Analytical approach for channel estimation in OFDM system based on kalman filtering

B.Padma Sirisha, Dr. I.Santi Prabha

***Abstract*— In this paper, Kalman and Wiener filtering is used for Multiple-Input-Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) channel estimation. The channel estimation is done using Least Square (LS) estimation. The Kalman and Wiener filtering estimation is based on estimation and prediction values. The proposed estimator outperforms the existing estimators in terms of Mean Square Error (MSE) and Signal to Noise Ratio (SNR). Finally, the performance is analyzed with the help of simulation results.**

***Index Terms*— OFDM, Kalman and Wiener filtering, channel estimation**

I. Introduction

Orthogonal frequency division multiplexing (OFDM)[1] is one of the multicarrier modulation schemes which modulate multiple carriers simultaneously. In addition, the subcarriers are overlapped to maximize the spectral efficiency. Since it is robust to inter symbol interference, it is widely used for broadband wireless communication systems. Orthogonal frequency division multiplexing too suffers from high peak to average power ratio which occurs due to insufficient power distribution. The input is binary sequence in which pilot insertion plays a very crucial role. The pilot symbols[2] are used to guard the data. The Inverse Discrete Fourier Transform (IDFT) is used to convert a signal from frequency domain into time domain. The cyclic prefix (CP) is used at regular periods in order to avoid inter-symbol interference (ISI).

II. LEAST SQUARE ESTIMATION

Channel estimation is done by inserting pilot symbols in both time and frequency domain. These pilot symbols provide an estimate of channel at certain locations within a subframe. Through interpolation technique it is possible to estimate the channel across an arbitrary number of subframes. The unique positioning of pilot symbols is used so that reliable estimate of complex gains can be achieved. The least squares estimates of the channel frequency response at the pilot symbols are calculated. The least squares[3] estimates are then averaged to reduce any unwanted noise from the pilot symbols. Virtual pilot symbols are created to aid the interpolation process near the edge of the sub frame where no pilot symbols may be located. Using the averaged pilot symbol estimates and the calculated virtual pilot symbols, interpolation is then carried out to estimate the entire sub frame.

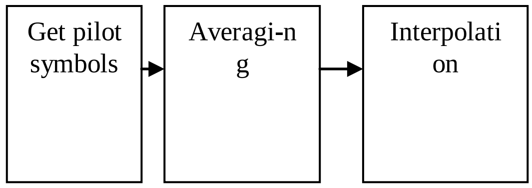

Fig 1. LS estimation

A. Get pilot symbols

The first step in determining the least squares estimate is to extract the pilot symbols from their known location within the received subframe. The value of these pilot symbols is known and therefore the channel response at these locations can be determined by using the least squares estimate which is obtained by dividing the received pilot symbols by their expected value.

$$Y(K) = X(K)H(K) + noise \quad (1)$$

$$\widehat{H}_{P,LS}(K) = \frac{Y_P(K)}{X_P(K)} + noise \quad (2)$$

Where Y(K) is received symbol value, X(K) is transmitted symbol value, H(K) is complex channel gain, $\widehat{H}_{P,LS}(K)$ is least squares estimate of pilot symbol locations, $Y_P(K)$ is received symbol value, $X_P(K)$ is known transmitted pilot symbol.

B. Noise

Noise is the keen parameter which is challenging task to maintain estimation of channel accuracy. The least-squares estimates and the averaged estimates contain the same data, apart from additive noise. Simply taking the difference between the two estimates will result in a value for the noise level on the least squares channel estimates at pilot symbol locations.

$$\widehat{H}_{P,LS}(K) = \frac{Y_P(K)}{X_P(K)} + noise \quad (3)$$

$$\widehat{H}_{P,AVG}(K) = \frac{Y_P}{X_P} \quad (4)$$

$$noise = \widehat{H}_{P,LS}(K) - \widehat{H}_{P,AVG}(K) \quad (5)$$

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C. Pilot average

To reduce the effects of noise on pilot estimates, the averaging is done. This method substantially reduces the levels of noise on pilot symbols which will further help in minimizing leakage.

D. Interpolation

Once the noise has been reduced or removed from the least squares pilot symbol averages and sufficient virtual pilots have been determined, it is possible to use interpolation to estimate the missing values from the channel estimation missing values from the channel estimation.

III. EXISTING WORK

Consider an OFDM system with N subcarriers in which U subcarriers with index set Ω_U are actually used, i.e., $\Omega_U \subset \Omega_N = \{0, 1 \ldots N\text{-}1\}$. Among Ω_U, P subcarriers with index set $\Omega_P \subset \Omega_U$ are used for pilot subcarriers. Here, V = (N−U)P/U subcarriers with index set $\Omega_V \subset \Omega_N \backslash \Omega_U$ can be considered as artificial pilot subcarriers. Also, a length-G cyclic prefix (CP) with index set $\Omega_G = \{0, 1, \ldots, G\text{-}1\}$ is used, it is assumed that G is larger than the maximum delay spread τ_{max} which is much larger than the maximum number of paths, L ,i.e., $L \lll \tau_{max} \lll G$. Also, P and Ω_P are assumed to be well designed for successful channel estimation.

Let the index set of the nonzero CIR taps be $\Omega\tau$. Then, the G× 1 CIR vector h is written as $h = [h(0)\;\; h(1) \ldots\ldots h(G-1)]^T$ with G×G covariance matrix $R \triangleq E\{hh^H\}$, where h(n) is the complex gain at the nth tap and nonzero only when $n \in \Omega\tau$. The received vector is written when CP is removed as

$$y = x \otimes h + n \qquad (6)$$

where x is the N× 1 transmitted OFDM symbol vector in the TD before the CP insertion, n is the N× 1 independent identically distributed (IID) complex white Gaussian noise vector in the TD with mean zero and covariance matrix $\sigma_n^2 n^2 I_N$ and ⊗ denotes the circular convolution. Let Ω_F and Ω_T be the FD and TD index sets of the DFT-based channel estimator repectively. Also, F denotes the N×N unitary DFT matrix with:

$$[F]_{m,n} \overset{\Delta}{=} e^{\frac{-j2\Pi mn}{N}}. \qquad (7)$$

The following steps are involved to minimize the error with respect to SNR.

A. Threshold setting

1. The initial threshold has been designed in such a way that P_{MD} is satisfied with values assumed as $|\Omega_\tau| = L, [R]_{n,n} = \frac{1}{L}$ for $n \in \Omega_\tau$, and $[R_{||}]_{n,n} = \frac{tr(LL^H)}{P^2G^2}$

- Initialization step : $\Omega_T \leftarrow \phi$
- candidate index set estimation: $\Omega_T \leftarrow f_G^{\gamma_i}(\hat{h})$
- Second step (recursion): while $k \leftarrow \arg\max_{n \epsilon \Omega_C} |\hat{h}(n)|$
- If $|\hat{h}(k)| > \gamma_r, \Omega_C \leftarrow \Omega_C\{k\}, \Omega_T \leftarrow \Omega_T \cup \{k\}$, and $(j) \leftarrow \hat{h}(j) - \frac{1}{P}\hat{h}(k)[L]_{j,k}\, for\; j \epsilon \Omega_C \backslash \{k\}$

From these assumptions, the initial threshold is obtained as:

$$\boldsymbol{\gamma}_i = \sqrt{\frac{1}{L}\left(\frac{1}{L} + \frac{1}{P^2G^2} tr(LL^H) + \frac{1}{\rho P}\right) ln\left(\frac{1}{1-P_{MD}}\right)} \qquad (8)$$

2. A successive MST selection and leakage cancellation is done with the recursive threshold γ_r. By assuming that the leakage is sufficiently suppressed, the recursive threshold can be directly used to minimize the MSE as

$$\boldsymbol{\gamma}_r = \sqrt{\frac{\ln((G-L)\rho P/L^2)}{\rho P - L}} \qquad (9)$$

The MSE of the estimator under the assumption, $|\Omega_\tau| = L, [R]_{n,n} = \frac{1}{L}$ for Ω_τ and $[R_{||}]_{n,n} = \sigma_r^2\; n \in \Omega_G$ can be obtained from:

$$\Gamma_{prop} = L\left(\frac{1-P_{MD}}{\rho P} + P_{MD}\left(\frac{1}{L} - \frac{\gamma_i^2}{\exp(L\gamma_i^2)-1}\right)\right) + (G-1)P_{FA}\left(\frac{U+\rho NP\gamma_i^2}{\rho NP}\right) \qquad (10)$$

Where $P_{MD} = 1 - \exp(-\frac{\rho P \gamma_r^2}{1+\rho P \sigma_{r+\frac{\rho P}{L}}^2})$ probability of is miss detection and $P_{FA} = \exp(-\frac{\rho P \gamma_r^2}{1+\rho P \sigma_r^2})$ is the probability of false alarm.The residual leakage power after successive cancellation is:

$$\sigma_r^2 \triangleq \frac{1+3\delta}{\rho P} \qquad (11)$$

where $\delta = \frac{VU}{NP}$.

B. Complexity Analysis

The complexity analysis depends on the number of complex multiplications which are summarized in Table I.

IV. PROPOSED WORK

Kalman filter is an estimator which focuses on instantaneous state of a system perturbed by white Gaussian noise. Kalman filtering predicts[4] and estimates the future values based on past values but whereas Wiener filter does only prediction. The estimation of Kalman filter equations fall into two categories:

1) Predictor equations
2) Corrector or estimator equations.

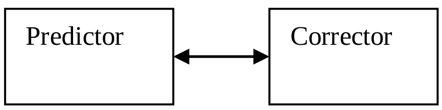

Fig 2.Kalman filter cycle

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TABLE 1
COMPLEXITY COMPARISION

Estimator	No of multiplications
Simple(MMSE)	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U}$
Conventional (LS)	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U} + VP$
Optimal(Ideal)	$\frac{N}{3}\log_2 N + P + UP$
Existing	$\frac{2N}{3}\log_2 N + P + \frac{NP}{3U}\log_2 \frac{NP}{U} + \lvert\Omega_T\rvert^2$

The first step is prediction stage

$$\hat{x}_{k,k-1} = F\hat{x}_{k,k-1} \quad (12)$$

$$P_{k,k-1} = FP_{k,k-1}F^T \quad (13)$$

Where $\hat{x}_{k,k-1}$ is estimated value of k using past value k-1 and $P_{k,k-1}$ is the predicted estimate covariance. The second step corrector stage is

$$r_k = r_k - H\hat{x}_{k,k-1} \quad (14)$$

$$S_k = HP_{k,k-1}H^T + R \quad (15)$$

$$K_k = P_{k,k-1}H^T S_k^{-1} \quad (16)$$

$$\hat{x}_{k,k} = \hat{x}_{k,k-1} + Kr_k \quad (17)$$

$$P_{k,k} = (I - K_k H)P_{k,k-1} \quad (18)$$

Where the observation r_k in (14) is either the real or imaginary part of the k-th sample entering the receiver in (15) is the variance, K_k is the optimal Kalman gain in (16),$\hat{x}_{k,k}$ is state estimate in (17) and $P_{k,k}$ is estimate covariance in (18).The variance R in (15) needs to be estimated from received samples.

Using these estimated values of both stages, the channel estimation is performed. Hence it is analyzed in the simulation as the SNR increases MSE decreases and hence improved when compared to Wiener filter estimation method.

V. EXPERIMENTAL RESULTS

The initial threshold γ_i and the recursive threshold γ_r are set to satisfy $P_{MD} = 10^{-3}$in (8) and (9). In Fig.3, the MSE performance is shown when $\delta = 0.4797$ .In Fig.4, the complexity i.e, number of multiplications required is seen. In Fig.5, the error is minimised as the signal to noise ratio increases using Wiener and Kalman filtering.

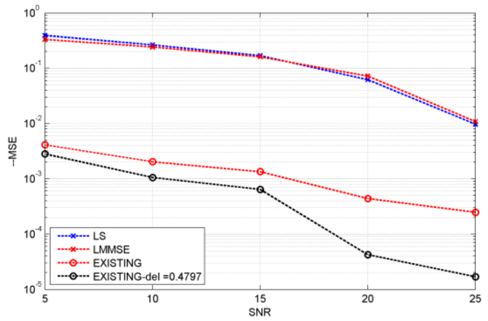

Fig.3 MSE performance versus SNR(ρ)

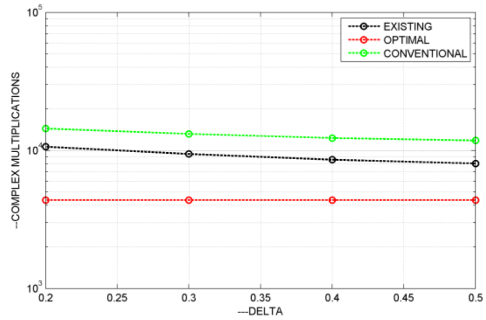

Fig.4 Complex multiplications

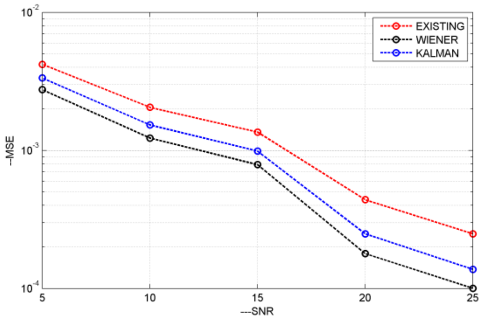

Fig.5. Estimation performance with Kalman and Wiener filtering.

VI. CONCLUSION

In this paper, the proposed estimations for Kalman and Wiener filtering have greatly improved the performance of mean square error with respect to signal to noise ratio. The prediction and corrector stages have made the process iterative so that state of system can be predicted. Hence this method provides enhanced performance in terms of power leakage and complexity. It is fruitful to develop a practical channel estimator suitable for LTE-advanced systems.

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