Some quadrature methods for approximate evaluation of definite integrals for Complex Variables

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Abstract

In this paper, two quadrature rules Weddle's transformed rule and Clenshaw-Curtis five point rule of same precision five are mixed up and a quadrature rule of higher precision seven is obtained. An asymptotic error estimate of the rule has been determined and the rule is numerically verified with suitable examples.

Keywords: Quadrature rule, Asymptotic error, Analytic function, Numerical integration, Taylor's series.
MSC2010: 65D30,65D32

1. Intruduction

Keeping the facts in mind [3,4,5,6,9,10,11] we have mixed up Weddle's transformed rule and Cleanshaw-Curtis five point rule each of same precision five and produced a mixed quadrature of precision seven. A new rule of precision seven is used for evaluating an integral of the form,

$$I(f)=\int_L f(z)dz. \qquad (1.1)$$

Where L is a directed line segment from the point (z_0-h) to (z_0+h) in the complex plane and $f(z)$ is analytic in certain domain Ω containing the line segment L. Lether [7] using the transformation $z=(z_0+th)$, where $t\in[-1,1]$ transformed the integral (1.1) to the integral

$$h\int_{-1}^{1} f(z_0+th)dt. \qquad (1.2)$$

2. Construction of mixed quadrature rule:

Weddle's transformed rule $R_W(f)$ is

$$I(f)\cong R_W(f)=\frac{h}{10}\left[\begin{array}{l} f(z_0-h)+f(z_0+h)+ \\ f\left(z_0-\frac{h}{3}\right)+f\left(z_0+\frac{h}{3}\right)+5f\left(z_0-\frac{2h}{3}\right) \\ +5f\left(z_0+\frac{2h}{3}\right)+6f(z_0)\end{array}\right] \qquad (2.1)$$

Expanding each term of eqn (2.1) using Taylor series about z_0

$$R_W(f)=2h\left[\begin{array}{l} f(z_0)+\frac{h^2}{3!}f''(z_0)+ \\ \frac{h^4}{5!}f^{iv}(z_0)+\frac{35h^6}{243\times 6!}f^{vi}(z_0)+ \\ \frac{1307h^8}{10935\times 8!}f^{viii}(z_0)+\ldots\ldots\end{array}\right] \qquad (2.2)$$

We can write eqn (1.1) using Taylor series expansion about z_0

$$I(f)=2h\left[\begin{array}{l} f(z_0)+\frac{h^2}{3!}f''(z_0)+ \\ \frac{h^4}{5!}f^{iv}(z_0)+\frac{h^6}{7!}f^{vi}(z_0)+ \\ \frac{h^8}{9!}f^{viii}(z_0)+\ldots\ldots\end{array}\right] \qquad (2.3)$$

Cleanshaw-curtis five point rule is

$$I(f)\cong R_{CC5}(f)=\frac{h}{15}\left[\begin{array}{l} f(z_0+h)+f(z_0-h)+ \\ 8f\left(z_0+\frac{h}{\sqrt{2}}\right)+ \\ 8f\left(z_0-\frac{h}{\sqrt{2}}\right)+12f(z_0)\end{array}\right] \qquad (2.4)$$

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3. Error Analysis

Now the error associated with Weddle's rule $R_W(f)$ is

$$E_W(f) = I(f) - R_W(f) = -\frac{4h^7}{1701\times 6!} f^{vi}(z_0) - \frac{184h^9}{10935\times 8!} f^{viii}(z_0) \quad (3.1)$$

Now the error associated with Cleanshaw-Curtis five point rule $R_{CC5}(f)$ is

$$E_{CC5}(f) = \frac{1}{315\times 5!} h^7 f^{vii}(z_0) + \frac{1}{360\times 7!} h^9 f^{viii}(z_0) + \dots\dots\dots\dots \quad (3.2)$$

Using Taylor series expansion of eq^n (2.1) and eq^n , (2.4) we get,

$$I(f) = R_W(f) + E_W(f) \quad (3.3)$$
$$I(f) = R_{CC5}(f) + E_{CC5}(f) \quad (3.4)$$

Where f is infinitely differentiable since it is assumed to be analytic in certain domain Ω containing the line segment L.

Now multiplying $\left(\frac{1}{81}\right)$ in eqn (3.4) and $\left(\frac{1}{10}\right)$ in eqn$(3.3)$and adding,

$$I(f) = R_{WCC5}(f) + E_{WCC5}(f) \quad (3.5)$$
$$R_{WCC5}(f) = \frac{1}{91}\left[10R_{CC5}(f) + 81R_W(f)\right] \quad (3.6)$$
$$E_{WCC5}(f) = \frac{1}{91}\left[10E_{CC5}(f) + 81E_W(f)\right] \quad (3.7)$$

Calculation of $E_{WCC5}(f)$

Substituting eqn (3.1) and eqn (3.2) in eqn (3.7)

$$E_{WCC5}(f) = -\frac{77}{49140\times 7!} h^9 f^{viii}(z_0) \quad (3.8)$$

From eqn (3.8), the degree of precision of $R_{WCC5}(f)$ is seven where as degree of precision o $R_W(f)$ and $R_{CC5}(f)$ are five.

4. Numerical Verification

Let us approximate the value of the following integrals I_1, I_2 , I_3 using $R_W(f)$, $R_{CC5}(f)$, $R_{WCC5}(f)$ quadrature rule

Where

$$I_1 = \int_{-i}^{i} e^z\, dz, \quad I_2 = \int_{-i}^{i} \cos z\, dz, \quad I_3 = \int_{-\frac{i}{3}}^{\frac{i}{3}} \cosh z\, dz$$

Table-1 (Comparison between approximate and exact results)

Exact value of the integrals	*Weddle's Transformed rule* $R_W(f)$	*Clenshaw-Curtis five point rule* $R_{CC5}(f)$
I_1=1.682941969615793i	1.682939111213524i	1.682967877663091i
I_2=2.350402387287603i	2.350406080572671i	2.350375376931479i
I_3=0.654389393592304i	0.654389392120058i	0.654389405660818i

Table-2 (Comparison between approximate and exact results)

Mixed quadrature rule $R_{WCC5}(f)$	$\lvert E_W(f)\rvert$	$\lvert E_{CC5}(f)\rvert$	$\lvert E_{WCC5}(f)\rvert$
1.682942272361828i	0.00000285	0.00002590	0.00000030
2.350402706546167i	0.00000369	0.00002701	0.00000031
0.654389393608054i	0.000000001	0.00000001	0.00000000001

From the Table-1 and Table-2 we compare the results as follows,

$$\lvert E_{WCC5}(f)\rvert \le \lvert E_W(f)\rvert \le \lvert E_{CC5}(f)\rvert$$

5. Conclusion

From the Table-1 and Table-2 it is evident that the mixed quadrature rule $R_{WCC5}(f)$ of degree of precision seven is giving us better result than the constituent rules $R_W(f)$ and $R_{CC5}(f)$ each of degree of precision five. Our quadrature rule is more efficient and numerically better convergent to exact result.

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