

AN IRREDUCIBLE SMP MODEL ON THE PERFORMANCE ANALYSIS OF BEACON MESSAGE DISSEMINATION IN DSRC SAFETY COMMUNICATION IEEE 802.11.

M. Reni Sagayaraj¹, C. Bazil Wilfred²

¹Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, India

²Department of Mathematics, Karunya University, Coimbatore, India

Abstract— We propose an analytical model for evaluation of periodic beacon message dissemination in the dedicated short range communications (DSRC) system on highways. In order to develop an analytical approximation, we develop an irreducible semi-Markov process model for the tagged vehicle to capture the periodic message generation, outdated message replacement, channel contention and backoff behavior in IEEE 802.11 broadcast ad hoc-networks. evaluate the performance measures.

Index Terms—Semi-Markov processes, DSRC, Irreducible SMP, Performance measures.

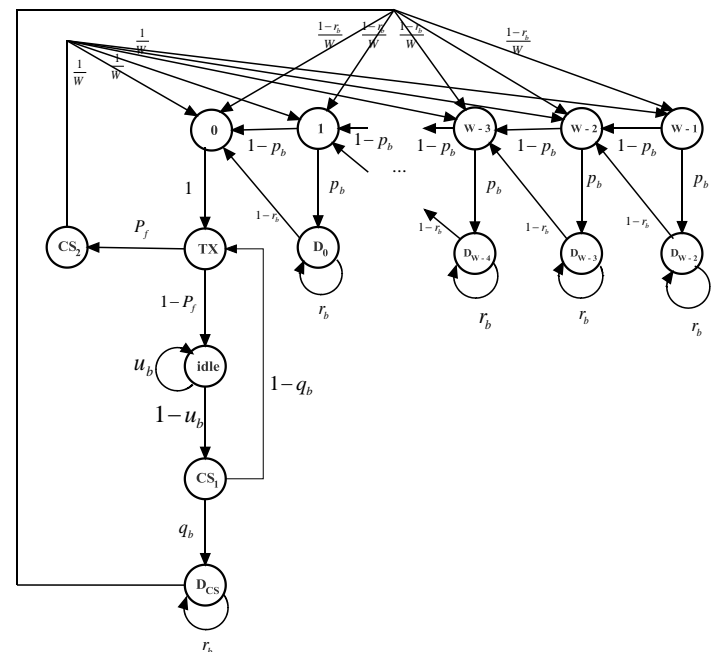
I. INTRODUCTION

Intelligent Transport System (ITS) has become an important component in the effective management of safety vehicular traffic in developed countries. Various safety messages such as collision avoidance (CA) and safety warning (SW) are issued through wireless vehicular communications to prevent and reduce traffic accidents. These safety messages called beacon messages are periodically transmitted between vehicle to vehicle (V2V) and vehicle to road side unit (RSU). Safety being the prime concern in transportation, successful transmissions of beacon messages take the highest priority. A beacon message contains the information such as the velocity, the position and the direction of the vehicle. Beacon messages are transmitted in a broadcast fashion in the Dedicated Short Range Communication (DSRC) band on the highway system and they are exchanged with the nearby vehicles and RSU. On the basis of the beacon message, requirements of safety applications are rendered to the drivers to act accordingly in prevention of collision and other emergencies. Accordingly, priority in transmission must be given to beacon messages in order to ensure the quality of service (QoS) for various applications. Since beacon messages are broadcasted in DSRC, their MAC level performance is a critical issue to be addressed. Several researchers have studied the MAC-level performance of the beacon message dissemination in DSRC system (For simulation studies see ElBatt et al. [1], Torrent-Moreno [2], for analytical studies see Ma et al. [3], Yin et al. [4], Yin et al. [5]). In simulation studies, based on a network simulator, the throughput, end to end delay and number of hops by changing the transmission range of wireless units and the penetration rate of equipped vehicles are analysed. On the other hand, in analytical models, the MAC layer behavior of beacon message transmission and important performance metrics are evaluated by using Markov models. Ma et al. [3] have used Poisson process to approximate the periodic generations of beacon messages. Bastani et al. [6] developed an analytical model to investigate the dynamics of two reliability metrics, namely the probability of message reception for the event-driven warning messages and the variability of the inter-reception time (IRT) for periodic messages. They considered periodic generation of beacon messages and replacement of old messages with the new messages. Yin et al. [7] have observed that the transmission time of a beacon message varies depending on the channel access property and therefore the message generation cannot be periodic. They

conclude that the process of message generation and the transmission behavior are not independent. Taking this correlated property, Yin et al. [7] have developed an analytical model based on semi-Markov process to incorporate both periodic generations and cancelling old messages with the new beacon messages. Their model is in fact a set of interacting D/G/1/1LCFS queues, one for each vehicle. Based upon the model, they have obtained all the MAC-level performance metrics including mean delay, packet delivery ratio (PDR), packet reception ratio (PRR) and normalized channel throughput. In their model, they have assumed that the sojourn times of all the channel sensing states are deterministic. They justify their assumption by stating that extensive simulation studies indicate that randomness in sojourn times has little impact on the system performance measures. However, it is felt that the MAC protocol embraces randomness in the channel access mechanism. Accordingly, in this chapter, we extend the model of Yin et al. [7] by proposing that the sojourn times for some of the channel sensing states are random and obtain the various system performance measures.

II. IRREDUCIBLE SMP MODEL

We characterize the behavior of a tagged vehicle using the irreducible SMP model shown in Figure



The tagged vehicle is in *idle* state with probability u_b if there is no packet. After a packet is generated, the vehicle goes from state *idle* to state CS_1 with probability $1-u_b$, here the vehicle senses channel activity for DIFS time period. If the channel is detected not busy during this period (DIFS) (with probability $1-q_b$), the vehicle goes from *idle* state to

TX state, which means that a packet is transmitting. Otherwise, the vehicle will defer until channel is idle for DIFS duration represented by state D_{CS} . Such deference behavior for the tagged vehicle includes two parts: waiting for the current packet in the channel finishing transmission and waiting for subsequent transmissions if any from other neighbors within its receiving range. The self-loop for state D_{CS} represents the phenomena that the tagged vehicle waits for the current packet in the channel finishing transmission, and then senses the channel for DIFS time, which seizes the transmission from another vehicle and leads to further deference for backoff procedure. The probability that the tagged vehicle detects another neighbors transmission during DIFS time is denoted as r_b . If no other neighbors transmission is detected, the tagged vehicle will start backoff procedure and randomly choose a backoff counter in the range $[0, w-1]$, where $W = CW + 1$ is the backoff window size. The backoff counter will be decreased by one if the channel is detected to be idle for a time slot of duration σ (with probability $1 - p_b$), which is captured by the transition from state $W - i$ to state $W - i - 1$. If the channel is busy during a backoff time slot of duration σ (i.e., another vehicle starts to transmit a packet during this time slot), the backoff counter of the tagged vehicle will be suspended, which represented by the transition from state $W - i$ to D_{W-i-1} with probability p_b . Similar to state D_{CS} , state D_{W-i-1} also contains self-loop because other neighbors transmission can lead to further deference of the tagged vehicle. When the backoff counter reaches zero, the packet will directly be transmitted (an SMP transition occurs from state 0 to state *TX* with probability one). In *TX* state, a packet is transmitting. To capture the out-dated packet replacement behavior, which can happen during any state except *idle* state, we simplify the model by considering the total replacement probability and placing it after state *TX*. If the current packet has not been replaced by the next packet (with probability $1 - P_f$), the SMP goes to state *idle*. Otherwise, this current packet is out-dated and replaced by the next incoming packet. Such simplification is reasonable since the packet transmission delay is usually much smaller than the packet generation interval and hence the replacement occurs extremely rare. Next, the tagged vehicle starts the service for the next packet immediately and senses the channel for DIFS time (state CS_2). A new backoff procedure is started subsequently for the new packet instead of inheriting the backoff state of the old message. This is mainly because the out-dated message may finish the backoff procedure and is replaced during its transmission. The SMP model proposed here captures more detailed DCF behavior for periodic beacon message transmission by adding more states and self-loop structure. In addition, out-dated message replacement behavior is incorporated into the model by the

newly introduced model parameter P_f . The sojourn time in state j is defined as T_j . The mean and variance of T_j , in the SMP model are:

$$E[T_j] = \tau_j = \begin{cases} A_1 & j = TX \\ A_2 & j = idle \\ A_3 & j = CS_1, CS_2 \\ A_4 & j = D_{CS} \\ A_5 & j = D_0, D_1, D_2, \dots, D_{w-2} \\ 0 & j = 0 \\ \sigma & j = 1, 2, 3, \dots, W - 1 \end{cases} \quad (2.1)$$

$$Var[T_j] = \theta_j^2 = \begin{cases} 0 & j \in U(\text{ set of states except } idle) \\ B_1 & j = idle \end{cases} \quad (2.2)$$

where

$$\begin{cases} A_1 = \frac{PL}{R_d} + T_H \\ A_2 = \tau - E[S] \\ A_3 = DIFS \\ A_4 = \frac{(A_1 + DIFS)}{2} \\ A_5 = A_1 + DIFS \end{cases}$$

and

$$B_1 = var[S]$$

PL represents the packet length. R_d represents the data rate. Hence $\frac{PL}{R_d}$ is the time to transmit the packet. T_H

represents the time to transmit the packet header including physical layer header and MAC layer header. $E[S]$ and $Var[S]$ are the mean and variance of the overall message service time, which will be derived later. The sojourn time in state *idle* is the packet inter arrival time excluding the packet service time. In addition, to simplify the model, the sojourn times for channel sensing states ($CS_1, CS_2, \dots, W - 1$) are modeled as deterministic using the upper bound channel sensing time (i.e., the sensing for each state only performs once). Such simplification may have impact on dense network in which channel contentions are severe. Moreover, the sojourn time in state i is different from that in D_i ($i = 0, 1, \dots, W - 2$) is because the packet transmission from another vehicle may already started before the new packet is generated from the tagged vehicle. Therefore, on average, the tagged vehicle only defers for a half of the packet transmission time plus an additional idle DIFS duration in state D_{CS} . In contrast, for state D_i ($i = 0, 1, \dots, W - 2$), the start point of packet transmission from another vehicle is detected within

the backoff time slot (state $i + 1$), and hence the tagged vehicle needs to defer for the whole packet transmission time plus an additional idle DIFS duration. The embedded DTMC is solved for its steady-state probabilities for each state:

III. BALANCE EQUATIONS FOR THE STATES.

For the State D_{CS}

$$(1 - r_b)v_{D_{CS}} + v_{D_{CS}}r_b = v_{D_{CS}}r_b + v_{CS_1}q_b$$

therefore

$$v_{D_{CS}} = \frac{v_{CS_1}}{(1 - r_b)}q_b \tag{3.1}$$

For the State CS_1

$$v_{CS_1}q_b + v_{CS_1}(1 - q_b) = v_{idle} \cdot (1 - u_b)$$

therefore

$$v_{CS_1} = v_{idle} \cdot (1 - u_b) \tag{3.2}$$

For the idle state

$$v_{idle}(1 - u_b) + v_{idle}u_b = v_{TX}(1 - p_f)v_{idle}u_b$$

$$v_{idle} = v_{TX} \frac{(1 - p_f)}{(1 - u_b)} \tag{3.3}$$

For the state TX

$$v_{TX} = v_{CS_1}(1 - q_b) + v_0 \tag{3.4}$$

For the state CS_2

$$v_{CS_2} \cdot 1 = v_{TX}p_f$$

therefore

$$v_{CS_2} = v_{TX}p_f \tag{3.5}$$

For the states 0 to $(w-1)$

$$v_0 \cdot 1 = v_{D_{CS}} \left(\frac{1 - r_b}{w} \right) + v_{CS_2} \frac{1}{w} + v_1(1 - p_b) + v_{D_0}(1 - r_b)$$

.....

$$v_j = v_{D_{CS}} \left(\frac{1 - r_b}{w} \right) + v_{CS_2} \frac{1}{w} + v_{j+1}(1 - p_b) + v_{D_j}(1 - r_b) \tag{3.6}$$

.....

$$v_{w-2} = v_{D_{CS}} \left(\frac{1 - r_b}{w} \right) + v_{CS_2} \frac{1}{w} + v_{w-1}(1 - p_b) + v_{D_{w-2}}(1 - r_b)$$

$$v_{w-1} = v_{D_{CS}} \left(\frac{1 - r_b}{w} \right) + v_{CS_2} \frac{1}{w} \tag{3.7}$$

Substituting equation (3.7) in (3.6) we get

$$v_j = v_{w-1} + v_{j+1}(1 - p_b) + v_{D_j}(1 - r_b), j = 0, 1, 2, \dots, w - 2$$

$$v_j - v_{j+1} = v_{w-1} - p_b v_{j+1} + (1 - r_b)v_{D_j}, j = 0, 1, 2, \dots, w - 2 \tag{3.8}$$

Substituting $j = 0, 1, 2, \dots, (w - 2)$ in equation (3.8) and summing up we get

$$v_0 - v_{w-1} = (w - 1)v_{w-1} - p_b(v_1 + \dots + v_{w-1}) + (1 - r_b)(v_{D_0} + \dots + v_{D_{w-2}}) \tag{3.9}$$

For the states D_0 to D_{w-2}

$$(1 - r_b)v_{D_0} + r_b v_{D_0} = v_{D_0}r_b + v_1 p_b$$

$$(1 - r_b)(v_{D_0} + v_{D_1} + \dots + v_{D_{w-2}}) = (v_1 + v_2 + \dots + v_{w-1})p_b \tag{3.10}$$

$$v_{D_j} = \frac{[w - j - 1]p_b}{(1 - r_b)}v_{w-1}, j = 0, 1, 2, \dots, w - 2 \tag{3.11}$$

To express all the states in terms of v_{w-1} .

We know that the total Probability = 1, therefore

$$v_{D_{CS}} + v_{CS_1} + v_{idle} + v_{TX} + v_{CS_2} + \sum_{j=0}^{w-1} v_j + \sum_{j=0}^{w-2} v_{D_j} = 1 \tag{3.12}$$

To find $\sum_{j=0}^{w-1} v_j$

$$\sum_{j=0}^{w-1} v_j = \left[\frac{w(w + 1)}{2} \right] v_{w-1} \tag{3.13}$$

To find $\sum_{j=0}^{w-2} v_{D_j}$

$$\sum_{j=0}^{w-2} v_{D_j} = \frac{p_b}{(1 - r_b)} \left[\frac{w(w - 1)}{2} \right] v_{w-1} \tag{3.14}$$

To find v_{TX}

$$v_{TX} = \frac{wv_{w-1}}{p_f + q_b(1 - p_f)} \tag{3.15}$$

To find v_{CS_1}

$$v_{CS_1} = v_{idle}(1 - u_b)$$

$$v_{CS_1} = \frac{wv_{w-1}}{p_f + q_b(1 - p_f)}(1 - p_f) \tag{3.16}$$

To find v_{idle}

$$v_{idle} = v_{TX} \frac{(1 - p_f)}{(1 - u_b)}$$

$$v_{idle} = \frac{wv_{w-1}}{p_f + q_b(1 - p_f)} \frac{(1 - p_f)}{(1 - u_b)} \tag{3.17}$$

To find v_{CS_2}

$$v_{CS_2} = v_{TX}p_f = \frac{w \cdot p_f}{[p_f + q_b(1 - p_f)]} v_{w-1} \tag{3.18}$$

To find $v_{D_{CS}}$

$$v_{D_{CS}} = v_{CS_1} \frac{q_b}{(1-r_b)} = \frac{q_b(1-p_f)w}{[p_f + (1-q_b)](1-r_b)} v_{w-1} \quad (3.19)$$

Substituting equations (3.13), (3.14), (3.15), (3.16), (3.17), (3.18) and (3.19) in equation (3.12) we get

$$v_{w-1} \left[\frac{2(1-r_b)(1-u_b)[p_f + q_b(1-p_f)]}{2[(1-r_b)[2(1-u_b) + (1-p_f)] + (1-p_f)(1-u_b)q_b] + (1-u_b)[p_f + q_b(1-p_f)][(w+1)(1-r_b) + (w-1)p_b]w} \right] = 1 \quad (3.20)$$

For the states $0, 1, 2, \dots, n$, the steady state probabilities are

$$v_j = \Pr\{X = j\}$$

$$\pi_j = \frac{v_j \tau_j}{\sum v_j \tau_j} \quad (3.21)$$

PL represents the packet length. R_d represents the data rate. $\frac{PL}{R_d}$ is the time to transmit the packet. T_H represents the time to transmit the packet header including physical layer header and MAC layer.

$$A_1 = \frac{PL}{R_d} + T_H = E[TX] = \tau_{TX}$$

Calculation of $v\tau$

$$v_j \tau_j = (w-j)v_{w-1} \sigma, j = 0, 1, 2, \dots, w-1 \quad (3.22)$$

$$v_{D_j} \tau_{D_j} = \frac{(w-j-1)p_b}{(1-r_b)} v_{w-1} A_5, \quad (3.23)$$

$$j = D_0, D_1, D_2, \dots, D_{w-2}$$

$$v_{D_{CS}} \tau_{D_{CS}} = \frac{q_b(1-p_f)w}{[p_f + q_b(1-p_f)](1-r_b)} v_{w-1} A_4 \quad (3.24)$$

$$v_{CS_1} \tau_{CS_1} = \frac{(1-p_f)w}{[p_f + q_b(1-p_f)]} v_{w-1} A_3 \quad (3.25)$$

$$v_{idle} \tau_{idle} = \frac{wv_{w-1}}{p_f + q_b(1-p_f)} \frac{(1-p_f)}{(1-u_b)} A_2 \quad (3.26)$$

$$v_{TX} \tau_{TX} = \frac{w}{[p_f + q_b(1-p_f)]} v_{w-1} A_1 \quad (3.27)$$

$$v_{CS_2} \tau_{CS_2} = \frac{p_f w}{[p_f + q_b(1-p_f)]} v_{w-1} A_3 \quad (3.28)$$

$$\sum_{j=0}^{w-1} v_j \tau_j = \sum_{j=0}^{w-1} (w-j)v_{w-1} \sigma \quad (3.29)$$

$$\sum_{j=0}^{w-1} v_j \tau_j = \frac{w(w+1)}{2} v_{w-1} \sigma \quad (3.29)$$

$$\sum_{j=0}^{w-2} v_{D_j} \tau_{D_j} = \frac{p_b}{(1-r_b)} \frac{w(w-1)}{2} v_{w-1} A_5 \quad (3.30)$$

Summing up Eqns (3.24), (3.25), (3.26), (3.27), (3.28), (3.29) and (3.30), we get,

$$\sum_{\alpha \in \text{State Space}} v_\alpha \tau_\alpha = \frac{q_b(1-p_f)wv_{w-1}}{p_f + q_b(1-p_f)} A_4 + \frac{(1-p_f)wv_{w-1}}{p_f + q_b(1-p_f)(1-u_b)} A_2$$

$$+ \frac{wv_{w-1}}{p_f + q_b(1-p_f)} A_1 + \frac{p_f wv_{w-1}}{p_f + q_b(1-p_f)} A_3$$

$$+ \frac{(w+1)wv_{w-1}}{2} \sigma + \frac{(p_b)}{(1-r_b)} \frac{(w-1)wv_{w-1}}{2} A_5$$

$$\sum_{\alpha \in \text{State Space}} v_\alpha \tau_\alpha = \frac{q_b(1-p_f)wv_{w-1}}{p_f + q_b(1-p_f)} A_4 + \frac{(1-p_f)wv_{w-1}}{p_f + q_b(1-p_f)(1-u_b)} A_2$$

$$+ \frac{wv_{w-1}}{p_f + q_b(1-p_f)} A_1 + \frac{p_f wv_{w-1}}{p_f + q_b(1-p_f)} A_3$$

$$+ \frac{(w+1)wv_{w-1}}{2} \sigma + \frac{(p_b)}{(1-r_b)} \frac{(w-1)wv_{w-1}}{2} A_5$$

$$\pi_{TX} = \frac{2(1-r_b)(1-u_b)}{\Delta_1 + \Delta_2} \quad (3.31)$$

Where

$$\Delta_1 = 2[(1-u_b)(1-p_f)q_b A_4 + (1-r_b)(1-u_b)[A_1 + A_3] + (1-p_f)(1-r_b)A_2]$$

$$\Delta_2 = (1-u_b)[p_f + q_b(1-p_f)][(w+1)(1-r_b)\sigma + p_b(w-1)A_5]$$

IV. PERFORMANCE INDICES

MAC-level Performance Metrics

Mean Transmission Delay

One of the important performance indices is the mean transmission delay of the beacon message. Different from the mean service time that takes into account all of the packets generated, the mean transmission delay only accounts for the packets transmitted and exclude those that have been replaced. Let $E[D]$ be the mean transmission delay that is also the mean service time of transmitted packets. Since the service time for the packet that has been replaced is τ and the mean service time is given in equation

$$E[S] = (1-P_f) \cdot E[D] + P_f \cdot \tau \quad (4.1)$$

Therefore, the mean transmission delay is given by

$$E[D] = \frac{E[S] - P_f \cdot \tau}{1 - P_f} \quad (4.2)$$

Packet Delivery Ratio

The PDR is the probability that all vehicles in the tagged vehicle’s transmission range successfully receive the broadcast packet from the tagged vehicle .i.e.,

$$PDR = PDR_{cc} \cdot PDR_{ht} \tag{4.3}$$

where

PDR_{cc} :is the non-concurrent transmission probability, i.e., two packets do not start transmission in the same time slot. Since the sojourn time in state 1 is one time slot, π_1 is equivalent to the probability that a vehicle starts to transmit in a time slot. Hence

$$PDR_{cc} = \sum_{i=0}^{\infty} (1 - \pi_1)^i \frac{(N_{CS} - 1)^i}{i!} e^{-(N_{CS}-1)} = e^{-(N_{CS}-1)\pi_1} \tag{4.4}$$

PDR_{ht} : is the event that a transmission from hidden terminals collides with the tagged vehicle’s transmission which happens only when hidden terminals start to transmit during the vulnerable period of duration $2 \cdot A_1$ The probability that a vehicle starts to transmit during the vulnerable period of hidden terminal transmissions is $2 \cdot \pi_{TX}$ and is given by

$$PDR_{ht} = \sum_{i=0}^{\infty} (1 - 2 \cdot \pi_{TX})^i \frac{(N_{ph})^i}{i!} e^{-N_{ph}} = e^{-2 \cdot N_{ph} \cdot \pi_{TX}} \tag{4.5}$$

Packet Reception Ratio

PRR : is defined as the percentage of nodes that successfully receive a packet from the tagged node among the receivers being investigated (i.e., vehicles within the receiving range of the tagged node) at the moment that the packet is sent out. Considering the impacts from both concurrent transmissions

PRR_{cc} and hidden terminals problem PRR_{ht} we have

$$PRR = PRR_{cc} PRR_{ht} \tag{4.6}$$

where the impact of the concurrent transmission is given by

$$PRR_{cc} = \frac{e^{-\beta R \pi_1}}{\beta R \pi_1} (1 - e^{-\beta R \pi_1})$$

and the impact of the hidden

terminal is

$$PRR_{ht} = \frac{1}{RC} (1 - e^{-RC}) \text{ where } C = 2\beta \cdot \pi_{TX}$$

Normalized Channel Throughput

The normalized channel throughput is the ratio of the time used for successful transmitted packets (i.e., the packet is received by all vehicles within the tagged vehicle’s transmission range) and the entire time. From a channel’s perspective, during one packet generation interval τ , $\$N_{tr}$ packets are generated in total for all vehicles sensing this channel, one packet from each vehicle. The transmission time for each packet is $\frac{PL}{R_d}$ (PL represents the

packet length and R_d represents the data rate). Hence, the normalized channel throughput is computed as

$$s = \frac{N_{tr}}{\tau} \cdot \frac{PL}{R_d} \cdot PDR$$

Conclusion

In this paper, considering the mathematical analysis for each state of the semi markov process and the behaviour of the beacon message contending for the channel resource, and also the steady state probability for the vehicle in transaction state, modelled by yin[7], an analytic model is developed to characterize the periodic beacon message dissemination in DSRC for highway safety communications. Instead of assuming Poisson arrivals and infinite queue as in most of the literature, the periodic packet generation and out-dated information replacement are taken into consideration. The service time distribution using SMP model is evaluated.

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