A Simultaneous Evaluation of Adaptive Design Parameters Policies for Hotelling's T^2 Charts

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Abstract

A *complectly adaptive* (CA) Hotelling's T^2 chart, that is a T^2 chart in which all the design parameters, viz, sampling interval, sample size, control limit, and warning limit are adaptive, each taking two values, is developed. The expressions for the statistical and operational performance measures for this chart are derived using a Markov chain approach. As any adaptive T^2 chart in which one or more of the design parameters are adaptive, each taking two values, is a particular case of the CA T^2 chart, the derived expressions are directly applicable to all such charts.These expressions can be used tocompute the performancemeasures for all such charts and thus to determine the most suitable adaptive T^2 chart for a given situation.

Key words:Average number of samples to signal, average number of observations to signal, average number of switches to signal, Multivariate Statistical process control, Steady-state average time to signal.

I. INTRODUCTION

Hotelling's T^2 chart is an effective on-line process control technique used to monitor simulataneouslytwo or more quality characteristics of a process. If all the design parameters of this chart are kept fixed throughout the period of monoitoring, it is called static T^2 chart while if at least one design parameter is variableand takes a value for a trial according to the location(s) of the sample points corresponding to the previous trial(s), the chart is called adaptive T^2 chart. The general principal of choosing values of the adaptive parameters for atrial is as follows. If the last plotted point(s) indicate possibility of a shift, choose short sampling interval and/or large sample size and/or narrow in-control limits for the next trial. On the other hand, if that indicate possibility of safe or in-control region, choose long sampling interval and/or small sample size and/or wide in-control region for the next trial. It has been shown that adapting one or more design parameters of a T^2 chart increases its statistical, operational, and economic performances significantly. See, for example,Aparasi[1], Aparasi andHaro[2, 3],Faraz and Moghadam[4],Mahadik and Shirke[5], Mahadik[6-10].

Recently, Mahadik[11]has developed a*complectly adaptive* (CA) $\bar{X}$ chart, that is an $\bar{X}$ chartin which all the design parameters, viz, sampling interval, sample size, control limits, and warning limits are adaptive, each taking two values. This idea has been extended for T^2 chart in this paper.

The following sections present the general description of a CA T^2 chart,derivations of the expressions for statistical and operationalperformance measures for this chart, numerical comparisons of the performances of various adaptive T^2 charts that are the particular cases of the CA T^2 chart,andconclusions.

II. A CA T^2 CHART

Suppose the $p > 1$ related quality characteristics $\boldsymbol{X} = (X_1, X_2, ..., X_p)'$ to be monitored together, follow p-variate normal distribution with mean vector $\boldsymbol{\mu}$ and known variance covariance matrix Σ. Let $\boldsymbol{\mu_0}$ be

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the target mean vector. An occurrence of an assignable cause shifts $\boldsymbol{\mu}$ from $\boldsymbol{\mu_0}$ to $\boldsymbol{\mu_1} \neq \boldsymbol{\mu_0}$. A CA$T^2$ chart to monitor $\boldsymbol{\mu}$ is as described below.

The control statistic is $T_i^2 = n(i)(\overline{\boldsymbol{X}}_i - \boldsymbol{\mu_0})'\Sigma^{-1}(\overline{\boldsymbol{X}}_i - \boldsymbol{\mu_0})$, where $\overline{\boldsymbol{X}}_i$, i = 1, 2, …, is the mean vector of the i^{th} sample of size $n(i)$ drawn on $\boldsymbol{X}$. Note that when $\boldsymbol{\mu} = \boldsymbol{\mu_0}$, T_i^2 follows central chi-square distribution with p degrees of freedom, and when $\boldsymbol{\mu} = \boldsymbol{\mu_1}$, for given $n(i) = n$, it follows non-central chi-square distribution with p degrees of freedom and non-centrality parameter $n(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\Sigma^{-1}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0}) = nd^2$, where $d = \sqrt{(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\Sigma^{-1}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})}$ is the Mahalanobis distance used to measure a change in the process mean vector. Let $t(i)$ be the length of sampling interval between the $(i-1)^{st}$ and i^{th}trials, i = 1, 2, …. Also, let $L(i)$ and $w(i)$ be the control and warning limits,respectively,for the i^{th} trial.

The values of ($t(i)$, $n(i)$,$L(i)$, $w(i)$) can be either (t_1, n_1, L_1, w_1) or (t_2, n_2, L_2, w_2), where t_1, t_2, n_1, n_2, L_1, L_2, w_1, and w_2 are such that $t_{\max} \geq t_1 \geq t_2 \geq t_{\min}$, $n_{\min} \leq n_1 \leq n_2 \leq n_{\max}$, $\infty > L_1 \geq L_2 > 0$, $0 < w_1 < L_1$, $0 < w_2 < L_2$, and $w_1 \geq w_2$.Here, $t_{\max}$ and $t_{\min}$ being the longest and shortest possible sampling interval lengths, respectively, while, $n_{\min}$ and $n_{\max}$ being the smallest and largest possible sample sizes, respectively,

When T_{i-1}^2 falls below$L(i-1)$, the values of ($t(i)$, $n(i)$, $L(i)$, $w(i)$), i = 2, 3, …, between (t_1, n_1, L_1, w_1) and (t_2, n_2, L_2, w_2) are chosen according to the following rule.

$$(t(i), n(i), L(i), w(i)) = \begin{cases} (t_1, n_1, L_1, w_1), & \text{if } T_{i-1}^2 \in I_1(i-1) \\ (t_2, n_2, L_2, w_2), & \text{if } T_{i-1}^2 \in I_2(i-1), \end{cases}$$

where

$$I_1(i-1) = [0, w(i-1)]$$

$$\text{and } I_2(i-1) = (w(i-1), L(i-1).$$

The chart signals an out-of-control stateat the i^{th} trial, i = 1, 2, …, if T_i^2 falls above $L(i)$.

The values of ($t(1)$, $n(1)$, $L(1)$, $w(1)$) can be chosen using an arbitrary probability distribution. In practice, it is recommended to choose the quadruplet(t_2, n_2, L_2, w_2) for that to provide additional protection against the problems that may exist initially. The trial following an out-of-control signal is again treated to be the first trial.

In the next section, expressions for performance measures for a CAT^2 chart are derived.

III. PERFORMANCE MEASURES

The measures of statistical performance of a CA T^2chart are *steady-state average time to signal* (SSATS),*average number of samples to signal* (ANSS), and *average number of observations to signal* (ANOS). SSATS is the expected value of the time between a shift that occurs at some random time after the process starts and the time the chart signals. ANSS and ANOS are the expected values of the number of samples and the number of observations, respectively taken from the time of a shift to the time the chart signals.Themeasure of operational performance is *average number of switches to signal* (ANSW), which is the expected value of the number of switches between the quadruplets of values of sampling interval length, sample size, control limit, and warning limit from a shift to the signal.

Let SSATS$_d$, ANSS$_d$, ANOS$_d$, and ANSW$_d$be the SSATS, ANSS, ANOS, and ANSW, respectively of a T^2chart when the process mean vector has shifted from $\boldsymbol{\mu_0}$ to $\boldsymbol{\mu_1}$in d units. In the following,the expressions for SSATS$_d$, ANSS$_d$, and ANOS$_d$are derived using a Markov chain approach.

Henceforth,thei^{th} trial refers to the i^{th} trial after a shift when $i > 0$ and the last trial before the shift when $i = 0$.Also, T_i^2 refers to the sample point corresponding to the i^{th} trial.

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Define the three states 1, 2, and 3 of a Markov chain corresponding to whether the sample pointcorresponding to the i^{th} trial is plotted in $I_1(i)$, $I_2(i)$, and $I_3(i)$ = [$L(i)$, ∞), respectively,i = 1, 2, ….. Note that state 3 is the absorbing state, as the control charting process is restarted when a sample point falls in region $I_3(i)$. The transition probability matrix is given by

$$\mathbf{P^d} = \begin{bmatrix} p_{11}^d & p_{12}^d & p_{13}^d \\ p_{21}^d & p_{22}^d & p_{23}^d \\ 0 & 0 & 1 \end{bmatrix},$$

where p_{jk}^d is the transition probability that j is the prior state and k is the current state, when the process mean vector has shifted by d units. For example,

$$p_{12}^d = \Pr_d[T_i^2 \in I_2(i) \mid T_{i-1}^2 \in I_1(i-1)]$$

$$= \Pr_d[T_i^2 \in I_2(i) \mid n(i) = n_1, L(i) = L_1, w(i) = w_1]$$

$$= \mathrm{F}_{\lambda_1}(L_1) - \mathrm{F}_{\lambda_1}(w_1)$$

where $\mathrm{F}_{\lambda_1}(\cdot)$ is the cumulative distribution function of non-central chi-square distribution with p degrees of freedom and non-centrality parameter $\lambda_1 = n_1 d^2$.

Then, $\mathrm{SSATS_d}$ and $\mathrm{ANSS_d}$ are given by

$$\mathrm{SSATS_d} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{t} - \mathrm{E}(U), \quad (1)$$

$$\mathrm{ANSS_d} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\mathbf{1},$$

and

$$\mathrm{ANOS_d} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{n},$$

where $\mathbf{I}$ is the identity matrix of order 2, $\mathbf{P_1^d}$ is the submatrix of $\mathbf{P^d}$ that contains the probabilities associated with the transient states only, $\boldsymbol{t}' = (t_1, t_2)$, $\mathbf{1}' = (1, 1)$, $\boldsymbol{n}' = (n_1, n_2)$, and $\boldsymbol{b}' = (b_1, b_2)$, b_j being the conditional probability that T_0^2 falls in $I_j(0)$ given that it falls below $L(0)$, j = 1, 2. We note that $b_2 = 1 - b_1$. The Expression for b_1 is derived by Mahadik[7] and is as given below.

$$b_1 = \frac{\dfrac{\mathrm{F}_0(w_2)}{\mathrm{F}_0(L_2)}}{1 - \dfrac{\mathrm{F}_0(w_1)}{\mathrm{F}_0(L_1)} + \dfrac{\mathrm{F}_0(w_2)}{\mathrm{F}_0(L_2)}},$$

where $\mathrm{F}_0(.)$ is the cumulative distribution function of central chi-square distribution with p degrees of freedom.

E(U) in equation (1) is the expected value of the time U between the 0^{th} trial and the shift. Assuming that an assignable cause of a process shift occurs according to a Poisson process, it can be shown that E(U) = $\mathrm{E}[t(1)]/2 = \boldsymbol{b}'\boldsymbol{t}/2$. Hence,

$$\mathrm{SSATS_d} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P_1^d})^{-1}\boldsymbol{t} - \boldsymbol{b}'\boldsymbol{t}/2.$$

Now, to derive the expression for $\mathrm{ANSW_d}$, let

$$Y_i = \begin{cases} 1, & \text{if } (T_{i-1}^2 \in I_1(i-1), T_i^2 \in I_2(i)) \\ 2, & \text{if } (T_{i-1}^2 \in I_2(i-1), T_i^2 \in I_1(i)) \\ 3, & \text{if } (T_{i-1}^2 \in I_1(i-1), T_i^2 \in I_1(i)) \\ 4, & \text{if } (T_{i-1}^2 \in I_2(i-1), T_i^2 \in I_2(i)) \\ 5, & \text{if } T_i^2 > L(i) \end{cases},$$

i = 1, 2, …

Note that { Y_i, i = 1, 2, … } is a Markov chain with transition probability matrix

$$\mathbf{Q^d} = \begin{bmatrix} 0 & p_{21}^d & 0 & p_{22}^d & p_{23}^d \\ p_{12}^d & 0 & p_{11}^d & 0 & p_{13}^d \\ p_{12}^d & 0 & p_{11}^d & 0 & p_{13}^d \\ 0 & p_{21}^d & 0 & p_{22}^d & p_{23}^d \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, $\mathrm{ANSW_d}$ is given by

$$\mathrm{ANSW_d} = \boldsymbol{a}'(\mathbf{I_1} - \mathbf{Q_1^d})^{-1}\boldsymbol{e},$$

where, $\mathbf{I_1}$ is the identity matrix of order 4, $\mathbf{Q_1^d}$ is the submatrix of $\mathbf{Q^d}$ that contains the probabilities associated with the transient states only, $\boldsymbol{e}$ = $(1, 1, 0, 0)'$,and $\boldsymbol{a}$ = $(a_1, a_2, a_3, a_4)'$, a_j being the initial probability of state j, j = 1, 2, 3, 4, given by

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$$a_j = \Pr_d[Y_1 = j] = \begin{cases} b_1 p_{12}^d \ , \ j=1 \\ b_2 p_{21}^d \ , \ j=2 \\ b_1 p_{11}^d \ , \ j=3 \\ b_2 p_{22}^d \ , \ j=4 \end{cases}.$$

In the next section, the above derieved expressions are used to compute the performance measures for a CA T^2 chart and for all the adaptive T^2 charts which are its particular cases.

IV. PERFORMANCE COMPARISON OF THE ADAPTIVE T^2 CHARTS

In this section, we simultaneously evaluate a CA T^2 chart and its all particular cases through numerical comparisons of their statistical and operational performances. For this, we have to design all these chart such that their in-control statistical performances match. Below is described the procedure of designing a CA T^2 chart whose in-control statistical performances match to that of a given static T^2 chart. Note that this procedure is also applicable to design all the charts, which are particular cases of a CA T^2 chart,such that their in-control statistical performances match to that of the given static T^2 chart.

Let t_0, n_0, and L_0 be the sampling interval length, sample size, and control limit of a static T^2 chart. Let SSATS$_0$(static), ANSS$_0$(static), and ANOS$_0$(static) be the in-control SSATS, ANSS, and ANOS, respectively of this chart. Then, we have

$$\text{SSATS}_0(\text{static}) = t_0\left(\frac{1}{1-\text{F}_0(-L_0)} - \frac{1}{2}\right)$$

$$\text{ANSS}_0(\text{static}) = \frac{1}{1-\text{F}_0(-L_0)}$$

and

$$\text{ANOS}_0(\text{static}) = \frac{n_0}{1-\text{F}_0(-L_0)}$$

Now, given t_0, n_0,and L_0, we have to choose the design parameters of a CA T^2 chart satisfying the following requirements.

$$\text{SSATS}_0(\text{CA}) = \text{SSATS}_0(\text{static})$$

$$\text{ANSS}_0(\text{CA}) = \text{ANSS}_0(\text{static})$$

and

$$\text{ANOS}_0(\text{CA}) = \text{ANOS}_0(\text{static})$$

Here,SSATS$_0$(CA), ANSS$_0$(CA), and ANOS$_0$(CA) are the in-control SSATS, ANSS, and ANOS, respectively of a CA T^2 chart.

Fixing any five among the design parameters (t_1, t_2, n_1, n_2, L_1, L_2, w_1, and w_2) of a CA T^2 chart, the above nonlinear equations can be solved for the remaining three parameters. This can be done, for example, using package rootSolve in R or using function 'fsolve' in Matlab.

The complete set of adaptive T^2 charts containing a CA T^2 chart and its all particular cases includes:

1. A *variable sampling interval* (VSI) T^2 chart
2. A *variable sample size* (VSS) T^2 chart
3. A *variable control limits* (VCL) T^2 chart
4. A *variable sample size and sampling interval* (VSSI) T^2 chart
5. A *variable sampling interval and control limits* (VSICL) T^2 chart
6. A *variable sampling interval and warning limits* (VSIWL) T^2 chart
7. A *variable sample size and control limits* (VSSCL) T^2 chart
8. A *variable sample size and warning limits* (VSSWL) T^2 chart
9. A *variable control limits and warning limits* (VCWL) T^2 chart
10. A *variable sample size, sampling interval, and control limits* (VSSICL) T^2 chart
11. A *variable sample size, sampling interval, and warning limits* VSSIWL) T^2 chart

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12. A *variable sampling interval, control limits, and warning limits* (VSICWL) T^2 chart
13. A *variable sample size, control limits, and warning limits* (VSSCWL) T^2 chart
14. A CA T^2 chart

Fixing the values of t_0, n_0, and L_0 and applying the procedure described above, all the charts in above set can be designed such that their in-control statistical performances match. Table 1 shows the design parameters of one suchset of matched charts while tables 2, 3, 4, and 5, respectively show the $SSATS_d$, $ANSS_d$,$ANOS_d$,and $ANSW_d$ performancesfor these charts for various values of*d*.Such tables are useful to determine the most suitable adaptive T^2 chart for a given situation. In general, one can see from tables 2 to 5 that CA T^2 chart is the best choice if one is interested in detecting only small shifts while VSI or VSIWL T^2 chartsare the best choices if the interest is in detecting only moderate to large shifts. However, in practice, one can choose the most suitable charts taking into consideration the practical constraints in deciding which of the design parameters of the charts can be adaptive.

V. CONCLUSIONS

The expressions for the statistical and operational performance measures for a CA T^2 chart are developed. These expressions are are directly applicable to any adaptive T^2 chart in which any of the design parameters are adaptive, each taking two values. The simultaneous numerical comparisons of the performances of all such charts indicate that in general, CA T^2 chart is the best chart for detecting small shifts while VSI or VSIWL T^2 charts are the best charts for detecting moderate to large shifts. In practice, such simultaneous comparisons guide to determine the most suitable T^2 chart satisfying the practical constraints in deciding which of the design parameters of the chart can be adaptive.

Table 1: Design parameters of the matched T^2 charts

Chart	Design Parameters							
	n_1	n_2	t_1	t_2	w_1	w_2	L_1	L_2
Static	5	5	1.00	1.00	14.86	14.86	0.00	0.00
VSI	5	5	1.79	0.20	14.86	14.86	3.36	3.36
VSS	2	10	1.00	1.00	14.86	14.86	4.21	4.21
VCL	5	5	1.00	1.00	16.42	13.93	3.36	3.36
VSSI	2	10	1.48	0.20	14.86	14.86	4.21	4.21
VSICL	5	5	1.79	0.20	16.42	13.93	3.36	3.36
VSIWL	5	5	1.79	0.20	14.86	14.86	4.03	2.75
VSSCL	2	10	1.00	1.00	17.35	13.15	4.21	4.21
VSSWL	2	10	1.00	1.00	14.86	14.86	4.89	3.30
VCWL	5	5	1.00	1.00	16.42	13.93	4.04	2.75
VSSICL	2	10	1.48	0.20	16.42	13.48	4.21	4.21
VSSIWL	2	10	1.48	0.20	14.86	14.86	4.66	3.57
VSICWL	5	5	1.56	0.20	16.42	13.63	4.04	3.78
VSSCWL	2	10	1.00	1.00	16.42	13.48	4.76	3.45
CA	2	10	1.48	0.20	16.42	13.48	4.88	3.30

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Table 2: $SSATS_d$ values for the matched T^2 charts

Chart	d							
	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
Static	126.73	48.69	17.57	6.95	1.63	0.69	0.52	0.50
VSI	118.98	38.10	10.52	3.21	0.78	0.54	0.50	0.50
VSS	120.47	33.12	8.26	3.11	1.31	0.99	0.80	0.64
VCL	123.45	44.86	15.53	6.12	1.56	0.70	0.52	0.50
VSSI	114.09	26.14	5.01	1.76	0.87	0.67	0.58	0.53
VSICL	115.93	35.15	9.37	2.89	0.77	0.54	0.50	0.50
VSIWL	117.42	36.23	9.61	2.93	0.77	0.54	0.50	0.50
VSSCL	96.98	23.09	6.24	2.66	1.31	1.05	0.88	0.71
VSSWL	119.78	31.97	8.07	3.18	1.36	1.00	0.80	0.64
VCWL	122.58	44.08	15.24	6.05	1.56	0.70	0.52	0.50
VSSICL	96.27	19.74	4.17	1.65	0.87	0.67	0.59	0.54
VSSIWL	112.70	24.66	4.75	1.83	0.93	0.69	0.58	0.53
VSICWL	115.80	35.12	9.42	2.93	0.77	0.54	0.50	0.50
VSSCWL	100.34	23.87	6.45	2.80	1.35	1.05	0.85	0.68
CA	93.85	18.15	3.99	1.78	0.96	0.71	0.60	0.54

Table 3: $ANSS_d$ values for the matched T^2 charts

Chart	d								
	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
Static	200	127.23	49.19	18.07	7.45	2.13	1.19	1.02	1.00
VSI	200	127.23	49.19	18.07	7.45	2.13	1.19	1.02	1.00
VSS	200	120.97	33.62	8.76	3.61	1.81	1.49	1.30	1.14
VCL	200	123.95	45.36	16.03	6.62	2.06	1.20	1.02	1.00
VSSI	200	120.97	33.62	8.76	3.61	1.81	1.49	1.30	1.14
VSICL	200	123.95	45.36	16.03	6.62	2.06	1.20	1.02	1.00
VSIWL	200	127.23	49.19	18.07	7.45	2.13	1.19	1.02	1.00
VSSCL	200	97.48	23.59	6.74	3.16	1.81	1.55	1.38	1.21
VSSWL	200	120.28	32.47	8.57	3.68	1.86	1.50	1.30	1.14
VCWL	200	123.08	44.58	15.74	6.55	2.06	1.20	1.02	1.00
VSSICL	200	102.01	25.22	7.06	3.23	1.81	1.53	1.35	1.18
VSSIWL	200	120.50	32.81	8.61	3.65	1.84	1.50	1.30	1.14
VSICWL	200	123.12	44.42	15.54	6.42	2.04	1.21	1.02	1.00
VSSCWL	200	100.84	24.37	6.95	3.30	1.85	1.55	1.35	1.18
CA	200	100.61	24.22	6.94	3.32	1.86	1.55	1.35	1.18

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Table 4: $ANOS_d$ values for the matched T^2 charts

Chart	d								
	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
Static	1000	636.15	245.97	90.35	37.25	10.67	5.97	5.10	5.00
VSI	1000	636.15	245.97	90.35	37.25	10.67	5.97	5.10	5.00
VSS	1000	644.70	211.70	64.20	26.50	11.80	9.40	7.90	6.40
VCL	1000	619.77	226.82	80.16	33.12	10.30	6.02	5.12	5.01
VSSI	1000	644.70	211.70	64.20	26.50	11.80	9.40	7.90	6.40
VSICL	1000	636.15	226.80	80.20	33.10	10.30	6.00	5.10	5.00
VSIWL	1000	636.20	246.00	90.40	37.30	10.70	6.00	5.10	5.00
VSSCL	1000	518.50	146.70	47.60	22.20	11.80	10.10	8.70	7.10
VSSWL	1000	649.60	211.87	63.96	26.56	11.82	9.43	7.83	6.37
VCWL	1000	615.40	222.90	78.70	32.80	10.30	6.00	5.10	5.00
VSSICL	1000	542.80	157.30	50.20	22.90	11.70	9.90	8.40	6.80
VSSIWL	1000	648.07	211.82	64.00	26.52	11.81	9.43	7.84	6.38
VSICWL	1000	615.60	222.10	77.70	32.10	10.20	6.00	5.10	5.00
VSSCWL	1000	542.27	156.11	50.06	22.99	11.79	9.89	8.37	6.79
CA	1000	542.16	155.91	50.05	23.03	11.81	9.89	8.37	6.79

Table 5: $ANSW_d$ values for the matched T^2 charts

Chart	d								
	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
Static	0	0	0	0	0	0	0	0	0
VSI	99.50	62.79	22.36	6.41	1.77	0.30	0.08	0.01	0.00
VSS	93.28	54.72	13.01	2.36	0.79	0.54	0.43	0.28	0.14
VCL	99.75	61.39	20.77	5.80	1.66	0.34	0.11	0.02	0.00
VSSI	93.28	54.72	13.01	2.36	0.79	0.54	0.43	0.28	0.14
VSICL	99.75	61.39	20.77	5.80	1.66	0.34	0.11	0.02	0.00
VSIWL	79.75	50.50	17.86	4.97	1.36	0.28	0.08	0.01	0.00
VSSCL	93.70	44.30	9.22	1.91	0.78	0.58	0.50	0.37	0.21
VSSWL	73.54	42.56	9.46	1.69	0.69	0.53	0.43	0.28	0.14
VCWL	63.50	40.86	14.51	3.80	0.96	0.20	0.05	0.01	0.00
VSSICL	93.59	46.30	9.83	1.98	0.78	0.57	0.48	0.34	0.18
VSSIWL	79.80	46.37	10.52	1.88	0.72	0.53	0.43	0.28	0.14
VSICWL	93.22	58.61	20.88	6.21	1.89	0.40	0.13	0.02	0.00
VSSCWL	77.39	37.60	7.58	1.54	0.71	0.56	0.47	0.33	0.18
CA	74.06	35.86	7.16	1.47	0.69	0.56	0.47	0.33	0.18

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