (Rotation parameter) a rapid rise in secondary velocity near the plate is noted, there after sudden decrease is observed. However an interesting point is noted near the plate i.e. R_m (Magnetic Reynolds number) first influences the secondary velocity profiles and then it gradually decreases it in figure. 2. The effect of R_m (Magnetic Reynolds number) and R (Rare-faction parameter) is to diminish shearing stress in both the cases, whereas Ω (Rotation parameter) is to influence the primary and secondary velocities.

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The effect of M (Magnetic parameter) and S_0 (Suction parameter) is to enhance the skin friction in primary case and to subside the same in secondary one. K (Permeability parameter) has adverse effects which are shown in Table 1 & 2.

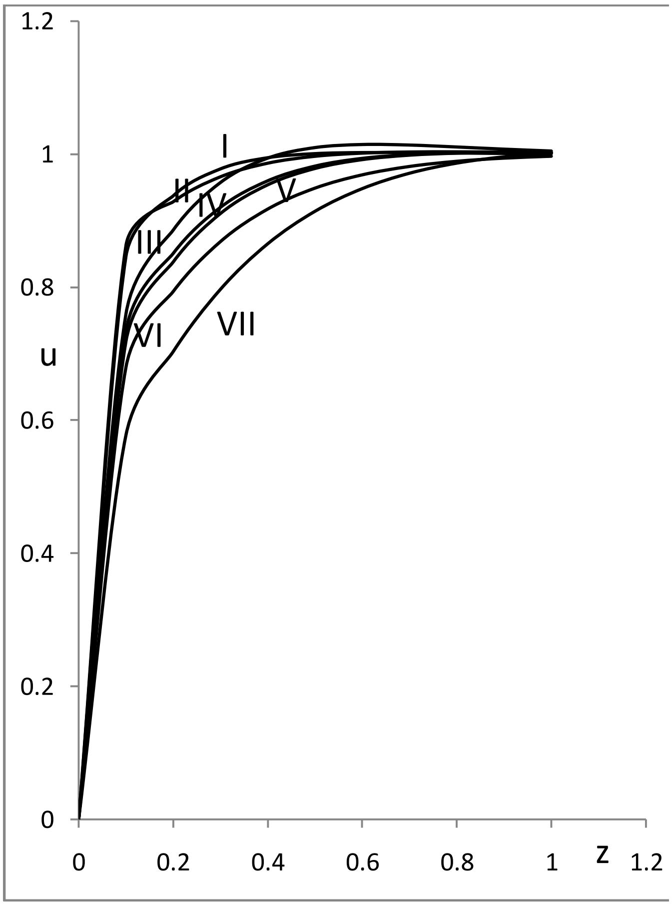

Figure.1 Primary velocity profiles

Fig.1(u)	R_m	R	M	S_0	K	Ω
I	0.05	0.1	4	3	1	2
II	0.05	0.2	2	3	1	2
III	0.05	0.1	2	3	1	4
IV	0.05	0.1	2	3	1	2
V	0.05	0.1	2	3	1000	2
VI	0	0.1	2	3	1	2
VII	0.05	0.1	2	1	1	2

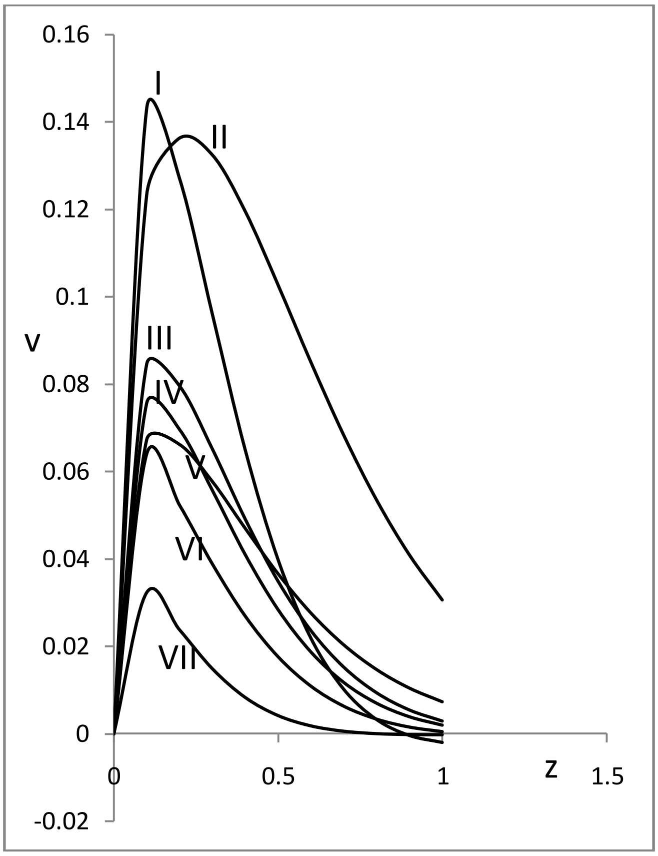

Figure. 2 Secondary velocity profiles

Fig. 2 (v)	R_m	R	M	S_0	K	Ω
I	0.05	0.1	2	3	1	4
II	0.05	0.1	2	1	1	2
III	0.05	0.1	2	3	1000	2
IV	0.05	0.1	2	3	1	2
V	0	0.1	2	3	1	2
VI	0.05	0.2	2	3	1	2
VII	0.05	0.1	4	3	1	2

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Table-1

R_m	τ_1	R	τ_1	M	τ_1	S_0	τ_1	K	τ_1	Ω	τ_1
0	3.00	0.1	2.66	2	2.66	3	2.66	1	2.66	2	2.66
0.05	2.66	0.2	2.11	4	3.25	6	3.31	100	2.59	4	2.79
0.1	2.35	0.3	1.75	6	3.78	9	3.75	1000	2.59	6	2.97

Table-2

R_m	τ_2	R	τ_2	M	τ_2	S_0	τ_2	K	τ_2	Ω	τ_2
0	2.29	0.1	0.15	2	0.152	3	0.15	1	0.15	2	0.15
0.05	0.15	0.2	-0.06	4	-0.001	6	-0.05	100	0.18	4	0.26
0.1	0.00	0.3	-0.14	6	-0.052	9	-0.12	1000	0.18	6	0.33

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