Design a crown wheel gear of differential gear box used in heavy vehicles for structural loads

T. Loknadh\textsuperscript{1}, DR. P.S.S. Vasu\textsuperscript{2}

Abstract:
In the present paper work Crown wheel gear of differential gear box are designed under structural condition and modeled. The main aim of the project is to focus on the design and structural analysis of crown wheel gear used in gear box when they transmit power at maximum speed 7200 rpm. The analysis is conducted to verify the best material (steel and Aluminum Alloys) for the crown wheel gear in the gear box at higher speed by analyzing static and dynamic conditions. Because the dynamic behavior of gears are very important in analyze the perfect structure. In this paper the design is developed in NX-CAD software and structural analysis id done by using ANSYS software.

Keywords: Analyzing, crown wheel, focus, NX-CAD, structure.

1. INTRODUCTION
In automobiles and other wheeled vehicles, the differential allows the outer drive wheel to rotate faster than the inner drive wheel during a turn. This is necessary when the vehicle turns, making the wheel that is traveling around the outside of the turning curve roll farther and faster than the other. The average of the rotational speed of the two driving wheels equals the input rotational speed of the drive shaft. An increase in the speed of one wheel is balanced by a decrease in the speed of the other. When used in this way, a differential (hereafter, diff) couples the input shaft (or prop shaft) to the Pinion, which in turn runs on the Crown wheel of the diff. This also works as reduction gearing to give the ratio. On rear wheel drive vehicles the diff may connect to half-shafts inside an axle casing or drive shafts that connect to the rear driving wheels. Front wheel drive vehicles tend to have the pinion on the end of the main-shaft of the gearbox and the diff is enclosed in the same casing as the gearbox. They have individual drive-shafts to each wheel. Older 4x4 vehicles and tractors usually have a solid front axle, the modern way can be a separate diff and drive shaft arrangement for the front.

2. PROBLEM DEFINITION AND METHODOLOGY
In the present work Crown wheel gear of differential gear box are designed under structural condition and modeled. The main aim of the project is to focus on the design and structural analysis of crown wheel gear used in gear box when they transmit power at maximum speed 7200 rpm.

The main objective of this project is to design and analysis of crown wheel gear, the design is developed in Pro-E software and structural analysis id done by using ANSYS software.

The methodology followed in this project is as follows:
\begin{itemize}
  \item Perform design calculations of crown wheel gear model.
  \item Create 3D model of the existing crown wheel gear from the 2D drawings. Pro-E software is used to do the 3D modelling.
  \item Convert the 3D model into parasolid format and import into ANSYS to do Finite element analysis.
  \item Perform Static analysis of the crown wheel gear by applying the transmit power at maximum speed 7200 rpm, and document the stresses and deflections.
  \item Perform Modal analysis of the crown wheel gear and calculate the natural frequencies in the operating range. Plot the frequencies and mode shapes.
  \item Perform harmonic analysis of the crown wheel gear in the operating range. Check the harmonic response of crown wheel gear at the natural frequencies and also check whether the stresses and deflections are within the limits.
  \item The analysis is conducted to verify the best material (steel and Aluminium Alloys) for the crown wheel gear in the gear box at higher speed by analyzing static and dynamic conditions.
\end{itemize}

3. DESIGN CALCULATIONS OF CROWN WHEEL GEAR
The two Primary function of a differential system is to allow the driven wheels to rotate at different angular velocities relative to each other and also to transfer power to them from transmission. The first function is important to allow smooth turning as can be seen in the following Fig.1 which illustrates a left hand run.

From Fig.1 it is clear that ro is larger than ri , therefore the outer wheel travelling along ro has to cover more distance than the inner wheel ri since both wheels are of the same diameter .the outer wheel has to complete more revolution than the inner wheel to accomplish this and hence spin faster. If Differentiation in speeds was not allowed and the wheels were locked together along a single drive shaft, then during turning the outer wheel would lose traction and will slip. This would result in reduced turning performance and increased tire wear. Below fig1. shows the crown wheel gear terminology used in this work.
3.1. Design calculations of gear for 7200 rpm speed:

Diameter of crown wheel \(D_G\) =465mm

Number of teeth on gear \(T_G\) =26

Number of teeth on pinion \(T_P\) =5

Module \(m = D_G / T_G = 465 / 26 = 17.8\)

Diameter of pinion \(D_P = m * T_P = 18 * 5 = 90\)

Material used for both pinion and gear is stainless Steel AISI 302.

Steel, stainless AISI 302 Mechanical Properties:
- Young’s modulus = 180Gpa
- Yield Strength = 509 Mpa
- Tensile Strength = 860 Mpa
- Density = 7850 kg/m³

Brinell hardness number (BHN)=140

Pressure angle of teeth is \(\phi = 20^0\) involutes’ system = \(\phi = 20^0\)

We know that velocity ratio

\[ V_{T} = \frac{D_G}{V_P} \]

\[ V_{R} = \frac{T_G}{T_P} = \frac{26}{5} = 5.2 \]

\[ V_{R} = \frac{NP}{NG} \]

\[ L = \frac{\sqrt{\frac{465^2}{2} + \frac{90^2}{2}}} \]

\[ L = \sqrt{54056 + 2025} \]

\[ L = 236.8 \]

The factor \((L-b/L)\) may be called as bevel factor.

For satisfactory operation of the bevel gear the face width should be from 5.2m to 10m so b is taken as 10m.

\[ b = 10*18 = 180 \]

\[ W_T = 250*0.081*180*0.0284*(236.8-180)/236.8 \]

\[ W_T = 5850*0.237 \]

\[ W_T = 1388N \]

4.3D MODELLING OF CROWN WHEEL GEAR USED IN GEAR BOX

The 3D model of the crown wheel gear is created using Pro-E software from the 2d drawings. Pro-E is the world’s leading 3D product development solution. This software enables designers and engineers to bring better products to the market faster. It takes care of the entire product definition to serviceability.

4.1. Crown wheel gear specifications for existing model used in heavy vehicle gear box:

Tooth form factor for gear

\[ Y_1 = 0.154 - 0.912 / T_{EG} \]

\[ = 0.154 - 0.912 / 138.2 \]

\[ = 0.0284 \]
5. FINITE ELEMENT ANALYSIS

5.1. Structural analysis of crown wheel gear

Finite Element Modelling (FEM) and Finite Element Analysis (FEA) are two most popular mechanical engineering applications offered by existing CAE systems. This is attributed to the fact that the FEM is perhaps the most popular numerical technique for solving engineering problems. The generality of the FEM fits the analysis requirements of today’s complex engineering systems and designs where closed form solutions are governing equilibrium equations are not available. In addition it is an efficient design tool by which designers can perform parametric design studying various cases (different shapes, material loads etc.) analyzing them and choosing the optimum design.

5.2. Material properties:

All the components of the crown wheel gear are made using hot-rolled structural steel stainless AISI 302. All the components of the crown wheel gear are assigned as per the below material properties.

**Steel stainless AISI 302 Mechanical Properties:**

- Young’s modulus = 180 Gpa
- Yield Strength = 509 Mpa
- Tensile Strength = 860 Mpa

### Finite Element Model

**Element Type Used:**

- Element type: Solid92
- No. of nodes: 10
- Degrees of freedom: 3 (UX, UY, UZ)

**Solid92:**

The element is defined by ten nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. SOLID92 has quadratic displacement behaviour and is well suited to model irregular meshes. 3D model of the crown wheel gear was developed in UNIGRAPHICS. The model was converted into a Para solid to import in ANSYS.

6. STATIC ANALYSIS OF CROWN WHEEL GEAR

**Objective:** Structural static analysis has been performed on the Hypoid gear structure by applying the axial force. The bolt is fixed in all dof. Deflection and stresses are observed and plotted. From the analysis results the factor of safety of the crown wheel gear at different locations are calculated.

**Boundary conditions for static analysis:**

- Force is applied on one teeth at once = 1679N
  - Crown wheel gear is arrested in all Dof at shaft location.
  - Tangential Force is applied on one gear teeth.
  - Gravity is also applied on Crown wheel gear in z direction.
Fig. 8. shows the Boundary conditions for static analysis of crown wheel gear

Results: Deflections

Fig. 9. Deflection in X-dir for static analysis

Fig. 10. Deflection in Y-dir for static analysis

Fig. 11. Deflection in Z-dir for static analysis

Fig. 12. Total Deflection for static analysis

Results: Stress

Fig. 13. 1st principal stress for static analysis

Fig. 14. 2nd principal stress for static analysis

Fig. 15. 3rd principal stress for static analysis

Fig. 16. Von Mises stress for static analysis
From the above results it is observed that:

- The Max Deflection 0.069mm observed on the crown wheel gear for operating loading conditions. The Max VonMises Stress observed 310Mpa on the crown wheel gear for operating loading conditions. And the Yield strength of the materials stainless steel is 509Mpa. Hence according to the Maximum Yield Stress Theory, the Von Mises stress is less than the yield strength of the material. Hence the design of crown wheel gear is safe under the above operating loads. And the FOS is 509/310 = 1.64.

7. DYNAMIC ANALYSIS OF CROWN WHEEL GEAR

Methodology:

- Develop a 3D model.
- The 3D model is created using UNIGRAPHICS-NX software.
- The 3D model is converted into parsaolid and imported into ANSYS to do modal analysis.
- Calculate natural frequencies and plot their mode shapes.

Natural Frequency:

Natural frequency is the frequency at which a system naturally vibrates once it has been set into motion. In other words, natural frequency is the number of times a system will oscillate (move back and forth) between its original position and its displaced position, if there is no outside interference.

The natural frequency is calculated from the formula given below. The natural frequencies depend on stiffness of the geometry and mass of the material.

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

MODAL ANALYSIS:

Modal analysis was carried out on Crown wheel gear to determine the natural frequencies and mode shapes of a structure in the frequency range of 0 - 1500 Hz.

Boundary Conditions:

- Crown wheel gear is arrested in all DoF at shaft location.

From the modal analysis, a total of 6 natural frequencies are observed in the frequency range of 0-2000 Hz. The total weight of the Crown wheel gear is 71kg. The mass participation of each of these 6 frequencies are listed in the below table.

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY</th>
<th>PARTIC.FACTO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X- Dir</td>
</tr>
<tr>
<td>1</td>
<td>1185.14</td>
<td>1.4E-04</td>
</tr>
<tr>
<td>2</td>
<td>1200.57</td>
<td>2.5E-02</td>
</tr>
<tr>
<td>3</td>
<td>1278.54</td>
<td>-5.4E-03</td>
</tr>
<tr>
<td>4</td>
<td>1398.52</td>
<td>9.2E-06</td>
</tr>
<tr>
<td>5</td>
<td>1408.81</td>
<td>-3.1E-03</td>
</tr>
</tbody>
</table>

The mode shapes for the above frequencies are plotted below.

**Results – **Mode 1 @ 1185 Hz

Fig.17. Shows Mode shape 1@1185Hz for Crown wheel gear

**Results – **Mode 2 @ 1201 Hz

Fig.18. Shows Mode shape 2@ 1201Hz for Crown wheel gear
Results – Mode 3 @ 1279Hz

Fig. 19. Shows Mode shape 3@1279Hz for Crown wheel gear

Results – Mode 4 @ 1399 Hz

Fig. 20. Shows Mode shape 4@1399Hz for Crown wheel gear

Results – Mode 5 @ 1409 Hz

Fig. 21. Shows Mode shape 5@ 1409Hz for Crown wheel gear

From the modal analysis,

- It is observed that the maximum mass participation of 0.00068 tone is observed in X-dir for the frequency of 1185Hz.
- It is observed that the maximum mass participation of 0.0023 tone is observed in Y-dir for the frequency of 1200Hz.
- It is observed that the maximum mass participation of 0.043 tone is observed in Z-dir for the frequency of 1279Hz.

To check the structure response at the mentioned frequency due to the operating loads, harmonic analysis is carried out on the Crown wheel gear.

7. HARMONIC ANALYSIS OF CROWN WHEEL GEAR

Any sustained cyclic load will produce a sustained cyclic response (a harmonic response) in a structural system. Harmonic response analysis gives you the ability to predict the sustained dynamic behaviour of your structures, thus enabling you to verify whether or not your designs will successfully overcome resonance, fatigue, and other harmful effects of forced vibrations.

Harmonic response analysis is a technique used to determine the steady-state response of a linear structure to loads that vary sinusoidal (harmonically) with time. The idea is to calculate the structure's response at several frequencies and obtain a graph of some response quantity (usually displacements) versus frequency. "Peak" responses are then identified on the graph and stresses reviewed at those peak frequencies.

The procedure for a full harmonic response analysis consists of three main steps:

1. Build the model.
2. Apply loads and obtain the solution.
3. Review the results.

Harmonic response occurs at forcing frequencies that match the natural frequencies of your structure. Before obtaining the harmonic solution, you should first determine the natural frequencies of your structure by obtaining a modal solution.

Harmonic analysis was carried out to determine the structure behavior due to operating loads at the above mentioned frequencies from modal analysis. Deflections and stress of a structure in the frequency range of 1100 -1400 Hz are recorded and plotted.

No. Of sub steps = 6

Boundary conditions for harmonic analysis:

Force is applied on one teeth at once = 1679N

- Crown wheel gear is arrested in all Dof at shaft location.
- Tangential Force is applied on one gear teeth.
- Gravity is also applied on Crown wheel gear in z direction.

Fig. 22. Shows the applied boundary conditions for harmonic analysis of Crown wheel gear
From modal analysis results:

Table 2. Frequencies and Mass Participation in kgs for Modes in the range of 0-1500Hz.

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQUENCY</th>
<th>PARTIC. FACTO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X- Dir</td>
</tr>
<tr>
<td>1</td>
<td>1185.14</td>
<td>1.4E-04</td>
</tr>
<tr>
<td>2</td>
<td>1200.57</td>
<td>2.5E-02</td>
</tr>
<tr>
<td>3</td>
<td>1278.54</td>
<td>-5.4E-03</td>
</tr>
<tr>
<td>4</td>
<td>1398.52</td>
<td>9.2E-06</td>
</tr>
<tr>
<td>5</td>
<td>1408.81</td>
<td>-3.1E-03</td>
</tr>
</tbody>
</table>

From the above table we can conclude that the 1278Hz frequency is critical frequency because the effective mass is higher than the other natural frequencies.

Harmonic analysis was carried out to determine the structure behavior due to operating loads at the above mentioned frequencies from modal analysis. Deflections and stress of a structure in the frequency range of 1100 -1400 Hz are recorded and plotted.

**GRAPHs: AMPLITUDE VS OPERATING FREQUENCY:**

1. Harmonic response at teethes

Fig. shows the harmonic response at teethes in linear and log scale

2. Harmonic response at teethes

Fig. shows the harmonic response at teethes in linear and log scale

3. Harmonic response at nearby fixed end

Fig. shows the harmonic response at nearby fixed end in linear and log scale
From the above table we can conclude that the 1278Hz frequency is critical frequency because the effective mass is higher than the other natural frequencies.

The deflections and stresses nearest to the above frequencies are plotted below.

1. Max. Deflection and stress of frequency @ 1300Hz

Deflection:

![Fig. 23. shows the max. Deflection of Crown wheel gear](image)

Von Mises stress:

![Fig. 24. shows the Von Mises stress of Crown wheel gear](image)

Table: shows the deflections and von mises stress for critical frequencies

<table>
<thead>
<tr>
<th>S.no</th>
<th>FREQUENCY(Hz)</th>
<th>DEFLECTIONS (mm)</th>
<th>VON MISES STRESS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1300</td>
<td>0.2</td>
<td>317</td>
</tr>
</tbody>
</table>

From the above results it is observed that the critical frequency 1300Hz is having stresses of 317 MPa respectively. The yield strength of the material used for Crown wheel gear is 509 MPa.

According to the VonMises Stress Theory, the VonMises stresses of Crown wheel gear at frequencies is less than the yield strength of the material used for Crown wheel gear.

Form the dynamic analysis results we can conclude that the natural frequency and operating frequency are calculated and the deflections and stresses are calculated at critical frequencies. Hence the design of Crown wheel gear is safe for the above operating loading conditions.

CONCLUSION

In the present work Crown wheel gear of differential gear box are designed under structural condition and modeled. The main aim of the project is to focus on the design and structural analysis of crown wheel gear used in gear box when they transmit power at maximum speed 7200 rpm. The analysis is conducted to verify the best material (two different Aluminum Alloys) for the crown wheel gear in the gear box at higher speed by analyzed static and dynamic conditions.

From the analysis results we can conclude that static results are deflections and vonmises stress values are below the design limit. Form the dynamic analysis results we can conclude that the natural frequency and operating frequency are calculated and the deflections and stresses are calculated at critical frequencies. Hence according to the Maximum Yield Stress Theory, the Von Mises stress is less than the yield strength of both the materials aluminum and steel.

Hence the design of crown wheel gear is safe under the above operating loading conditions. But the steel material model factor of safety (2.9) is better than the aluminum material model FOS (1.64). Hence we can conclude that the steel material design is better than the aluminum material design.

REFERENCES

- Design, manufacturing & analysis of differential crown gear and pinion for MFWD axle by S. H. Gawande, S.V. Khandagale,
- Design and stress strain analysis of composite differential gear box by Nitin Kapoor, Virender Upneja in July 2014 ,
- Design and structural analysis of differential gear box at different loads by C. Veeranjaneyulu, U. Hari Babu
- Parametric modeling and weight analysis of glass filled polyamide composite differential gearbox by Nitin Kapoor, Pardeep Kumar, Rahul Garg and Ram Bhool,
- Analysis of process parameters during press quenching of bevel gear parts by Mats Bagge, Bengt Lindberg,
- Computer Aided Design and Analysis of Gear Tooth Geometry by S.H. Chang and R.L. Huston, in May 1987,
- Failure investigation of crown wheel and pinion by A. Bensely, S. Stephen Jayakumar, D. Mohan Lal in 9 February 2006,
- Modeling and contact analysis of crowned spur gear teeth by Ramalingam Gurumani, Subramaniam Shanmugam,

