# DESIGN OF DISCRETE PID x (N-2) STAGE PD CASCADE CONTROLLER FOR TYPE1, 5TH ORDER SISO SYSTEMS

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## ABSTRACT:

This paper presents a design technique for the control system using $PID \times (n-2)$ stage PD as a cascade controller for a SISO system of $n^{th}$ order plants. Specifications based on root locus approach. This controller can be used instead of a conventional PID controller for the higher order plant to obtain better performances. By this design technique, the controlled system is approximated as a second order system, and the desired performances are satisfied. The closed-loop poles are located on the left half of the plane.

The Nth order plant to be controlled by using the standard PID and PD to be cascaded by (n-2) stage controller is designed by using the root locus technique. in this project to design the higher order plant. This paper presents a design technique for the control system using $PID \times (n-2)$ stage PD as a cascade SISO systems in this proposal method is to increase the type and order of the plant in this method good performance.

Keywords-PID controller, root locus technique

### 1. INTRODUCTION

Feed back control system may be classified according to their ability to fallow the step input, ramp inputs and so on. consider the following open loop transfer function

G(S)H(S):

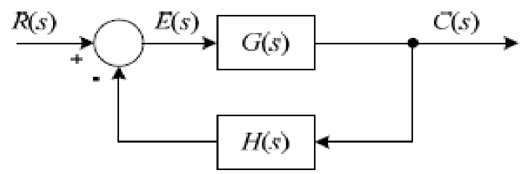


Fig. 1 Closed-loop system

Consider the following open loop transfer function :

$$G(s)H(s) = \frac{k(T_a s+1)(T_b s+1)...(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)...(T_p s+1)}$$

It involves the term $s^N$ in the denominator, representing a pole of multiplicity N at the origin. So this classification is based on the number of integrations indicated by the open loop transfer function. If N=0,N=1,N=2,……a system is called type0 type1,type2,….,respectively. The PID (Proportional –Integral-Derivative) is widely used by applying the well-known ziegler Nichlos tuning method. It is clear that the pid controller is properly applied in the typical second order plant. But it is quite difficult to use only the PID controller for the 3rd or higher order plant because the order of the plant is greater than the number of zeros provided by the PID controllers. The standing PID and pd to be connected in cascaded by (n-2) stage controllers transfer function Kpid(s), Kpd(s) and their equivalent form for the Nth order plant are assumed to be given below.



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$$K\,pid(s)=K\,p\left(1+\frac{1}{T_1 S}+T_D S\right)$$

$$k_{pid}(s)=k_{pid}\ \frac{(s+z_1)(s+z_2)}{s}$$

$$k_{pid}\ \frac{(s+z_{pid})^2}{s}$$

$$k_{pd}(s)=k_p(1+T_D s),$$
$$k_{pd}(s)=k_{pd}(s+z_{pd}),$$

Where

- $K_p$ is the proportional gain.
- $T_1$ is the integral time.
- $T_d$ is the derivative time.

PID controller (n-2) PD

$$\frac{k_{(n-2)}(s)G_n(s)=k_{pid}(s+z_1)(s+z_2)\times k_{pd}(s+z_{pd}).}{s(s+p_1)(s+p_2)....(s+p_p)}$$

Where, K= $K_{pid}$ $K_{pd}$ …. Kn then

$$K_{(n-2)}(s)G_n(s)=K\frac{(s+z_1)(s+z_2)\times(s+z_{pd}).....}{s(s+p_1)(s+p_2)....(s+p_p)}$$
=G(s)H(s)

The characteristic equation is

F(s )= 1+G(s)H(s)=0

Since G(s)H(s) is a complex quantity can be split into two equations are as follows

in this angle condition is:

$\angle G(s)H(s)=\pm(2k+1)\pi$ , k=0,1,2…….

In this magnitude condition is:

$$|G(S)H(S)|=1$$

The basic idea behind the root locus method is that the value of s that make the transfer function around the loop equal -1 must be satisfy the characteristic equation of the system.

## 2. METHODOLGY

### 2.1.DESIGN FOR CONTINUOUS TIME SYSTEMS

The statement of PID×(n-2) stage PD cascade controller design problem is to find out the locations of their zeros that the desired specifications are to be achieved.

Where as, the desired specifications to be designed are usually specified in terms of transient response and steady state response characteristics of a control system to a unit step input exhibited by a pair of complex-conjugate dominant closed loop poles sd as follows:

$$\left.\begin{aligned}&\text{Percent Overshoot }(P.O.)=e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)}\times 100\%,\\&\text{Settling Time }(t_s)=\frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_n},(\pm 2\%).\end{aligned}\right\}$$

Associate professor Dr. kitti tirasesth proposed the simplest way to determines all locations of

These zeros known as kitti's method. In this method easily find out the zeros

This method can be applied to the type 0,3rd order plant

$$G(s)=\frac{15}{(s+1)(s+3)(s+5)(s+7)}$$

With the given specifications are peek over shoot $\leq 5$ %, $ts(\pm 2\%)\leq 1$ sec and $e_{ss}(t)=0$ As the following steps:

Step1: find the damping ratio from the given percent over shoot

$$\zeta=\sqrt{\frac{\left[\ln\left(\frac{p.o}{100}\right)\right]}{\pi^2+\left[\ln\left(\frac{p.o.}{100}\right)\right]^2}}\ =0.69$$



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Step2: find the un damped natural frequency $\omega$ n from the given of the settling time

$$ts(\pm 2\%) = -\ln \left(0.02\sqrt{1-\zeta^2}\right) \Big/ \omega n\zeta$$

Step3: Find the location of dominant closed loop pole

= - 4.235+j4.442

here, the open loop transfer function with (n-2) stage cascade PD controller is

$$K(n-2)(s)Gn(s) = 15KpidKs + d\,\frac{(s+z_1)(s+z_2)\times(s+z_{pd})}{s(s+1)(s+3)(s+5)(s+7)}$$

Step4: Mark the location of Sd first, the locate all poles and (n-1) zeros of the on the s plane $K_{(n-1)}(s)G_n(s)$

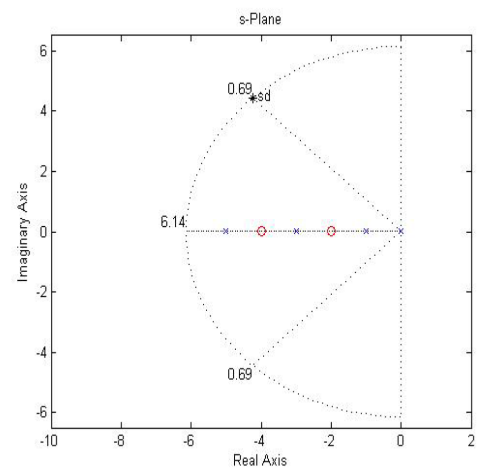


Fig.2 pole-zero map

Step5: find sum of the angles at Sd with all of the open loop poles and the (n-1) zeros of by graphical

Step7:determine the gain $k_{pid}k_{pd}$ at Sd from(7)

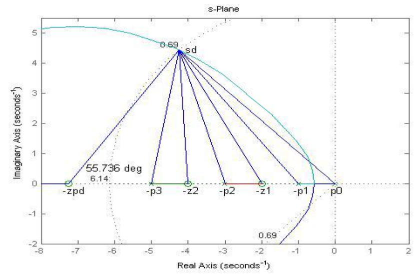


Fig3.Determination of the angle of PD,s zero

$$\theta_{p0} = \angle(s_d) = 133.639^\circ$$
$$\theta_{p1} = \angle(s_d + p_1) = \angle(s_d + p_1) = 126.071^\circ$$
$$\theta_{p2} = \angle(s_d + p_2) = \angle(s_d + p_2) = 15.543^\circ$$
$$\theta_{p3} = \angle(s_d + p_3) = \angle(s_d + p_3) = 80.232^\circ$$
$$\theta_{p4} = \angle(s_d + p_4) = \angle(s_d + p_4) = 58.099^\circ$$
$$\Sigma\theta p = \theta_{p0} + \theta_{p1} + \theta_{p3} + \theta_{p4} = 504.584^\circ$$
$$\theta_{z1} = \angle(s_d + z_1) = \angle(s_d + z_1) = \angle(s_d + 2) = 116.715^\circ$$
$$\theta_{z2} = \angle(s_d + z_2) = \angle(s_d + z_2) = \angle(s_d + 4) = 93.034^\circ$$
$$_{z3} = \angle(s_d + z_3) = \angle(s_d + z_3) = \angle(s_d + 6) = 68.329^\circ$$
$$\theta_{z_{pd}} = \angle(s_d + z_{pd}) = ?^\circ$$
$$\Sigma\theta_z = \theta_{z_1} + \theta_{z_2} + \theta_{z3} + \theta_{z_{pd}} = 277.783^\circ + \theta_{z_{pd}}$$
$$\Sigma\theta_z - \Sigma\theta_p = (277.783^\circ + \theta_{z_{pd}}) - (504.584^\circ)$$
$$-225.801 + \theta_{z_{pd}} = \pm 180^\circ(2k+1), k = 0,1,2.....$$
$$\therefore \theta_{z_{pd}} = 45.801^\circ$$

Step6: determine the location of the zero $(s + z_{pd})$ using the $\theta_{z\,pd} = \angle(s_d + z_{pd})$ angle $=45.801^0$ determination of the locations of PD's of zeros

$$\tan\theta = \frac{opposite}{adjiacent} = \frac{|\mathrm{Im}(s_d)|}{x}, x = \frac{|\mathrm{Im}(s_d)|}{\tan(\theta_{z.pd})}$$

$$z_{pd} = |\mathrm{Re}(s_d)| + x = |\mathrm{Re}s_d| + \frac{|\mathrm{Im}s_d|}{\tan(\theta_{z_{pd}})} = 8.319$$

K=15 $k_{pid}k_{pd}$ =5.895



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$$\left|15k_{pid}k_{pd}\frac{(s_d+2)(s_d+4)(s_d+6)(s_d+8.31951)}{s_d(s_d+1)(s_d+3)(s_d+5)(s_d+7)}\right|=1$$

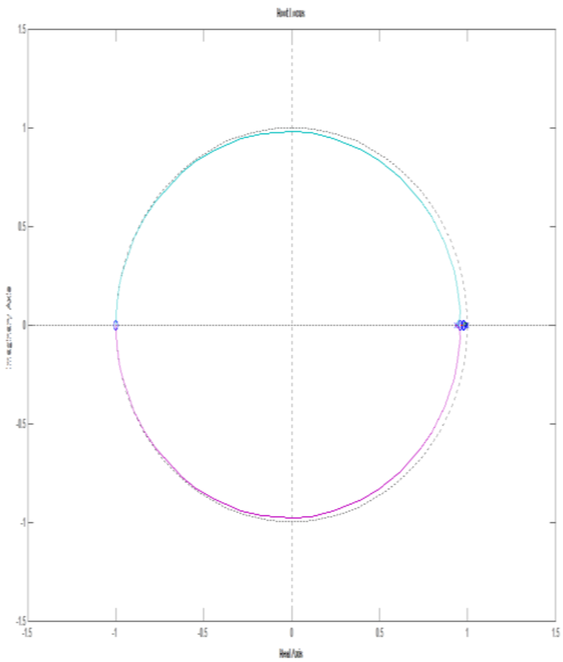

Fig4. Root locus of continuous system

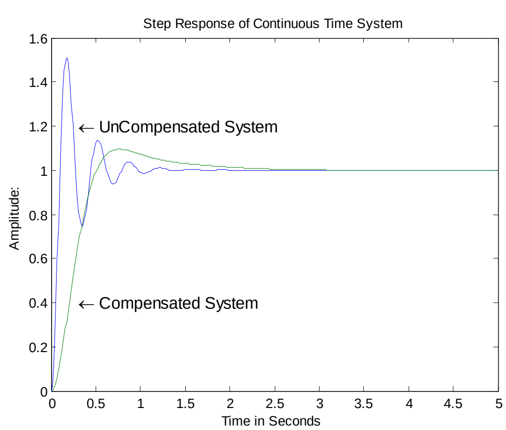


fig5.the unit step response of the control system

note that the resultant percentage overshoot about 15% it is greater than the desired specification of $P.O.. \leq 5$ %.to obtain the smaller of the percent overshoot, kitti's method just increase the gain $k_{pid} or k_{pd}$ in $k_{pid}k_{pd}$ from k=5.895 to 10 times, all desire specifications are then obtained .This salient feature of kitti's method is intended to extensively design of discrete-time control systems.

## 2.2 Design for Discrete-time system:

There are several ways for converting from continuous-time to discrete-time system. The method in [2] zero order hold (ZOD),which the PID and the cascaded PD controllers transfer

$$k_{PID} = k_P + k_I\left(\frac{z}{z-1}\right) + k_D\left(\frac{z-1}{z}\right)$$

$$\frac{(k_p+k_I+k_D)z^2-(k_p+2k_D)z+k_D}{z(z-1)}$$

$$\frac{k_{pid}(z-z_1)(z-z_2)}{z(z-1)}$$

$$=\frac{k_{pid}(z-z)^2}{z(z-1)}$$

$$k_{PD}(z)=\frac{(k_p+k_D)z-k_D}{z}$$

$$=\frac{k_{pd}(z-z_{pd})}{z}$$

$$k_{(n-2)}(z)=\frac{k_{pid}(z-z_1)(z-z_2)}{z(z-1)}\times\frac{k_{pd}(z-z_{pd})}{z}$$

However, in discrete-time control ,integration and discretization are equivalent. [4] amoung these following three methods , euler method(forward rectangular



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rule), the Tustin method (trapezoidal integration) and the back word rectangular rule. these three approximations to the integral are as shown in fig.9, and summarized as written in table

| Method | Discrete equivalent |
|---|---|
| Forward rectangular ruel(eulet) | $\frac{1}{s} \to \frac{T_s}{z-1}$ |
| Backward rectangular rule | $\frac{1}{s} \to \frac{T_s z}{z-1}$ |
| Trapezoidal method (Tustin method) | $\frac{1}{s} \to \frac{T_s}{2}\frac{z+1}{z-1}$ |

In order to obtain the response of sampled system close to continuous or un sampled or un sampled system as much as possible, this paper is then propose the approximate transformation using the bilinear (or Tustin) transformation. The transfer function of PID and PD controllers can be written as follows,

$$k_{PID}(z) = k_p + k_I\left(\frac{T(z+1)}{2(z-1)}\right) + k_D\left(\frac{2(z-1)}{T(z+1)}\right),$$

$$\frac{(2k_pT - k_IT^2 + 4k_D)Z^2 + (2k_1T^2 - 8k_D)z}{T(z-1)(z+1)} + \frac{(-2k_pT + k_1T^2 + 4k_D)}{T(z-1)(z+1)}$$

$$\frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{(z-1)(z+1)}$$

$$= \frac{k_{pid}(z-z_1)(z-z_2)}{(z-1)(z+1)} = \frac{k_{pid}(z-z_{pid})^2}{(z-1)(z+1)}$$

$$k_{PD}(z) = k_p + k_D\left(\frac{2(z-1)}{T(z+1)}\right)$$

$$=\text{k}\frac{(k_pT + 2k_D)z + (k_pT - 2k_D)}{T(z+1)}$$

$$\frac{\alpha_1 z + \alpha_0}{(z+1)} = \frac{k_{pd}(z - z_{pd})}{(z+1)}$$

$$k_{(n-1)}(z) = \frac{k_{pid}(z-z_1)(z-z_2)}{(z-1)(z+1)} \times \frac{k_{pd}(z-z_{pd})}{(z+1)}$$

Here, discritized plant Gn(z) with sampling time T= $\frac{1}{1000}$ sec/sample is

$$G_3(z) = \frac{10^{-8}(z+z_3)(z+z_4)(z+z_5)(z+z_7)}{(z-p_1)(z-p_2)(z-p_3)(z-p_4)}$$

Where,

$z_3 = 1.0049 + j0.0085, z_4 = 1.0049 - j0.0085,$
$z_5 = 0.9902 + j0.0085, z_6 = 1.0049 - j0.0085$

$p_1 = 0.999, p_2 = 0.997, and$
$p_3 = 0.995, p_4 = 0.993$

the corresponding dominant closed loop pole Zd on z-plane is located at

$z_d = e^{ts_d} = e^{T\left(-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)}$ =0.996+j4.423 $\times 10^{-3}$

Now, it is ready to apply kitti's method for discrete PID × (n-2) stage PD cascade controller design

Problem to the following open loop transfer function where

$$k_{(n-2)}(z) = k\frac{(z-z_1)(z-z_2)(z-z_{pd})}{(z+1)(z+1)(z-1)}$$

$$G_n(z) = (10^{-8})\frac{(z+z_3)(z+z_4)(z+z_5)}{(z-p_1)(z-p_2)(z-p_3)}$$

There are 4 unknown parameters to be solved here as follows $z_{pd,} z_1, z_2$, and parameter K.



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Since the root loci on the real axis are determined by open loop poles and zeros lying on it. the complex-conjugate poles or zeros of the open loop transfer function have no effect on the location of root loci on the real axis because the angle contribution of a pair of complex-conjugate poles or zeros is $360^{\theta}$ on the real axis. Each of the root locus on the real axis extends over a range from a pole or zero to another pole or zero.

In order to force a couple of real poles at z =1 and z=0.999 to become a pair of dominant closed loop complex –conjugate poles by loop gain or parameter K.

It is reasonably places $z_1 = 0.998$ and $z_2$ =0.994, $z_3$ =0.996 the same way as continuous-time case. Then, the remaining $z_{pd}$ =0.995 and K($10^{-8}$)=$3.276 \times 10^{-3}$ are obtained.

When compare between the ZOH and Tustin method to the continuous-time case. Here, the sampling time T=$\frac{1}{000}$ sec/samples, the results from simulations in fig show that the response from Tustin method is more closer to the continuous-time case than the ZOH method

DISCRETE TIME SYSTEMS:

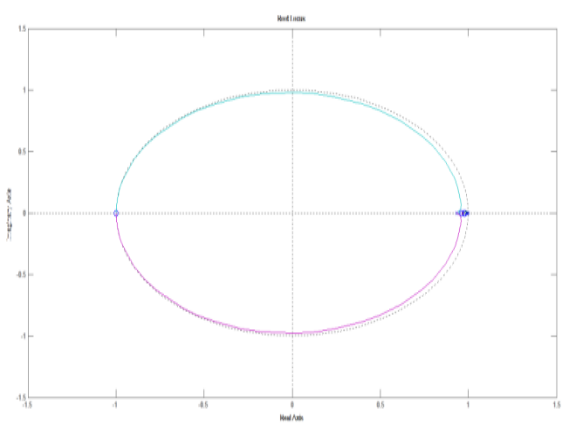

Fig6. Root locus of discrete

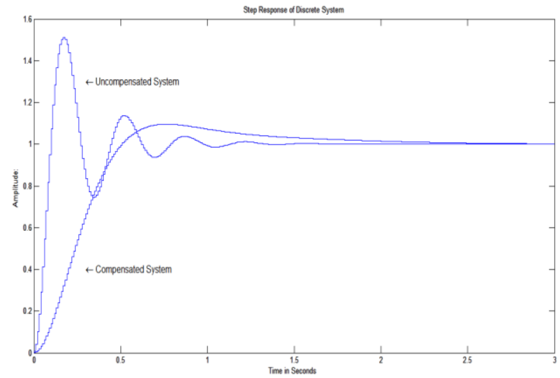


Fig7.Unit step response of discrete system

## 3. CONCLUSION:

The discrete PIDx(n-2) stage PD cascade controller designed by using the root locus technique for the n^th Order plant has been proposed. All desired specifications are easily obtained with the better than the previous time of using ZOH method, in discrete-time control system design via kitti's method. The implementation of the proposed controller on the type 1 and 5^th order plant is carried out. The resulting graphs are explained

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## BIOGRAPHIES

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